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SIMULATIONS AND ANALYSIS OF COVID-19 AS A FRACTIONAL MODEL WITH DIFFERENT KERNELS

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Abstract

Recently, Atangana proposed new operators by combining fractional and fractal calculus. These recently proposed operators, referred to as fractal–fractional operators, have been widely used to study complex dynamics. In this paper, the COVID-19 model is considered via Atangana–Baleanu fractal-fractional operator. The Lyapunov stability for the model is derived for first and second derivative. Numerical results have developed through Lagrangian-piecewise interpolation for the different fractal–fractional operators. We develop numerical outcomes through different differential and integral fractional operators like power-law, exponential law, and Mittag-Leffler kernel. To get a better outcome of the proposed scheme, numerical simulation is made with different kernels having the memory effects with fractional parameters.

Keywords: COVID-19 Model; Stability; Power-Law; Exponential Law; Mittag-Leffler; Fractional Parameters.

1. INTRODUCTION

Science and mathematics have strong bonding and both are significant to each other. Mathematics is an internal ingredient of science, and the importance of mathematics cannot be denied in several fields of sciences like Electrical engineering, physics, biology and medicine. Mutually, science stimulates and energizes mathematics, produces advanced inquiry, and generates the modern strategy of mathematics. In the 21st century, due to several applications of Mathematical biology, researchers have shown exceptional interest in this sector. So, in the case of mathematical modeling, their vital concern is to express contagious diseases and control them. Brownlee[1] arranged a solid foundation of mathematical biology. For the spread of infection,[2] he suggested a law with the help of probabilistic techniques.

Mathematics presents techniques for ordering and organizing know-how so that, whilst applied to the era, permits scientists and engineers to provide systematic, reproducible, and transmitted information. In the same manner, the involvement of mathematics is obligatory for its increase and improvement. New innovations are achievable with the main contribution to mathematics.[3–5] This system of improvement and change brought about new searches, and it has become obligatory to try progressive approaches. One of these revolutionary techniques in mathematics that we use at a great level is modeling. Mathematical models can assist us in understanding and exploring the means of equations or functional relationships. Initially, to approximate the tendency of the epidemic, Wang *et al.*[6] established the SEIR model. Further, Wu *et al.*[7] also worked on the same model. They approximated the exponential extension rate of COVID and the time of doubling rate of the epidemic. With the assistance of Mittag-Leffler kernel,[8] Aslam *et al.* provided the generalized form of the fractional-order COVID-19 model. They used the Sumudu transformation for the existence and uniqueness of the model. Also, Farman *et al.* build up the solution of the epidemic model with the support of several fractal operators.[9] Moreover, they presented the property of positivity and boundness. Singh *et al.* introduced[10] a new approach toward the solution of the local fractional partial differential equations (LFPDEs) describing fractal vehicular traffic flow. They obtained a revised version of LFVIM to analyze and procure the non-differentiable solutions of linear, nonlinear, and non-homogeneous LFPDEs occurring in shape traffic flow through illustrative examples. Matar *et al.* measured the p-Laplacian nonperiodic boundary value problem in the sense of generalized Caputo fractional derivatives.[11] Baleanu *et al.* came up with an extension for the second-order differential equation of a thermostat model to the fractional hybrid equation and inclusion versions.[12] Further, Atangna and Baleanu gave the new idea of fractional derivatives with the non-local and non-singular kernel.[13] Several real-life applications of fractional operators can be seen in these articles.[14–20]

A probabilistic method is proposed to predict the spreading profile of COVID-19 in the United States through time-variant reliability analysis.[21] They transformed the problem into a reliability model and analyzed it through the Monte Carlo sampling. For computer security, Singh *et al.* examined the moderate epidemiological model to describe computer viruses with an arbitrary order derivative having a non-singular kernel.[22] With the help of the iterative method, they attain the results of the issue. For the sake of forecasting the upcoming actions of COVID-19, several mathematics models could be applied for simulation. Among them, some have been recommended for analysis.[23–27] These models depend a lot on the implementation of differential and integral operators. One key motive force of the unfold is direct contact with inflamed patients, items or corpses from COVID-19. Even as the sector continues to be waiting for a possible vaccine, measures have been initiated in nations around the world and among them are lockdown and social distancing. Models in papers[28,29] provide more information about COVID-19 under fractional-order differential equation. In Iran, the second gesture of COVID-19, Ghanbari, was directed to forecast the contaminated people.[30] An examination of stability and numerical outcomes for the COVID-19 model is given by Alkahtani *et al.* that showed the expansion of disease in Italy.[31] The Omicron effect is investigated with different fractional parameters, and simulation has been made to understand the actual behavior of the virus.[32] With the help of the Caputo fractional derivative, a mathematical model for the transmission of COVID-19 is presented in the paper.[33] Further, for the prediction of spread in Iran, an SEIR epidemic model for the expansion of COVID-19 is given by Rezapour.[34]

We arranged the paper as follows: In Sec. 2, we presented our model in component and set up the equilibria of the model. In Sec. 3, we built a stability analysis of the model via first and second derivative of Lyapunov. In Secs. 4 and 5, new fractal–fractional operators are provided while their applications are provided in Sec. 6. We discussed the simulation of the model in Sec. 7. In Sec. 8, we presented its conclusion.

2. FRACTIONAL-ORDER COVID-19 MODEL

The affected system has countless parameters responsible for the existence and eradication of COVID-19. Thus, a sole model cannot distinguish the disease globally. The connection of isolated communities with the expansion of COVID-19 has direct relation. There are two categories of isolation that are commonly known: susceptible and contaminated isolation. Here, we discuss contaminated isolation.

We reviewed the model presented by Misra *et al.*[35] Here, the model is split into six compartments: $S(t)$ susceptible, $U(t)$ uncovered or exposed, A_1 home quarantine, $C(t)$ infectious, A_2 medical quarantine, and $R(t)$ recovered population. By using fractional parameters $0 < \omega \leq 1$, we reproduced the model by fractionalization.

$$
\begin{aligned}
{}^{\mathrm{FFM}}D_{0,t}^{\Omega}S(t) &= \overline{\aleph} - \overline{\varepsilon}S(t)C(t) - \overline{\nu_1}S(t) + \overline{\psi_2}A_1(t),\\
{}^{\mathrm{FFM}}D_{0,t}^{\Omega}U(t) &= \overline{\varepsilon}S(t)C(t) - (\overline{\varphi_1} + \overline{\varphi_2} + \overline{\nu_1})U(t),\\
{}^{\mathrm{FFM}}D_{0,t}^{\Omega}A_1(t) &= \overline{\varphi_1}U(t) - (\overline{\psi_1} + \overline{\psi_2} + \overline{\nu_1})A_1(t),\\
{}^{\mathrm{FFM}}D_{0,t}^{\Omega}C(t) &= \overline{\psi_1}A_1(t) - (\overline{\sigma_1} + \overline{\sigma_2} + \overline{\nu_1} + \overline{\nu_2})C(t)\\
&\quad + \overline{\varphi_2}U(t),
\end{aligned}
$$

$$
\begin{aligned}
{}^{\mathrm{FFM}}D_{0,t}^{\Omega}A_2(t) &= \overline{\sigma_1}C(t) - (\overline{\nu_1} + \overline{\nu_3} + \overline{\xi})A_2(t),\\
{}^{\mathrm{FFM}}D_{0,t}^{\Omega}R(t) &= \overline{\xi}A_2(t) + \overline{\sigma_2}C(t) - \overline{\nu_1}R(t),
\end{aligned}
\tag{1}
$$

with initial condition

$$
\begin{gathered}
S(0) \geq 0, \quad U(0) \geq 0, \quad A_1(0) \geq 0,\\
C(0) \geq 0, \quad A_2(0) \geq 0, \quad R(0) \geq 0.
\end{gathered}
$$

Here, $\overline{\aleph}$ is the birth-rate, $\overline{\varepsilon}$ is the transfer rate of disease, $\overline{\nu_1}$ is the real demise rate, $\overline{\nu_2}$ is the demise rate of infectious population, $\overline{\nu_3}$ is the demise rate of medical quarantine, $\overline{\psi_1}$ is the transfer rate of susceptible to infectious population, $\overline{\psi_2}$ is the rate of transfer of susceptible, $\overline{\varphi_1}$ is the transfer rate of uncover to susceptible under home quarantine, $\overline{\varphi_2}$ is the transfer rate of uncover to infectious population, $\overline{\sigma_1}$ is the rate of transfer of infectious to medical quarantine after positive test, $\overline{\sigma_2}$ is the rate of transfer of infectious to recovered population, and rate of recovery from medical quarantine is $\overline{\xi}$.

2.1. Equilibria of the Model

Corona-free equilibria:

$$
M_1 = (S^1, U^1, A_1^1, C^1, A_2^1, R^1) = \left(\frac{\overline{\aleph}}{\overline{\nu_1}}, 0, 0, 0, 0, 0\right),
$$

and the corona existing equilibria are as follows:

$$
M_2 = (S^*, U^*, A_1^*, C^*, A_2^*, R^*).
$$

So, we get $S^*, U^*, A_1^*, C^*, A_2^*, R^*$ as

$$
\begin{aligned}
S^* &= \frac{\overline{\aleph}R_0\overline{\psi_1} - (\overline{\varepsilon}\overline{\psi_1} + R_0(\overline{\nu_1} + \overline{\nu_2} + \overline{\sigma_1} + \overline{\sigma_2}))C^*}{R_0\overline{\nu_1}\overline{\psi_1}},\\
U^* &= \frac{\overline{\varepsilon}C^*}{R_0(\overline{\varpi_1} + \overline{\varpi_2} + \overline{\nu_1})},\\
A_1^* &= \frac{\overline{\varepsilon}C^*}{R_0(\overline{\nu_1} + \overline{\nu_2} + \overline{\sigma_1} + \overline{\sigma_2})},\\
C^* &= \frac{\overline{\psi}A_1^*}{R_0(\overline{\nu_1} + \overline{\nu_2} + \overline{\sigma_1} + \overline{\sigma_2})},\\
A_2^* &= \frac{\overline{\sigma_1}C^*}{\overline{\xi} + \overline{\nu_1} + \overline{\nu_2}},\\
R^* &= \frac{\overline{\xi}\overline{\sigma_1} + \overline{\sigma_2}(\overline{\xi} + \overline{\nu_1} + \overline{\nu_3})C^*}{\overline{\nu_1}(\overline{\xi} + \overline{\nu_1} + \overline{\nu_3})}.
\end{aligned}
\tag{2}
$$

For the derivation, the reproduction number R_0 is the spectral radius of $Y * Z^{-1}$, where Y and Z are the transmission and the transition matrices, respectively. Thus, we get

$$
R_0 = \frac{\overline{\varepsilon}\overline{\aleph}[\overline{\varphi_1}\overline{\psi_1} + \overline{\varphi_2}(\overline{\psi_1} + \overline{\psi_2} + \overline{\nu_1})]}{(\overline{\varphi_1} + \overline{\varphi_2} + \overline{\nu_1})(\overline{\psi_1} + \overline{\psi_2} + \overline{\nu_1}) \times (\overline{\sigma_1} + \overline{\sigma_2} + \overline{\nu_1} + \overline{\nu_2})}. \tag{3}
$$

3. STABILITY OF THE SUGGESTED MODEL

For the endemic Lyapunov function,[36] we set all independent variables in the model, in our case, $\{S, U, A_1, C, A_2, R\}$, $\dot{L} < 0$ is the harmful equilibrium E^*.

3.1. First Derivative of Lyapunov

Theorem 1. *If the reproductive number $R_0 > 1$, the endemic equilibrium points of harmful impact equilibrium points E^* of the survival of fractional calculus model are globally asymptotically stable.*

Proof. To prove theorem, we consider the Lyapunov function as follows:

$$
\begin{aligned}
L(S^*, U^*, A_1^*, C^*, A_2^*, R^*) &= \left(S - S^* - S^*\log\frac{S^*}{S}\right)\\
&\quad + \left(U - U^* - U^*\log\frac{U^*}{U}\right)\\
&\quad + \left(A_1 - A_1^* - A_1^*\log\frac{A_1^*}{A_1}\right)\\
&\quad + \left(C - C^* - C^*\log\frac{C^*}{C}\right)\\
&\quad + \left(A_2 - A_2^* - A_2^*\log\frac{A_2^*}{A_2}\right)\\
&\quad + \left(R - R^* - R^*\log\frac{R^*}{R}\right).
\end{aligned}
\tag{4}
$$

Therefore, applying the derivative with respect to t on both sides, we get the following:

$$
\begin{aligned}
\frac{dL}{dt} = \frac{d}{dt}\Bigg[&\left(S - S^* - S^*\log\frac{S^*}{S}\right)\\
&+ \left(U - U^* - U^*\log\frac{U^*}{U}\right)\\
&+ \left(A_1 - A_1^* - A_1^*\log\frac{A_1^*}{A_1}\right)
\end{aligned}
$$

$$+\left(C - C^* - C^* \log \frac{C^*}{C}\right)$$
$$+\left(A_2 - A_2^* - A_2^* \log \frac{A_2^*}{A_2}\right)$$
$$+\left(R - R^* - R^* \log \frac{R^*}{R}\right)\Bigg], \qquad (5)$$

and

$$\frac{dL}{dt} = \left(\frac{S-S^*}{S}\right)\dot{S} + \left(\frac{U-U^*}{U}\right)\dot{U} + \left(\frac{A_1-A_1^*}{A_1}\right)\dot{A}_1 + \left(\frac{C-C^*}{C}\right)\dot{C} + \left(\frac{A_2-A_2^*}{A_2}\right)\dot{A}_2 + \left(\frac{R-R^*}{R}\right)\dot{R}. \qquad (6)$$

Now, we put values in the above equation for derivatives

$$\frac{dL}{dt} = \left(\frac{S-S^*}{S}\right)(\overline{\aleph} - \overline{\varepsilon}S(t)C(t) - \overline{\nu_1}S(t) + \overline{\psi_2}A_1(t)) + \left(\frac{U-U^*}{U}\right)$$
$$\times (\overline{\varepsilon}S(t)C(t) - (\overline{\varphi_1} + \overline{\varphi_2} + \overline{\nu_1})U(t)) + \left(\frac{A_1-A_1^*}{A_1}\right)$$
$$\times (\overline{\varphi_1}U(t) - (\overline{\psi_1} + \overline{\psi_2} + \overline{\nu_1})A_1(t)) + \left(\frac{C-C^*}{C}\right)$$
$$\times (\overline{\psi_1}A_1(t) - (\overline{\sigma_1} + \overline{\sigma_2} + \overline{\nu_1} + \overline{\nu_2})C(t) + \overline{\varphi_2}U(t)) + \left(\frac{A_2-A_2^*}{A_2}\right)(\overline{\sigma_1}C(t)$$
$$- (\overline{\nu_1} + \overline{\nu_3} + \overline{\xi})A_2(t)) + \left(\frac{R-R^*}{R}\right)$$
$$\times (\overline{\xi}A_2(t) + \overline{\sigma_2}C(t) - \overline{\nu_1}R(t)). \qquad (7)$$

Replacing $S = S - S^*$, $U = U - U^*$, $A_1 = A_1 - A_1^*$, $C = C - C^*$, $A_2 = A_2 - A_2^*$ and $R = R - R^*$ we can have the following:

$$\frac{dL}{dt} = \left(\frac{S-S^*}{S}\right)(\overline{\aleph} - \overline{\varepsilon}(S-S^*)(C-C^*) - \overline{\nu_1}(S-S^*) + \overline{\psi_2}(A_1 - A_1^*)) + \left(\frac{U-U^*}{U}\right)$$
$$\times (\overline{\varepsilon}(S-S^*)(C-C^*) - (\overline{\varphi_1} + \overline{\varphi_2} + \overline{\nu_1}) \times (U-U^*)) + \left(\frac{A_1-A_1^*}{A_1}\right)$$
$$\times (\overline{\varphi_1}(U-U^*) - (\overline{\psi_1} + \overline{\psi_2} + \overline{\nu_1}) \times (A_1-A_1^*)) + \left(\frac{C-C^*}{C}\right)(\overline{\psi_1}(A_1-A_1^*)$$
$$- (\overline{\sigma_1} + \overline{\sigma_2} + \overline{\nu_1} + \overline{\nu_2}) \times (C-C^*) + \overline{\varphi_2}(U-U^*)) + \left(\frac{A_2-A_2^*}{A_2}\right)$$
$$\times (\overline{\sigma_1}(C-C^*) - (\overline{\nu_1} + \overline{\nu_3} + \overline{\xi})(A_2-A_2^*)) + \left(\frac{R-R^*}{R}\right)(\overline{\xi}(A_2-A_2^*) + \overline{\sigma_2}(C-C^*)$$
$$- \overline{\nu_1}(R-R^*)). \qquad (8)$$

The above equality can be rearranged as

$$\frac{dL}{dt} = \overline{\aleph} - \frac{\overline{\aleph}S^*}{S} - \frac{\overline{\varepsilon}C(S-S^*)^2}{S} + \frac{\overline{\varepsilon}C^*(S-S^*)^2}{S}$$
$$- \frac{\overline{\nu_1}(S-S^*)}{S} + \overline{\psi_2}A_1 - \overline{\psi_2}A_1^* - \frac{\overline{\psi_2}A_1S^*}{S}$$
$$+ \frac{\overline{\psi_2}A_1^*S^*}{S} + \overline{\varepsilon}SC - \overline{\varepsilon}SC^* - \overline{\varepsilon}S^*C + \overline{\varepsilon}S^*C^*$$
$$- \frac{\overline{\varepsilon}SCU^*}{U} + \frac{\overline{\varepsilon}SC^*U^*}{U} + \frac{\overline{\varepsilon}S^*CU^*}{U} - \frac{\overline{\varepsilon}S^*C^*U^*}{U}$$
$$- (\overline{\varphi_1} + \overline{\varphi_2} + \overline{\nu_1})\frac{(U-U^*)^2}{U} + \overline{\varphi_1}U - \overline{\varphi_1}U^*$$
$$- \frac{\overline{\varphi_1}UA_1^*}{A_1} + \frac{\overline{\varphi_1}U^*A_1^*}{A_1} - (\overline{\psi_1} + \overline{\psi_2} + \overline{\nu_1})$$
$$\times \frac{(A_1-A_1^*)^2}{A_1} + \overline{\psi_1}A_1 - \overline{\psi_1}A_1^* - \frac{\overline{\psi_1}A_1C^*}{C}$$
$$+ \frac{\overline{\psi_1}A_1^*C^*}{C} - (\overline{\sigma_1} + \overline{\sigma_2} + \overline{\nu_1} + \overline{\nu_2})$$
$$\times \frac{(C-C^*)^2}{C} + \overline{\varphi_2}U - \overline{\varphi_2}U^* - \frac{\overline{\varphi_2}UC^*}{C}$$
$$+ \frac{\overline{\varphi_2}U^*C^*}{C} + \overline{\sigma_1}C - \overline{\sigma_1}C^* - \frac{\overline{\sigma_1}CA_2^*}{A_2}$$
$$+ \frac{\overline{\sigma_1}C^*A_2^*}{A_2} - (\overline{\nu_1} + \overline{\nu_3} + \overline{\xi})\frac{(A_2-A_2^*)^2}{A_2}$$
$$+ \overline{\xi}A_2 - \overline{\xi}A_2^* - \frac{\overline{\xi}A_2R^*}{R} + \frac{\overline{\xi}A_2^*R^*}{R}$$

$$
+\overline{\sigma_2}C - \overline{\sigma_2}C^* - \frac{\overline{\sigma_2}CR^*}{R} + \frac{\overline{\sigma_2}C^*R^*}{R}
- \overline{\nu_1}\frac{(R-R^*)^2}{R}. \tag{9}
$$

For simplicity, we can rewrite the above equality as follows:

$$
\frac{dL}{dt} = \Lambda_1 - \Lambda_2, \tag{10}
$$

where

$$
\begin{aligned}
\Lambda_1 = {} & \overline{\aleph} + \frac{\overline{\varepsilon}C^*(S-S^*)^2}{S} + \overline{\psi_2}A_1 + \frac{\overline{\psi_2}A_1^*S^*}{S} + \overline{\varepsilon}SC \\
& + \overline{\varepsilon}S^*C^* + \frac{\overline{\varepsilon}SC^*U^*}{U} + \frac{\overline{\varepsilon}S^*CU^*}{U} \\
& + \overline{\varphi_1}U + \frac{\overline{\varphi_1}U^*A_1^*}{A_1} + \overline{\nu_1}\frac{(A_1-A_1^*)^2}{A_1} + \overline{\psi_1}A_1 \\
& + \frac{\overline{\psi_1}A_1^*C^*}{C} + \overline{\varphi_2}U + \frac{\overline{\varphi_2}U^*C^*}{C} + \overline{\sigma_1}C \\
& + \frac{\overline{\sigma_1}C^*A_2^*}{A_2} + \overline{\xi}A_2 + \frac{\overline{\xi}A_2^*R^*}{R} + \overline{\sigma_2}C + \frac{\overline{\sigma_2}C^*R^*}{R},
\end{aligned} \tag{11}
$$

and

$$
\begin{aligned}
\Lambda_2 = {} & \frac{\overline{\aleph}S^*}{S} + \frac{\overline{\varepsilon}C(S-S^*)^2}{S} + \frac{\overline{\nu_1}(S-S^*)}{S} + \overline{\psi_2}A_1^* \\
& + \frac{\overline{\psi_2}A_1S^*}{S} + \overline{\varepsilon}SC^* + \overline{\varepsilon}S^*C + \frac{\overline{\varepsilon}SCU^*}{U} \\
& + \frac{\overline{\varepsilon}S^*C^*U^*}{U} + (\overline{\varphi_1} + \overline{\varphi_2} + \overline{\nu_1})\frac{(U-U^*)^2}{U} \\
& + \overline{\varphi_1}U^* + \frac{\overline{\varphi_1}UA_1^*}{A_1} + (\overline{\psi_1} + \overline{\psi_2} + \overline{\nu_1}) \\
& \times \frac{(A_1-A_1^*)^2}{A_1} + \overline{\psi_1}A_1^* + \frac{\overline{\psi_1}A_1C^*}{C} \\
& + (\overline{\sigma_1} + \overline{\sigma_2} + \overline{\nu_1} + \overline{\nu_2})\frac{(C-C^*)^2}{C} + \overline{\varphi_2}U^* \\
& + \frac{\overline{\varphi_2}UC^*}{C} + \overline{\sigma_1}C^* + \frac{\overline{\sigma_1}CA_2^*}{A_2} + (\overline{\nu_1} + \overline{\nu_3} + \overline{\xi}) \\
& \times \frac{(A_2-A_2^*)^2}{A_2} + \overline{\xi}A_2^* + \frac{\overline{\xi}A_2R^*}{R} + \overline{\sigma_2}C^* \\
& + \frac{\overline{\sigma_2}CR^*}{R} + \overline{\nu_1}\frac{(R-R^*)^2}{R}.
\end{aligned} \tag{12}
$$

It can be easily seen that if $\Lambda_1 < \Lambda_2$, this yields $\frac{dL}{dt}$, however

$$
\Lambda_1 - \Lambda_2 = 0 \Rightarrow \frac{dL}{dt} = 0. \tag{13}
$$

If $S = S^*$, $U = U^*$, $A_1 = A_1^*$, $C = C^*$, $A_2 = A_2^*$ and $R = R^*$, then we follow that the largest compact invariant set for the suggested model in

$$
\left\{(S^*, U^*, A_1^*, C^*, A_2^*, R^*) \in \Xi : \frac{dL}{dt} = 0\right\}, \tag{14}
$$

is the point E^*, the endemic equilibrium of the considered model. From Lasalle's invariance concept, we can conclude that $E*$ is globally asymptotically stable in Ξ if $\Lambda_1 < \Lambda_2$. □

3.2. Second Derivative of Lyapunov

The global stability of endemic equilibrium points is evaluated by means of the first derivative of the Lyapunov function. Without loss of generality, the analysis of the first derivative provides us with useful information that can be completed by the analysis of the second derivative. For example, the second derivative provides us with the curvature accordingly to its sign, while the first derivative of these Lyapunov functions gives us some useful information about the spread of the disease. We strongly believe that its second derivative could possibly add more information.

$$
\begin{aligned}
\frac{d^2L}{dt^2} = {} & \frac{d}{dt}\left\{\left(1-\frac{S^\star}{S}\right)\dot{S} + \left(1-\frac{U^\star}{U}\right)\dot{U}\right. \\
& + \left(1-\frac{A_1^\star}{A_1}\right)\dot{A}_1 + \left(1-\frac{C^\star}{C}\right)\dot{C} \\
& \left. + \left(1-\frac{A_2^\star}{A_2}\right)\dot{A}_2 + \left(1-\frac{R^\star}{R}\right)\dot{R}\right\}
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
= {} & \left(\frac{\dot{S}}{S}\right)^2 S^* + \left(\frac{\dot{U}}{U}\right)^2 U^* + \left(\frac{\dot{A}_1}{A_1}\right)^2 A_1^* \\
& + \left(\frac{\dot{C}}{C}\right)^2 C^* + \left(\frac{\dot{A}_1}{A_2}\right)^2 A_2^* + \left(\frac{\dot{R}}{R}\right)^2 R^* \\
& + \left(1-\frac{S^*}{S}\right)\ddot{S} + \left(1-\frac{U^*}{U}\right)\ddot{U} \\
& + \left(1-\frac{A_1^*}{A_1}\right)\ddot{A}_1 + \left(1-\frac{C^*}{C}\right)\ddot{C} \\
& + \left(1-\frac{A_2^*}{A_2}\right)\ddot{A}_2 + \left(1-\frac{R^*}{R}\right)\ddot{R}.
\end{aligned} \tag{16}
$$

Here,

$$
\ddot{S} = -\overline{\varepsilon}\dot{S}C - \overline{\varepsilon}S\dot{C} - \overline{\nu_1}\dot{S} + \overline{\psi_2}\dot{A}_1,
$$

$$
\ddot{U} = \overline{\varepsilon}\dot{S}C + \overline{\varepsilon}S\dot{C} - (\overline{\varphi_1} + \overline{\varphi_2} + \overline{\nu_1})\dot{U},
$$

$$\begin{aligned}
\ddot{A}_1 &= \overline{\varphi_1}\dot{U} - (\overline{\psi_1} + \overline{\psi_2} + \overline{\nu_1})\dot{A}_1,\\
\ddot{C} &= \overline{\psi_1}\dot{A}_1 - (\overline{\sigma_1} + \overline{\sigma_2} + \overline{\nu_1} + \overline{\nu_2})\dot{C} + \overline{\varphi_2}\dot{U},\\
\ddot{A}_2 &= \overline{\sigma_1}\dot{C} - (\overline{\nu_1} + \overline{\nu_3} + \overline{\xi})\dot{A}_2,\\
\ddot{R} &= \overline{\xi}\dot{A}_2 + \overline{\sigma_2}\dot{C} - \overline{\nu_1}\dot{R}.
\end{aligned} \tag{17}$$

$$\begin{aligned}
\frac{d^2L}{dt^2} &= \left(\frac{\dot{S}}{S}\right)^2 S^* + \left(\frac{\dot{U}}{U}\right)^2 U^* + \left(\frac{\dot{A}_1}{A_1}\right)^2 A_1^*\\
&\quad + \left(\frac{\dot{C}}{C}\right)^2 C^* + \left(\frac{\dot{A}_1}{A_2}\right)^2 A_2^* + \left(\frac{\dot{R}}{R}\right)^2 R^*\\
&\quad + \left(1 - \frac{S^*}{S}\right)(-\overline{\varepsilon}\dot{S}C - \overline{\varepsilon}S\dot{C} - \overline{\nu_1}\dot{S} + \overline{\psi_2}\dot{A}_1)\\
&\quad + \left(1 - \frac{U^*}{U}\right)(\overline{\varepsilon}\dot{S}C + \overline{\varepsilon}S\dot{C}\\
&\quad - (\overline{\varphi_1} + \overline{\varphi_2} + \overline{\nu_1})\dot{U}) + \left(1 - \frac{A_1^*}{A_1}\right)\\
&\quad \times (\overline{\varphi_1}\dot{U} - (\overline{\psi_1} + \overline{\psi_2} + \overline{\nu_1})\dot{A}_1) + \left(1 - \frac{C^*}{C}\right)\\
&\quad \times (\overline{\psi_1}\dot{A}_1 - (\overline{\sigma_1} + \overline{\sigma_2} + \overline{\nu_1} + \overline{\nu_2})\dot{C} + \overline{\varphi_2}\dot{U})\\
&\quad + \left(1 - \frac{A_2^*}{A_2}\right)(\overline{\sigma_1}\dot{C} - (\overline{\nu_1} + \overline{\nu_3} + \overline{\xi})\dot{A}_2)\\
&\quad + \left(1 - \frac{R^*}{R}\right)(\overline{\xi}\dot{A}_2 + \overline{\sigma_2}\dot{C} - \overline{\nu_1}\dot{R}),
\end{aligned} \tag{18}$$

and

$$\begin{aligned}
&= \prod(S, U, A_1, C, A_2, R) + \left(1 - \frac{S^*}{S}\right)\\
&\quad \times (-\overline{\varepsilon}\dot{S}C - \overline{\varepsilon}S\dot{C} - \overline{\nu_1}\dot{S} + \overline{\psi_2}\dot{A}_1) + \left(1 - \frac{U^*}{U}\right)\\
&\quad \times (\overline{\varepsilon}\dot{S}C + \overline{\varepsilon}S\dot{C} - (\overline{\varphi_1} + \overline{\varphi_2} + \overline{\nu_1})\dot{U})\\
&\quad + \left(1 - \frac{A_1^*}{A_1}\right)(\overline{\varphi_1}\dot{U} - (\overline{\psi_1} + \overline{\psi_2} + \overline{\nu_1})\dot{A}_1)\\
&\quad + \left(1 - \frac{C^*}{C}\right)(\overline{\psi_1}\dot{A}_1 - (\overline{\sigma_1} + \overline{\sigma_2}\\
&\quad + \overline{\nu_1} + \overline{\nu_2})\dot{C} + \overline{\varphi_2}\dot{U})\\
&\quad + \left(1 - \frac{A_2^*}{A_2}\right)(\overline{\sigma_1}\dot{C} - (\overline{\nu_1} + \overline{\nu_3} + \overline{\xi})\dot{A}_2)\\
&\quad + \left(1 - \frac{R^*}{R}\right)(\overline{\xi}\dot{A}_2 + \overline{\sigma_2}\dot{C} - \overline{\nu_1}\dot{R}).
\end{aligned} \tag{19}$$

Now replacing $\dot{S}$, $\dot{U}$, $\dot{A}_1$, $\dot{C}$, $\dot{A}_2$ and $\dot{R}$ with their respective formula and then putting together positive and negative factors, we have

$$\frac{d^2L}{dt^2} = \Lambda_3 - \Lambda_4, \tag{20}$$

then

$$\begin{aligned}
&\text{if } \Lambda_3 > \Lambda_4 \quad \text{then } \frac{d^2L}{dt^2} > 0,\\
&\text{if } \Lambda_3 < \Lambda_4 \quad \text{then } \frac{d^2L}{dt^2} < 0,\\
&\text{if } \Lambda_3 = \Lambda_4 \quad \text{then } \frac{d^2L}{dt^2} = 0.
\end{aligned} \tag{21}$$

4. NEW FRACTAL FRACTIONAL DIFFERENTIAL AND INTEGRAL OPERATORS

To seize the dynamical structures of infectious disease, we can use the model of classical differential whilst the handiest initial situations are used to expect destiny behaviors of the unfold. However, whilst the scenario is unforeseeable which can be because of variabilities related to actual-world troubles, classical differentiation, and related critical operators reveal inadequate. Particularly, if we look into COVID-19, there are numerous uncertainties, innumerable concealed, and plenty of incorrect information, making it very tough to offer an appropriate mathematical model with classical differentiation. If there is a power law, crossover effects, or fading memory, then nonlocal operators are appropriate due to the ability to catch non-localities and some reminiscence consequences. But, similarly there are more complicated behaviors that could not be replicated. For that, we can use lately brought fractal–fractional operators that are a more appropriate apparatus to deal with such actions.[37,38] In the beginning, as the operators were unable to control time, so many queries were lifted, and so far, new operators were acknowledged to be enough in modeling complicated troubles. In this stage, we establish fractal–fractional operators with earlier settled strategies such as Newton and Lagrange polynomial. Some definitions are given in paper[38] which we used for analysis of the system.

Definition 4.1. Suppose a function $f(t)$, may or may not be differentiable. Assume $0 < \omega \leq 1$ and $0 < \varpi \leq 1$, where ω is a fractional order and ϖ is fractal dimension. We can define a power-law kernel with the help of fractal fractional derivative that has order $0 \leq \omega, \varpi \leq 1$

$$ {}_0^{\mathrm{FFP}}D_t^{\omega,\varpi} f(t) = \frac{1}{\Gamma(1-\omega)} \frac{d}{dt^{\varpi}} \int_0^t f(\Upsilon)(t-\Upsilon)^{-\omega} d\Upsilon, \quad (22) $$

where

$$ \frac{df(t)}{dt^{\varpi}} = \lim_{t \to t_1} \frac{f(t) - f(t_1)}{t^{2-\varpi} - t_1^{2-\varpi}} (2-\varpi). \quad (23) $$

The fractal–fractional derivative with exponential decay kernel is defined as

$$ {}_0^{\mathrm{FFE}}D_t^{\omega,\varpi} f(t) = \frac{M(\omega)}{1-\omega} \frac{d}{dt^{\varpi}} \int_0^t f(\Upsilon) \times \exp\left[-\frac{\omega}{1-\omega}(t-\Upsilon)\right] d\Upsilon. \quad (24) $$

The fractal–fractional derivative with Mittag-Leffler kernel is defined as

$$ {}_0^{\mathrm{FFM}}D_t^{\omega,\varpi} f(t) = \frac{AB(\omega)}{1-\omega} \frac{d}{dt^{\varpi}} \int_0^t f(\Upsilon) E_\omega \times \left[-\frac{\omega}{1-\omega}(t-\Upsilon)\right] d\Upsilon. \quad (25) $$

5. NEW FRACTAL FRACTIONAL DIFFERENTIAL AND INTEGRAL OPERATORS

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If there is a power law, crossover effects, or fading memory, then nonlocal operators are appropriate due to the ability to catch non-localities and some reminiscence consequences. But, if similarly there are more complicated behaviors that couldn't be replicated. For that, we can use lately brought fractal-fractional operators that are more appropriate apparatus to deal with such actions.[37,38] In the beginning, as the operators were unable to control time, so many queries were lifted, and so far, new operators were acknowledged to be enough in modeling complicated troubles. In this stage, we establish fractal-fractional operators with earlier settled strategies such as Newton and Lagrange polynomial. Some definitions are given in[38] which we used for analysis of the system.

Definition 5.1. Suppose a function $f(t)$, may or may not be differentiable. Assume $0 < \omega \leq 1$ and $0 < \varpi \leq 1$, where ω is a fractional order and ϖ is fractal dimension. We can define a power-law kernel with the help of fractal fractional derivative that has order $0 \leq \omega, \varpi \leq 1$;

$$ {}_0^{\mathrm{FFP}}D_t^{\omega,\varpi} f(t) = \frac{1}{\Gamma(1-\omega)} \frac{d}{dt^{\varpi}} \int_0^t f(\Upsilon)(t-\Upsilon)^{-\omega} d\Upsilon, \quad (26) $$

where

$$ \frac{df(t)}{dt^{\varpi}} = \lim_{t \to t_1} \frac{f(t) - f(t_1)}{t^{2-\varpi} - t_1^{2-\varpi}} (2-\varpi). \quad (27) $$

The fractal-fractional derivative with exponential decay kernel is defined: as;

$$ {}_0^{\mathrm{FFE}}D_t^{\omega,\varpi} f(t) = \frac{M(\omega)}{1-\omega} \frac{d}{dt^{\varpi}} \int_0^t f(\Upsilon) \times \exp\left[-\frac{\omega}{1-\omega}(t-\Upsilon)\right] d\Upsilon. \quad (28) $$

The fractal-fractional derivative with Mittag-Leffler kernel is defined as

$$ {}_0^{\mathrm{FFM}}D_t^{\omega,\varpi} f(t) = \frac{AB(\omega)}{1-\omega} \frac{d}{dt^{\varpi}} \int_0^t f(\Upsilon) E_\omega \times \left[-\frac{\omega}{1-\omega}(t-\Upsilon)\right] d\Upsilon. \quad (29) $$

6. APPLICATION OF NEW OPERATORS TO CORONA MODEL

In this segment, we apply the operators[38] to the proposed model. Here, our focus is to replace the classical operators with the power-law, exponential decay, and Mittag-Leffler kernels. Moreover, we impose the variable order version too. Initially, for the exponential decay kernel

$$ {}_0^{\mathrm{FFE}}D_t^{\omega,\varpi} S(t) = \overline{\aleph} - \overline{\varepsilon} S(t)C(t) - \overline{\nu_1} S(t) + \overline{\psi_2} A_1(t), $$
$$ {}_0^{\mathrm{FFE}}D_t^{\omega,\varpi} U(t) = \overline{\varepsilon} S(t)C(t) - (\overline{\varphi_1} + \overline{\varphi_2} + \overline{\nu_1}) U(t), $$

$$
\begin{aligned}
{}_0^{\mathrm{FFE}}D_t^{\omega,\varpi} A_1(t) &= \overline{\varphi_1}U(t) - (\overline{\psi_1} + \overline{\psi_2} + \overline{\nu_1})A_1(t),\\
{}_0^{\mathrm{FFE}}D_t^{\omega,\varpi} C(t) &= \overline{\psi_1}A_1(t) - (\overline{\sigma_1} + \overline{\sigma_2} + \overline{\nu_1} + \overline{\nu_2})\\
&\quad \times C(t) + \overline{\varphi_2}U(t),\\
{}_0^{\mathrm{FFE}}D_t^{\omega,\varpi} A_2(t) &= \overline{\sigma_1}C(t) - (\overline{\nu_1} + \overline{\nu_3} + \overline{\xi})A_2(t),\\
{}_0^{\mathrm{FFE}}D_t^{\omega,\varpi} R(t) &= \overline{\xi}A_2(t) + \overline{\sigma_2}C(t) - \overline{\nu_1}R(t).
\end{aligned}
$$

For the sake of clarity, we express the model as

$$
\begin{aligned}
{}_0^{\mathrm{FFE}}D_t^{\omega,\varpi} S &= S_1(t, S, U, A_1, C, A_2, R),\\
{}_0^{\mathrm{FFE}}D_t^{\omega,\varpi} U &= U_1(t, S, U, A_1, C, A_2, R),\\
{}_0^{\mathrm{FFE}}D_t^{\omega,\varpi} A_1 &= A_{1_1}(t, S, U, A_1, C, A_2, R),\\
{}_0^{\mathrm{FFE}}D_t^{\omega,\varpi} C &= C_1(t, S, U, A_1, C, A_2, R),\\
{}_0^{\mathrm{FFE}}D_t^{\omega,\varpi} A_2 &= A_{2_1}(t, S, U, A_1, C, A_2, R),\\
{}_0^{\mathrm{FFE}}D_t^{\omega,\varpi} R &= R_1(t, S, U, A_1, C, A_2, R),
\end{aligned}
$$

where

$$
\begin{aligned}
&S_1(t, S, U, A_1, C, A_2, R)\\
&\quad = \overline{\aleph} - \overline{\varepsilon}S(t)C(t) - \overline{\nu_1}S(t) + \overline{\psi_2}A_1(t),\\
&U_1(t, S, U, A_1, C, A_2, R)\\
&\quad = \overline{\varepsilon}S(t)C(t) - (\overline{\varphi_1} + \overline{\varphi_2} + \overline{\nu_1})U(t),\\
&A_{1_1}(t, S, U, A_1, C, A_2, R)\\
&\quad = \overline{\varphi_1}U(t) - (\overline{\psi_1} + \overline{\psi_2} + \overline{\nu_1})A_1(t),\\
&C_1(t, S, U, A_1, C, A_2, R)\\
&\quad = \overline{\psi_1}A_1(t) - (\overline{\sigma_1} + \overline{\sigma_2} + \overline{\nu_1} + \overline{\nu_2})C(t)\\
&\qquad + \overline{\varphi_2}U(t),\\
&A_{2_1}(t, S, U, A_1, C, A_2, R)\\
&\quad = \overline{\sigma_1}C(t) - (\overline{\nu_1} + \overline{\nu_3} + \overline{\xi})A_2(t),\\
&R_1(t, S, U, A_1, C, A_2, R)\\
&\quad = \overline{\xi}A_2(t) + \overline{\sigma_2}C(t) - \overline{\nu_1}R(t).
\end{aligned}
$$

By applying fractal–fractional integral with exponential kernel, we have

$$
\begin{aligned}
S(t_{\delta+1}) &= S(t_\delta) + \frac{1-\omega}{M(\omega)}[t_\delta^{1-\varpi} S_1(t_\delta, S(t_\delta), U(t_\delta),\\
&\quad A_1(t_\delta), C(t_\delta), A_2(t_\delta), R(t_\delta))\\
&\quad - t_{\delta-1}^{1-\varpi} S_1(t_{\delta-1}), S(t_{\delta-1}), U(t_{\delta-1}),\\
&\quad A_1(t_{\delta-1}), C(t_{\delta-1}), A_2(t_{\delta-1}), R(t_{\delta-1})]\\
&\quad + \frac{\omega}{M(\omega)}\int_{t_\delta}^{t_{\delta+1}} S_1(\Upsilon, S, U, A_1, C, A_2, R)\\
&\quad \times \Upsilon^{1-\varpi} d\Upsilon,\\
U(t_{\delta+1}) &= U(t_\delta) + \frac{1-\omega}{M(\omega)}[t_\delta^{1-\varpi} U_1(t_\delta, S(t_\delta), U(t_\delta),\\
&\quad A_1(t_\delta), C(t_\delta), A_2(t_\delta), R(t_\delta)) - t_{\delta-1}^{1-\varpi} U_1\\
&\quad \times (t_{\delta-1}), S(t_{\delta-1}), U(t_{\delta-1}), A_1(t_{\delta-1}),\\
&\quad C(t_{\delta-1}), A_2(t_{\delta-1}), R(t_{\delta-1})]\\
&\quad + \frac{\omega}{M(\omega)}\int_{t_\delta}^{t_{\delta+1}} U_1(\Upsilon, S, U, A_1, C, A_2, R)\\
&\quad \times \Upsilon^{1-\varpi} d\Upsilon,\\
A_1(t_{\delta+1}) &= A_1(t_\delta) + \frac{1-\omega}{M(\omega)}[t_\delta^{1-\varpi} A_{1_1}(t_\delta, S(t_\delta),\\
&\quad U(t_\delta), A_1(t_\delta), C(t_\delta), A_2(t_\delta), R(t_\delta))\\
&\quad - t_{\delta-1}^{1-\varpi} A_{1_1}(t_{\delta-1}), S(t_{\delta-1}), U(t_{\delta-1}),\\
&\quad A_1(t_{\delta-1}), C(t_{\delta-1}), A_2(t_{\delta-1}), R(t_{\delta-1})]\\
&\quad + \frac{\omega}{M(\omega)}\int_{t_\delta}^{t_{\delta+1}} A_{1_1}(\Upsilon, S, U, A_1, C, A_2, R)\\
&\quad \times \Upsilon^{1-\varpi} d\Upsilon,\\
C(t_{\delta+1}) &= C(t_\delta) + \frac{1-\omega}{M(\omega)}[t_\delta^{1-\varpi} I_1(t_\delta, S(t_\delta), U(t_\delta),\\
&\quad A_1(t_\delta), C(t_\delta), A_2(t_\delta), R(t_\delta)) - t_{\delta-1}^{1-\varpi} C_1\\
&\quad \times (t_{\delta-1}), S(t_{\delta-1}), U(t_{\delta-1}), A_1(t_{\delta-1}),\\
&\quad C(t_{\delta-1}), A_2(t_{\delta-1}), R(t_{\delta-1})]\\
&\quad + \frac{\omega}{M(\omega)}\int_{t_\delta}^{t_{\delta+1}} C_1(\Upsilon, S, U, A_1, C, A_2, R)\\
&\quad \times \Upsilon^{1-\varpi} d\Upsilon,\\
A_2(t_{\delta+1}) &= A_{2_1}(t_\delta) + \frac{1-\omega}{M(\omega)}[t_\delta^{1-\varpi} A_{2_1}(t_\delta, S(t_\delta),\\
&\quad U(t_\delta), A_1(t_\delta), C(t_\delta), A_2(t_\delta), R(t_\delta))\\
&\quad - t_{\delta-1}^{1-\varpi} A_{2_1}(t_{\delta-1}), S(t_{\delta-1}), U(t_{\delta-1}),\\
&\quad A_1(t_{\delta-1}), C(t_{\delta-1}), A_2(t_{\delta-1}), R(t_{\delta-1})]\\
&\quad + \frac{\omega}{M(\omega)}\int_{t_\delta}^{t_{\delta+1}} A_{2_1}(\Upsilon, S, U, A_1, C, A_2, R)\\
&\quad \times \Upsilon^{1-\varpi} d\Upsilon,
\end{aligned}
$$

$$\begin{aligned}R(t_{\delta+1}) &= R(t_\delta) + \frac{1-\omega}{M(\omega)}[t_\delta^{1-\varpi} T_1(t_\delta, S(t_\delta), U(t_\delta),\\ &\quad A_1(t_\delta), C(t_\delta), A_2(t_\delta), R(t_\delta))\\ &\quad - t_{\delta-1}^{1-\varpi} R_1(t_{\delta-1}), S(t_{\delta-1}), U(t_{\delta-1}),\\ &\quad A_1(t_{\delta-1}), C(t_{\delta-1}), A_2(t_{\delta-1}), R(t_{\delta-1})]\\ &\quad + \frac{\omega}{M(\omega)} \int_{t_\delta}^{t_{\delta+1}} R_1(\Upsilon, S, U, A_1, C, A_2, R)\\ &\quad \times \Upsilon^{1-\varpi} d\Upsilon.\end{aligned} \tag{30}$$

For the proposed model, we have the scheme as

$$\begin{aligned}S^{\delta+1} &= S^\delta + \frac{1-\omega}{M(\omega)}[t_\delta^{1-\varpi} S_1(t_\delta, S(t_\delta), U(t_\delta), A_1(t_\delta),\\ &\quad C(t_\delta), A_2(t_\delta), R(t_\delta)) - t_{\delta-1}^{1-\varpi} S_1(t_{\delta-1}),\\ &\quad S(t_{\delta-1}), U(t_{\delta-1}), A_1(t_{\delta-1}), C(t_{\delta-1}),\\ &\quad A_2(t_{\delta-1}), R(t_{\delta-1})]\\ &\quad + \frac{\omega}{M(\omega)}\left[\frac{23}{12} t_\delta^{1-\varpi} S_1(t_\delta, S(t_\delta), U(t_\delta), A_1(t_\delta),\right.\\ &\quad C(t_\delta), A_2(t_\delta), R(t_\delta))\Delta t - \frac{4}{3} t_{\delta-1}^{1-\varpi} S_1\\ &\quad \times (t_{\delta-1}), S(t_{\delta-1}), U(t_{\delta-1}), A_1(t_{\delta-1}),\\ &\quad C(t_{\delta-1}), A_2(t_{\delta-1}), R(t_{\delta-1})\Delta t + \frac{5}{12} t_{\delta-1}^{1-\varpi}\\ &\quad \times S_1(t_{\delta-2}, S(t_{\delta-2}), U(t_{\delta-2}), A_1(t_{\delta-2}),\\ &\quad \left. C(t_{\delta-2}), A_2(t_{\delta-2}), R(t_{\delta-2}))\Delta t\right],\end{aligned}$$

$$\begin{aligned}U^{\delta+1} &= U^\delta + \frac{1-\omega}{M(\omega)}[t_\delta^{1-\varpi} U_1(t_\delta, S(t_\delta), U(t_\delta), A_1(t_\delta),\\ &\quad C(t_\delta), A_2(t_\delta), R(t_\delta)) - t_{\delta-1}^{1-\varpi} U_1\\ &\quad \times (t_{\delta-1}), S(t_{\delta-1}), U(t_{\delta-1}), A_1(t_{\delta-1}),\\ &\quad C(t_{\delta-1}), A_2(t_{\delta-1}), R(t_{\delta-1})]\\ &\quad + \frac{\omega}{M(\omega)}\left[\frac{23}{12} t_\delta^{1-\varpi} U_1(t_\delta, S(t_\delta), U(t_\delta), A_1(t_\delta),\right.\\ &\quad C(t_\delta), A_2(t_\delta), R(t_\delta))\Delta t - \frac{4}{3} t_{\delta-1}^{1-\varpi} U_1\\ &\quad \times (t_{\delta-1}), S(t_{\delta-1}), U(t_{\delta-1}), A_1(t_{\delta-1}),\\ &\quad \left. C(t_{\delta-1}), A_2(t_{\delta-1}), R(t_{\delta-1})\Delta t\right]\\ &\quad + \frac{5}{12} t_{\delta-1}^{1-\varpi} U_1(t_{\delta-2}, S(t_{\delta-2}), U(t_{\delta-2}),\\ &\quad A_1(t_{\delta-2}), C(t_{\delta-2}), A_2(t_{\delta-2}), R(t_{\delta-2}))\Delta t],\end{aligned}$$

$$\begin{aligned}A_1^{\delta+1} &= A_1^\delta + \frac{1-\omega}{M(\omega)}[t_\delta^{1-\varpi} A_{1_1}(t_\delta, S(t_\delta), U(t_\delta),\\ &\quad A_1(t_\delta), C(t_\delta), A_2(t_\delta), R(t_\delta))\\ &\quad - t_{\delta-1}^{1-\varpi} A_{1_1}(t_{\delta-1}), S(t_{\delta-1}), U(t_{\delta-1}),\\ &\quad A_1(t_{\delta-1}), C(t_{\delta-1}), A_2(t_{\delta-1}), R(t_{\delta-1})]\\ &\quad + \frac{\omega}{M(\omega)}\left[\frac{23}{12} t_\delta^{1-\varpi} A_{1_1}(t_\delta, S(t_\delta), U(t_\delta),\right.\\ &\quad A_1(t_\delta), C(t_\delta), A_2(t_\delta), R(t_\delta))\Delta t\\ &\quad - \frac{4}{3} t_{\delta-1}^{1-\varpi} A_{1_1}(t_{\delta-1}), S(t_{\delta-1}), U(t_{\delta-1}),\\ &\quad A_1(t_{\delta-1}), C(t_{\delta-1}), A_2(t_{\delta-1}),\\ &\quad R(t_{\delta-1})\Delta t + \frac{5}{12} t_{\delta-1}^{1-\varpi} A_{1_1}\\ &\quad \times (t_{\delta-2}, S(t_{\delta-2}), U(t_{\delta-2}), A_1(t_{\delta-2}),\\ &\quad \left. C(t_{\delta-2}), A_2(t_{\delta-2}), R(t_{\delta-2}))\Delta t\right],\end{aligned}$$

$$\begin{aligned}C^{\delta+1} &= C^\delta + \frac{1-\omega}{M(\omega)}[t_\delta^{1-\varpi} C_1(t_\delta, S(t_\delta), U(t_\delta),\\ &\quad A_1(t_\delta), C(t_\delta), A_2(t_\delta), R(t_\delta))\\ &\quad - t_{\delta-1}^{1-\varpi} C_1(t_{\delta-1}), S(t_{\delta-1}), U(t_{\delta-1}),\\ &\quad A_1(t_{\delta-1}), C(t_{\delta-1}), A_2(t_{\delta-1}), R(t_{\delta-1})]\\ &\quad + \frac{\omega}{M(\omega)}\left[\frac{23}{12} t_\delta^{1-\varpi} C_1(t_\delta, S(t_\delta), U(t_\delta),\right.\\ &\quad A_1(t_\delta), C(t_\delta), A_2(t_\delta), R(t_\delta))\Delta t\\ &\quad - \frac{4}{3} t_{\delta-1}^{1-\varpi} C_1(t_{\delta-1}), S(t_{\delta-1}), U(t_{\delta-1}),\\ &\quad A_1(t_{\delta-1}), C(t_{\delta-1}), A_2(t_{\delta-1}), R(t_{\delta-1})\Delta t\\ &\quad + \frac{5}{12} t_{\delta-1}^{1-\varpi} C_1(t_{\delta-2}, S(t_{\delta-2}), U(t_{\delta-2}),\\ &\quad A_1(t_{\delta-2}), C(t_{\delta-2}), A_2(t_{\delta-2}),\\ &\quad \left. R(t_{\delta-2}))\Delta t\right],\end{aligned}$$

$$\begin{aligned}A_2^{\delta+1} &= A_2^\delta + \frac{1-\omega}{M(\omega)}[t_\delta^{1-\varpi} A_{2_1}(t_\delta, S(t_\delta), U(t_\delta),\\ &\quad A_1(t_\delta), C(t_\delta), A_2(t_\delta), R(t_\delta)) - t_{\delta-1}^{1-\varpi}\\ &\quad A_{2_1}(t_{\delta-1}), S(t_{\delta-1}), U(t_{\delta-1}), A_1(t_{\delta-1}),\\ &\quad C(t_{\delta-1}), A_2(t_{\delta-1}), R(t_{\delta-1})]\\ &\quad + \frac{\omega}{M(\omega)}\left[\frac{23}{12} t_\delta^{1-\varpi} A_{2_1}(t_\delta, S(t_\delta), U(t_\delta),\right.\end{aligned}$$

$$
\begin{aligned}
&A_1(t_\delta), C(t_\delta), A_2(t_\delta), R(t_\delta))\Delta t\\
&-\frac{4}{3}t_{\delta-1}^{1-\varpi}A_{2_1}(t_{\delta-1}), S(t_{\delta-1}), U(t_{\delta-1}),\\
&A_1(t_{\delta-1}), C(t_{\delta-1}), A_2(t_{\delta-1}), R(t_{\delta-1})\Delta t\\
&+\frac{5}{12}t_{\delta-1}^{1-\varpi}A_{2_1}(t_{\delta-2}, S(t_{\delta-2}), U(t_{\delta-2}),\\
&\left.A_1(t_{\delta-2}), C(t_{\delta-2}), A_2(t_{\delta-2}), R(t_{\delta-2}))\Delta t\right],
\end{aligned}
$$

$$
\begin{aligned}
R^{\delta+1} = R^\delta &+ \frac{1-\omega}{M(\omega)}[t_\delta^{1-\varpi}R_1(t_\delta, S(t_\delta), U(t_\delta),\\
&A_1(t_\delta), C(t_\delta), A_2(t_\delta), R(t_\delta))\\
&-t_{\delta-1}^{1-\varpi}R_1(t_{\delta-1}), S(t_{\delta-1}), U(t_{\delta-1}), A_1(t_{\delta-1}),\\
&C(t_{\delta-1}), A_2(t_{\delta-1}), R(t_{\delta-1})]\\
&+\frac{\omega}{M(\omega)}\left[\frac{23}{12}t_\delta^{1-\varpi}R_1(t_\delta, S(t_\delta), U(t_\delta), A_1(t_\delta),\right.\\
&C(t_\delta), A_2(t_\delta), R(t_\delta))\Delta t\\
&-\frac{4}{3}t_{\delta-1}^{1-\varpi}R_1(t_{\delta-1}), S(t_{\delta-1}), U(t_{\delta-1}),\\
&A_1(t_{\delta-1}), C(t_{\delta-1}), A_2(t_{\delta-1}), R(t_{\delta-1})\Delta t\\
&+\frac{5}{12}t_{\delta-1}^{1-\varpi}R_1(t_{\delta-2}, S(t_{\delta-2}), U(t_{\delta-2}),\\
&\left.A_1(t_{\delta-2}), C(t_{\delta-2}), A_2(t_{\delta-2}), R(t_{\delta-2}))\Delta t\right].
\end{aligned}
\tag{31}
$$

For Mittag-Leffler kernel, we have

$$
\begin{aligned}
S(t_{\delta+1}) = S(t_\delta) &+ \frac{1-\omega}{AB(\omega)}t_\delta^{1-\varpi}S_1(t_\delta, S(t_\delta), U(t_\delta),\\
&A_1(t_\delta), C(t_\delta), A_2(t_\delta), R(t_\delta))\\
&+\frac{\omega}{AB(\omega)\Gamma(\omega)}\sum_{l=2}^{\delta}\int_{t_\delta}^{t_{\delta+1}}S_1(\Upsilon, S, U, A_1,\\
&C, A_2, R)\Upsilon^{1-\varpi}(t_{\delta+1}-\Upsilon)^{\omega-1}d\Upsilon.
\end{aligned}
\tag{32}
$$

For the numerical scheme, we have

$$
\begin{aligned}
S^{\delta+1} = &\frac{1-\omega}{AB(\omega)}t_\delta^{1-\varpi}S_1(t_\delta, S^\delta, U^\delta, A_1^\delta, C^\delta, A_2^\delta, R^\delta)\\
&+\frac{\omega(\Delta t)^\omega}{AB(\omega)\Gamma(\omega+1)}\Sigma_{l=2}^{\delta}t_{l-2}^{1-\varpi}S_1(t_{l-2},\\
&S^{l-2}, U^{l-2}, A_1^{l-2}, C^{l-2}, A_2^{l-2}, R^{l-2})\\
&\times[(\delta-l+1)^\omega-(\delta-l)^\omega]\\
&+\frac{\omega(\Delta t)^\omega}{AB(\omega)\Gamma(\omega+2)}\Sigma_{l=2}^{\delta}[t_{l-1}^{1-\varpi}S_1(t_{l-1},\\
&S^{l-1}, U^{l-1}, A_1^{l-1}, C^{l-1}, A_2^{l-1}, R^{l-1})\\
&-t_{l-2}^{1-\varpi}S_1(t_{l-2}, S^{l-2}, U^{l-2},\\
&A_1^{l-2}, C^{l-2}, A_2^{l-2}, R^{l-2})]\\
&\times[(\delta-l+1)^\omega(\delta-l+3+2\omega)\\
&-(\delta-l)^\omega(\delta-l+3+3\omega)]\\
&+\frac{\omega(\Delta t)^\omega}{2AB(\omega)\Gamma(\omega+3)}\sum_{l=2}^{\delta}[t_l^{1-\varpi}S_1(t_l, S^l, U^l,\\
&A_1^l, C^l, A_2R^l, R^l)-2t_{l-1}^{1-\varpi}S_1\\
&\times(t_{l-1}, S^{l-1}, U^{l-1}, A_1^{l-1}, C^{l-1}, A_2^{l-1}, R^{l-1})\\
&+t_{l-2}^{1-\varpi}S_1(t_{l-2}, S^{l-2}, U^{l-2}, A_1^{l-2}, C^{l-2},\\
&A_2^{l-2}, R^{l-2})]\times[(\delta-l+1)^\omega[2(\delta-l)^2\\
&+(3\omega+10)(\delta-l)+2\omega^2+9\omega+12]\\
&-(\delta-l)^\omega[2(\delta-l)^2+(5\omega+10)(\delta-l)\\
&+6\omega^2+18\omega+12].
\end{aligned}
\tag{33}
$$

Similarly, we can develop generalized form for $U(t), A_1(t), C(t), A_2(t)$ and $R(t)$.

For power-law kernel, we have

$$
\begin{aligned}
S(t_{\delta+1}) &= \frac{1}{\Gamma(\omega)}\sum_{l=2}^{\delta}\int_{t_l}^{t_{l+1}}S_1(\Upsilon, S, U, A_1, C, A_2, R)\\
&\quad\times\Upsilon^{1-\varpi}(t_{\delta+1}-\Upsilon)^{\omega-1}d\Upsilon,\\
U(t_{\delta+1}) &= \frac{1}{\Gamma(\omega)}\sum_{l=2}^{\delta}\int_{t_l}^{t_{l+1}}I_1(\Upsilon, S, U, A_1, C, A_2, R)\\
&\quad\times\Upsilon^{1-\varpi}(t_{\delta+1}-\Upsilon)^{\omega-1}d\Upsilon,\\
A_1(t_{\delta+1}) &= \frac{1}{\Gamma(\omega)}\sum_{l=2}^{\delta}\int_{t_l}^{t_{l+1}}D_1(\Upsilon, S, U, A_1, C, A_2, R)\\
&\quad\times\Upsilon^{1-\varpi}(t_{\delta+1}-\Upsilon)^{\omega-1}d\Upsilon,\\
C(t_{\delta+1}) &= \frac{1}{\Gamma(\omega)}\sum_{l=2}^{\delta}\int_{t_l}^{t_{l+1}}A_1(\Upsilon, S, U, A_1, C, A_2, R)\\
&\quad\times\Upsilon^{1-\varpi}(t_{\delta+1}-\Upsilon)^{\omega-1}d\Upsilon,
\end{aligned}
$$

$$A_2(t_{\delta+1}) = \frac{1}{\Gamma(\omega)} \sum_{l=2}^{\delta} \int_{t_l}^{t_{l+1}} R_1(\Upsilon, S, U, A_1, C, A_2, R) \times \Upsilon^{1-\varpi}(t_{\delta+1} - \Upsilon)^{\omega-1} d\Upsilon,$$
$$R(t_{\delta+1}) = \frac{1}{\Gamma(\omega)} \sum_{l=2}^{\delta} \int_{t_l}^{t_{l+1}} T_1(\Upsilon, S, U, A_1, C, A_2, R) \times \Upsilon^{1-\varpi}(t_{\delta+1} - \Upsilon)^{\omega-1} d\Upsilon. \tag{34}$$

Now for the numerical scheme, we have

$$\begin{aligned} S^{\delta+1} &= \frac{(\Delta t)^\omega}{\Gamma(\omega+1)} \sum_{l=2}^{\delta} t_{l-2}^{1-\varpi} S_1(t_{l-2}, S^{l-2}, U^{l-2}, A_1^{l-2}, C^{l-2}, A_2^{l-2}, R^{l-2}) \times [(\delta-l+1)^\omega \\ &\quad - (\delta-l)^\omega] + \frac{(\Delta t)^\omega}{\Gamma(\omega+2)} \sum_{l=2}^{\delta} [t_{l-1}^{1-\varpi} S_1(t_{l-1}, S^{l-1}, U^{l-2}, A_1^{l-2}, C^{l-2}, A_2^{l-1}, R^{l-1}) \\ &\quad - t_{l-2}^{1-\varpi} S_1(t_{l-2}, S^{l-2}, U^{l-2}, A_1^{l-2}, C^{l-2}, A_2^{l-2}, R^{l-2})] \\ &\quad \times [(\delta-l+1)^\omega(\delta-l+3+2\omega) - (\delta-l)^\omega(\delta-l+3+3\omega)] + \frac{(\Delta t)^\omega}{2\Gamma(\omega+3)} \\ &\quad \times \sum_{l=2}^{\delta} [t_l^{1-\varpi} S_1(t_l, S^l, U^l, A_1^l, C^l, A_2^l, R^l) \\ &\quad - 2t_{l-1}^{1-\varpi} S_1(t_{l-1}, S^{l-1}, U^{l-2}, A_1^{l-2}, C^{l-2}, A_2^{l-1}, R^{l-1}) + t_{l-2}^{1-\varpi} S_1(t_{l-2}, S^{l-2}, U^{l-2}, A_1^{l-2}, C^{l-2}, A_2^{l-2}, R^{l-2})] \\ &\quad \times [(\delta-l+1)^\omega[2(\delta-l)^2 + (3\omega+10)(\delta-l) + 2\omega^2 + 9\omega + 12] - (\delta-l)^\omega[2(\delta-l)^2 \\ &\quad + (5\omega+10)(\delta-l) + 6\omega^2 + 18\omega + 12]]. \end{aligned} \tag{35}$$

Similarly, we can develop generalized form for $U(t), A_1(t), C(t), A_2(t)$ and $R(t)$.

Now for exponential kernel, by adopting the same procedure as mentioned above, we have

$$\begin{aligned} {}_0^{\mathrm{FFE}}D_t^{\omega,\varpi} S &= S_1(t, S, U, A_1, C, A_2, R), \\ {}_0^{\mathrm{FFE}}D_t^{\omega,\varpi} U &= U_1(t, S, U, A_1, C, A_2, R), \\ {}_0^{\mathrm{FFE}}D_t^{\omega,\varpi} A_1 &= A_{1_1}(t, S, U, A_1, C, A_2, R), \\ {}_0^{\mathrm{FFE}}D_t^{\omega,\varpi} C &= C_1(t, S, U, A_1, C, A_2, R), \\ {}_0^{\mathrm{FFE}}D_t^{\omega,\varpi} A_2 &= A_{2_1}(t, S, U, A_1, C, A_2, R), \\ {}_0^{\mathrm{FFE}}D_t^{\omega,\varpi} R &= R_1(t, S, U, A_1, C, A_2, R). \end{aligned}$$

The above expression has constant fractional and variable fractal dimension. So we can reformulate the above expression as

$$\begin{aligned} S(t_{\delta+1}) &= S(t_\delta) + \frac{1-\omega}{M(\omega)} \Bigg[t_\delta^{2-\varpi(t_\delta)} \times \left(-\frac{\varpi(t_{\delta+1}) - \varpi(t_\delta)}{\Delta t} \ln t_\delta + \frac{2-\varpi(t_\delta)}{t_\delta} \right) \times S_1(t_\delta, S(t_\delta), U(t_\delta), A_1(t_\delta), C(t_\delta), A_2(t_\delta), R(t_\delta)) \\ &\quad - t_{\delta-1}^{2-\varpi(t_{\delta-1})} \left(-\frac{\varpi(t_\delta) - \varpi(t_{\delta-1})}{\Delta t} \ln t_{\delta-1} + \frac{2-\varpi(t_{\delta-1})}{t_{\delta-1}} \right) \\ &\quad \times S_1(t_{\delta-1}), S(t_{\delta-1}), U(t_{\delta-1}), A_1(t_{\delta-1}), C(t_{\delta-1}), A_2(t_{\delta-1}), R(t_{\delta-1}) \Bigg] \\ &\quad \times \frac{\omega}{M(\omega)} \int_{t_\delta}^{t_{\delta+1}} S_1(\Upsilon, S, U, A_1, C, A_2, R) \times \left[\acute{\varpi}(\Upsilon) \ln(\Upsilon) + \frac{2-\varpi(\Upsilon)}{\Upsilon} \right] \times \Upsilon^{2-\varpi(\Upsilon)} D\Upsilon. \end{aligned} \tag{36}$$

Similarly, we can develop generalized form for $U(t), A_1(t), C(t), A_2(t)$ and $R(t)$.

If we repeat the procedure then we have numerical approximation as

$$\begin{aligned} S(t_{\delta+1}) &= S(t_\delta) + \frac{1-\omega}{M(\omega)} \Bigg[t_\delta^{2-\varpi(t_\delta)} \times \left(-\frac{\varpi(t_{\delta+1}) - \varpi(t_\delta)}{\Delta t} \ln t_\delta + \frac{2-\varpi(t_\delta)}{t_\delta} \right) \times S_1(t_\delta, S(t_\delta), U(t_\delta), A_1(t_\delta), C(t_\delta), A_2(t_\delta), R(t_\delta)) \\ &\quad - t_{\delta-1}^{2-\varpi(t_{\delta-1})} \left(-\frac{\varpi(t_\delta) - \varpi(t_{\delta-1})}{\Delta t} \ln t_{\delta-1} \right. \end{aligned}$$

$$
\begin{aligned}
&+\frac{2-\varpi(t_{\delta-1})}{t_{\delta-1}}\Bigg)(S_1(t_{\delta-1}),S(t_{\delta-1}),\\
&U(t_{\delta-1}),A_1(t_{\delta-1}),C(t_{\delta-1}),\\
&A_2(t_{\delta-1}),R(t_{\delta-1}))\Bigg]\frac{\omega}{M(\omega)}\Bigg[\frac{23}{12}t_\delta^{2-\varpi(t_\delta)}\\
&\times\Bigg(-\frac{\varpi(t_{\delta+1})-\varpi(t_\delta)}{\Delta t}\ln t_\delta\\
&+\frac{2-\varpi(t_\delta)}{t_\delta}\Bigg)\\
&\times S_1(t_\delta,S(t_\delta),U(t_\delta),A_1(t_\delta),C(t_\delta),\\
&A_2(t_\delta),R(t_\delta))-\frac{4}{3}t_{\delta-1}^{2-\varpi(t_{\delta-1})}\\
&\times\Bigg(-\frac{\varpi(t_\delta)-\varpi(t_{\delta-1})}{\Delta t}\ln t_{\delta-1}\\
&+\frac{2-\varpi(t_{\delta-1})}{t_{\delta-1}}\Bigg)\times S_1(t_{\delta-1},S(t_{\delta-1}),\\
&U(t_{\delta-1}),A_1(t_{\delta-1}),C(t_{\delta-1}),A_2(t_{\delta-1}),\\
&R(t_{\delta-1}))+\frac{5}{12}t_{\delta-2}^{2-\varpi(t_{\delta-2})}\\
&\times\Bigg(-\frac{\varpi(t_{\delta-1})-\varpi(t_{\delta-2})}{\Delta t}\ln t_{\delta-2}\\
&+\frac{2-\varpi(t_{\delta-2})}{t_{\delta-2}}\Bigg)\\
&\times S_1(t_{\delta-2},S(t_{\delta-2}),U(t_{\delta-2}),A_1(t_{\delta-2}),\\
&C(t_{\delta-2}),A_2(t_{\delta-2}),R(t_{\delta-2}))\Bigg]. \qquad (37)
\end{aligned}
$$

Similarly, we can develop generalized form for $U(t), A_1(t), C(t), A_2(t)$ and $R(t)$.

For Mittag-Leffler kernel

$$
\begin{aligned}
S(t_{\delta+1})={}&\frac{1-\omega}{AB(\omega)}t_\delta^{2-\varpi(t_\delta)}\Bigg(-\frac{\varpi(t_{\delta+1})-\varpi(t_\delta)}{\Delta t}\\
&\ln t_\delta+\frac{2-\varpi(t_\delta)}{t_\delta}\Bigg)\times S_1(t_\delta,S(t_\delta),U(t_\delta),\\
&A_1(t_\delta),C(t_\delta),A_2(t_\delta),R(t_\delta))\\
&+\frac{\omega}{AB(\omega)\Gamma(\omega)}\int_0^t S_1(\Upsilon,S,U,A_1,\\
&C,A_2,R)(t_{\delta+1}-s)^{\omega-1}\\
&\times\left[-\acute{\varpi}(\Upsilon)\ln(\Upsilon)+\frac{2-\varpi(\Upsilon)}{\Upsilon}\right]\\
&\times\Upsilon^{2-\varpi(\Upsilon)}d\Upsilon. \qquad (38)
\end{aligned}
$$

So, for the numerical solution we have scheme as

$$
\begin{aligned}
S(t_{\delta+1})={}&\frac{1-\omega}{AB(\omega)}t_\delta^{2-\varpi(t_\delta)}\Bigg(-\frac{\varpi(t_{\delta+1})-\varpi(t_\delta)}{\Delta t}\ln t_\delta\\
&+\frac{2-\varpi(t_\delta)}{t_\delta}\Bigg)\times S_1(t_\delta,S^\delta,U^\delta,A_1^\delta,C^\delta,\\
&A_2^\delta,R^\delta)+\frac{\omega(\Delta t)^\omega}{AB(\omega)\Gamma(\omega+1)}\sum_{l=2}^{\delta}t_{l-2}^{2-\varpi(t_{l-2})}\\
&\times\Bigg(-\frac{\varpi(t_{l-1})-\varpi(t_{l-2})}{\Delta t}\ln t_{l-2}\\
&+\frac{2-\varpi(t_{l-2})}{t_{l-2}}\Bigg)S_1(t_{l-2},S^{l-2},U^{l-2},A_1^{l-2},\\
&C^{l-2},A_1^{l-2},R^{l-2})[(\delta-l+1)^\omega\\
&-(\delta-l)^\omega]+\frac{\omega(\Delta t)^\omega}{AB(\omega)\Gamma(\omega+2)}\\
&\times\sum_{l=2}^{\delta}\Bigg[t_{l-1}^{2-\varpi(t_{l-1})}\Bigg(-\frac{\varpi(t_l)-\varpi(t_{l-1})}{\Delta t}\\
&\times\ln t_{l-1}+\frac{2-\varpi(t_{l-1})}{t_{l-1}}\Bigg)\\
&\times S_1(t_{l-1},S(t_{l-1}),U(t_{l-1}),\\
&A_1(t_{l-1}),C(t_{l-1}),A_2(t_{l-1}),R(t_{l-1}))\\
&-t_{l-2}^{2-\varpi(t_{l-2})}\Bigg(-\frac{\varpi(t_{l-2})-\varpi(t_{l-2})}{\Delta t}\\
&\times\ln t_{l-2}+\frac{2-\varpi(t_{l-2})}{t_{l-2}}\Bigg)\\
&\times S_1(t_{l-2},S(t_{l-2}),U(t_{l-2}),A_1(t_{l-2}),\\
&C(t_{l-2}),A_2(t_{l-2}),R(t_{l-2}))\Bigg]\\
&\times[(\delta-l+1)^\omega(\delta-l+3+2\omega)\\
&-(\delta-l)^\omega(\delta-l+3+3\omega)]\\
&+\frac{\omega(\Delta t)^\omega}{2AB(\omega)\Gamma(\omega+3)}\sum_{l=2}^{\delta}\Bigg[t_l^{2-\varpi(t_l)}\\
&\times\Bigg(-\frac{\varpi(t_{l+1})-\varpi(t_l)}{\Delta t}\ln t_l+\frac{2-\varpi(t_l)}{t_l}\Bigg)\\
&\times S_1(t_l,S^l,U^l,A_1^l,C^l,A_2^l,R^l)-2t_{l-1}^{2-\varpi(t_\delta)}\\
&\times\Bigg(-\frac{\varpi(t_l)-\varpi(t_{l-1})}{\Delta t}\ln t_{l-1}
\end{aligned}
$$

$$+\frac{2-\varpi(t_{l-1})}{t_{l-1}}\bigg)\times S_1(t_{l-1},S(t_{l-1}),$$
$$U(t_{l-1}),A_1(t_{l-1}),C(t_{l-1}),A_2(t_{l-1}),$$
$$R(t_{l-1}))+t_{l-2}^{2-\varpi(t_{l-2})}$$
$$\times\bigg(-\frac{\varpi(t_{l-1})-\varpi(t_{l-2})}{\Delta t}\ln t_{l-2}$$
$$+\frac{2-\varpi(t_{l-2})}{t_{l-2}}\bigg)\times S_1(t_{l-2},S(t_{l-2}),$$
$$U(t_{l-2}),A_1(t_{l-2}),C(t_{l-2}),A_2(t_{\delta-1}),$$
$$\times[(\delta-l+1)^{\omega}[2(\delta-l)^2+(3\omega+10)$$
$$\times(\delta-l)+2\omega^2+9\omega+12]-(\delta-l)^{\omega}$$
$$\times[2(\delta-l)^2+(5\omega+10)(\delta-l)$$
$$+6\omega^2+18\omega+12]]. \tag{39}$$

Similarly, we can develop generalized form for $U(t), A_1(t), C(t), A_2(t)$ and $R(t)$.

For power-law kernel

$$S(t_{\delta+1})=S_0+\frac{1}{\Gamma(\omega)}\int_0^t S_1(\Upsilon,S,U,A_1,C,A_2,R)$$
$$\times(t_{\delta+1}-s)^{\omega-1}$$
$$\times\left[-\acute{\varpi}(\Upsilon)\ln(\Upsilon)+\frac{2-\varpi(\Upsilon)}{\Upsilon}\right]$$
$$\times\Upsilon^{2-\varpi(\Upsilon)}d\Upsilon.$$

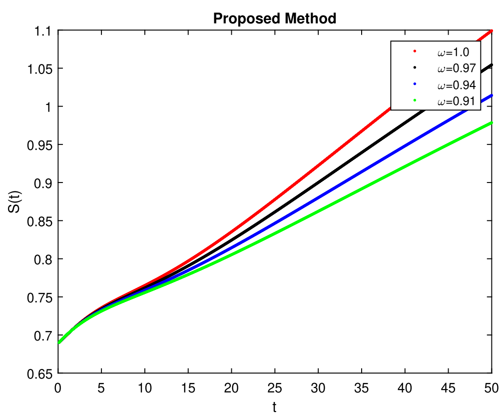
So, for the numerical solution we have scheme as

$$S(t_{\delta+1})=\frac{\omega(\Delta t)^{\omega}}{\Gamma(\omega+1)}\sum_{l=2}^{\delta}t_{l-2}^{2-\varpi(t_{l-2})}$$
$$\bigg(-\frac{\varpi(t_{l-1})-\varpi(t_{l-2})}{\Delta t}$$
$$\times\ln t_{l-2}+\frac{2-\varpi(t_{l-2})}{t_{l-2}}\bigg)$$
$$\times S_1(t_{l-2},S^{l-2},U^{l-2},A_1^{l-2},$$
$$C^{l-2},A_2^{l-2},R^{l-2})[(\delta-l+1)^{\omega}-(\delta-l)^{\omega}]$$
$$+\frac{\omega(\Delta t)^{\omega}}{\Gamma(\omega+2)}\sum_{l=2}^{\delta}\bigg[t_{l-1}^{2-\varpi(t_{l-1})}$$
$$\times\bigg(-\frac{\varpi(t_l)-\varpi(t_{l-1})}{\Delta t}\ln t_{l-1}$$
$$+\frac{2-\varpi(t_{l-1})}{t_{l-1}}\bigg)S_1(t_{l-1},S(t_{l-1}),$$
$$U(t_{l-1}),A_1(t_{l-1}),C(t_{l-1}),A_2(t_{l-1}),$$
$$R(t_{l-1}))-t_{l-2}^{2-\varpi(t_{l-2})}$$
$$\times\bigg(-\frac{\varpi(t_{l-2})-\varpi(t_{l-2})}{\Delta t}\ln t_{l-2}$$
$$+\frac{2-\varpi(t_{l-2})}{t_{l-2}}\bigg)S_1(t_{l-2},S(t_{l-2}),$$
$$U(t_{l-1}),A_1(t_{l-2}),C(t_{l-2}),A_2(t_{l-2}),$$
$$R(t_{l-2}))\bigg]$$
$$\times[(\delta-l+1)^{\omega}(\delta-l+3+2\omega)$$
$$-(\delta-l)^{\omega}(\delta-l+3+3\omega)]$$
$$+\frac{\omega(\Delta t)^{\omega}}{2\Gamma(\omega+3)}\sum_{l=2}^{\delta}\bigg[t_l^{2-\varpi(t_l)}$$
$$\times\bigg(-\frac{\varpi(t_{l+1})-\varpi(t_l)}{\Delta t}\ln t_l+\frac{2-\varpi(t_l)}{t_l}\bigg)$$
$$\times S_1(t_l,S^l,U^l,A_1^l,C^l,A_2^l,R^l)$$
$$-2t_{l-1}^{2-\varpi(t_{\delta})}\bigg(-\frac{\varpi(t_l)-\varpi(t_{l-1})}{\Delta t}\ln t_{l-1}$$
$$+\frac{2-\varpi(t_{l-1})}{t_{l-1}}\bigg)$$
$$\times S_1(t_{l-1},S(t_{l-1}),U(t_{l-1}),A_1(t_{l-1}),$$
$$C(t_{l-1}),A_2(t_{l-1}),R(t_{l-1}))$$
$$+t_{l-2}^{2-\varpi(t_{l-2})}\bigg(-\frac{\varpi(t_{l-1})-\varpi(t_{l-2})}{\Delta t}\ln t_{l-2}$$
$$+\frac{2-\varpi(t_{l-2})}{t_{l-2}}\bigg)\times S_1(t_{l-2},S(t_{l-2}),$$
$$U(t_{l-2}),A_1(t_{l-2}),C(t_{l-2}),A_2(t_{\delta-1}),$$
$$R(t_{l-2}))\bigg]\times[(\delta-l+1)^{\omega}[2(\delta-l)^2$$
$$+(3\omega+10)(\delta-l)+2\omega^2+9\omega+12]$$
$$-(\delta-l)^{\omega}[2(\delta-l)^2+(5\omega+10)(\delta-l)$$
$$+6\omega^2+18\omega+12]]. \tag{40}$$

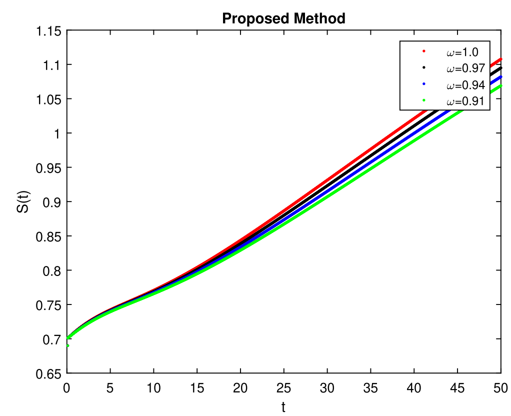
Similarly, we can develop generalized form for $U(t), A_1(t), C(t), A_2(t)$ and $R(t)$.

7. SIMULATION AND DISCUSSION

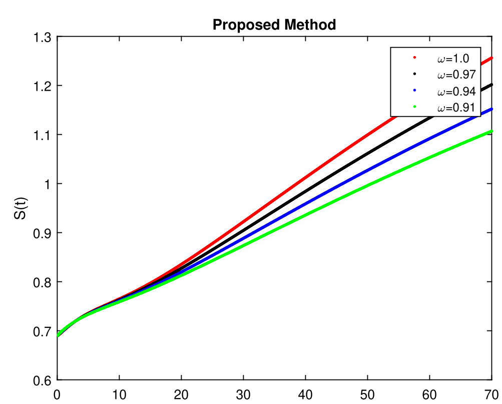
Here, we used the techniques of the power law, exponential law, and Mittag-Leffler kernel. At first, we present the values of the parameters used in the simulation.[35] The birth rate $(\overline{\aleph})$ is 0.02, while the demise rate of real $(\overline{\nu_1})$, infectious $(\overline{\nu_2})$, and quarantine $(\overline{\nu_3})$ is $0.01, 0.018$, and 0.015, respectively. The constant rate of susceptible population $(\overline{\psi_1})$ due to home quarantine is 0.3, while rate of transfer of suspected population $(\overline{\psi_2})$ after a negative test is 0.02. The transfer rate of infectious to medical $(\overline{\sigma_1})$ and recovered $(\overline{\sigma_2})$ ones are 0.07 and 0.1. The transfer rate of uncovered to susceptible $(\overline{\varphi_1})$ and infectious $(\overline{\varphi_2})$ populations are 0.3 and 0.05. The constant rate $(\overline{\varepsilon})$ is 0.1 for the susceptible population infected by the infectious population, and 0.02 is the rate at which hospitalized quarantine $\overline{\xi}$ population gets recovered. We obtain the numerical simulation, and the graphs act for the numerical simulation results for different values of $\alpha = 0.91, 0.94, 0.97, 1.0$. Initially (Fig. 1), we can see that in all cases, as the fractional-order derivative grows, the susceptible community also spreads, and the uncovered community reduces as the fractional-order value spreads (Fig. 2). Further, in Figs. 3 and 4, we discuss the cases of home quarantine and infection. Initially, the fractional-order derivative extends, and both communities also contract. But later, we noted that if the fractional order expands, both communities contract. We see that, the infection tends to zero as the population follows preventive measures like lockdown and masks. In Fig. 5, we can observe that the fractional-order derivative behaves slightly differently in all cases of medical quarantine. In power-law kernel and exponential-law kernel, fractional-order derivative and medical quarantine are directly proportional, while they are inversely proportional later. In the Mittag-Leffler



(a)



(b)



(c)

Fig. 1 Simulation of $S(t)$ with fractal fractional operator **(a)** power-law kernel, **(b)** exponential-law kernel and **(c)** Mittag-Leffler kernel.

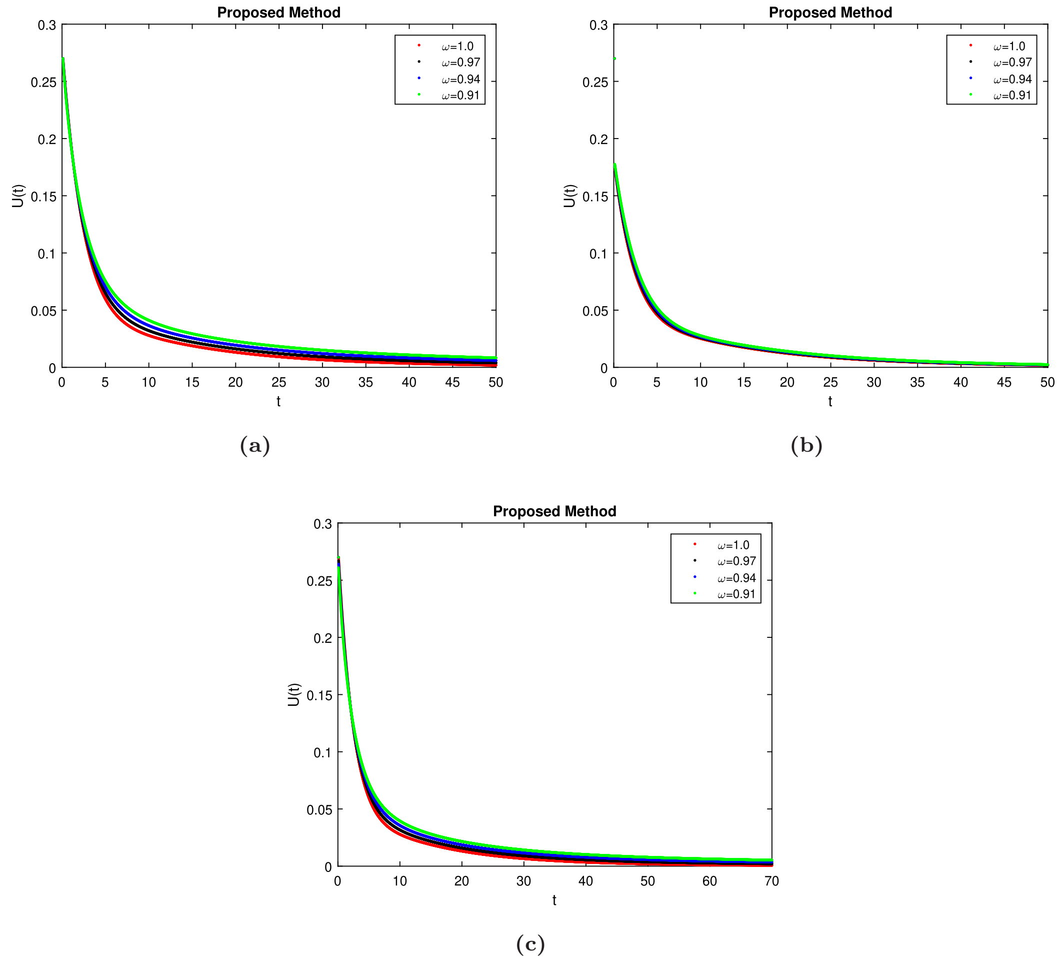


Fig. 2 Simulation of $U(t)$ with fractal fraction operator **(a)** power-law kernel, **(b)** exponential-law kernel and **(c)** Mittag-Leffler kernel.

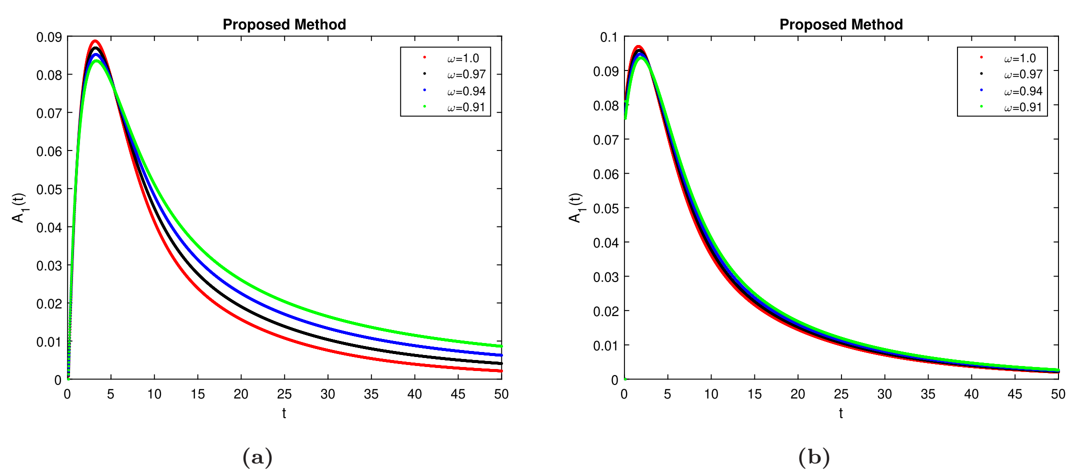
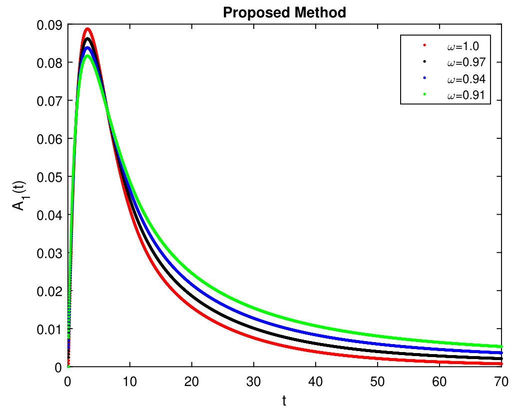
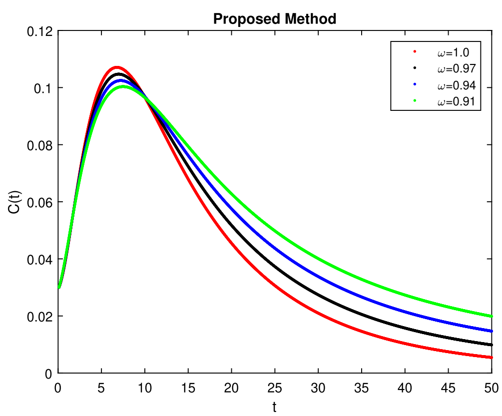


Fig. 3 Simulation of $A(t)$ with fractal fraction operator **(a)** power-law kernel, **(b)** exponential-law kernel and **(c)** Mittag-Leffler kernel.

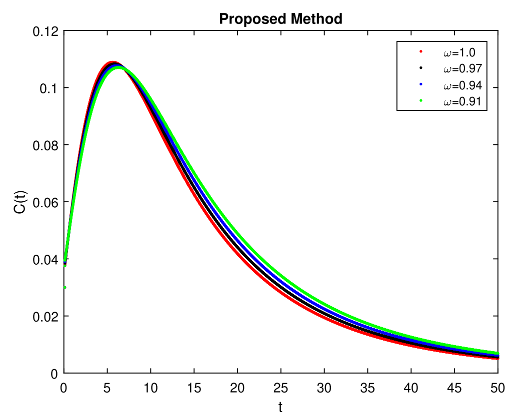


(c)

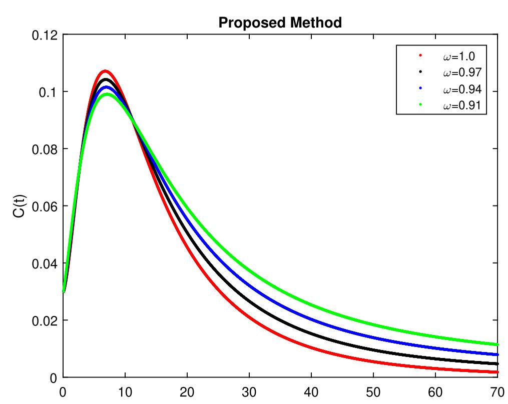
Fig. 3 (*Continued*)



(a)



(b)



(c)

Fig. 4 Simulation of $C(t)$ with fractal fraction operator **(a)** power-law kernel, **(b)** exponential-law kernel and **(c)** Mittag-Leffler kernel.

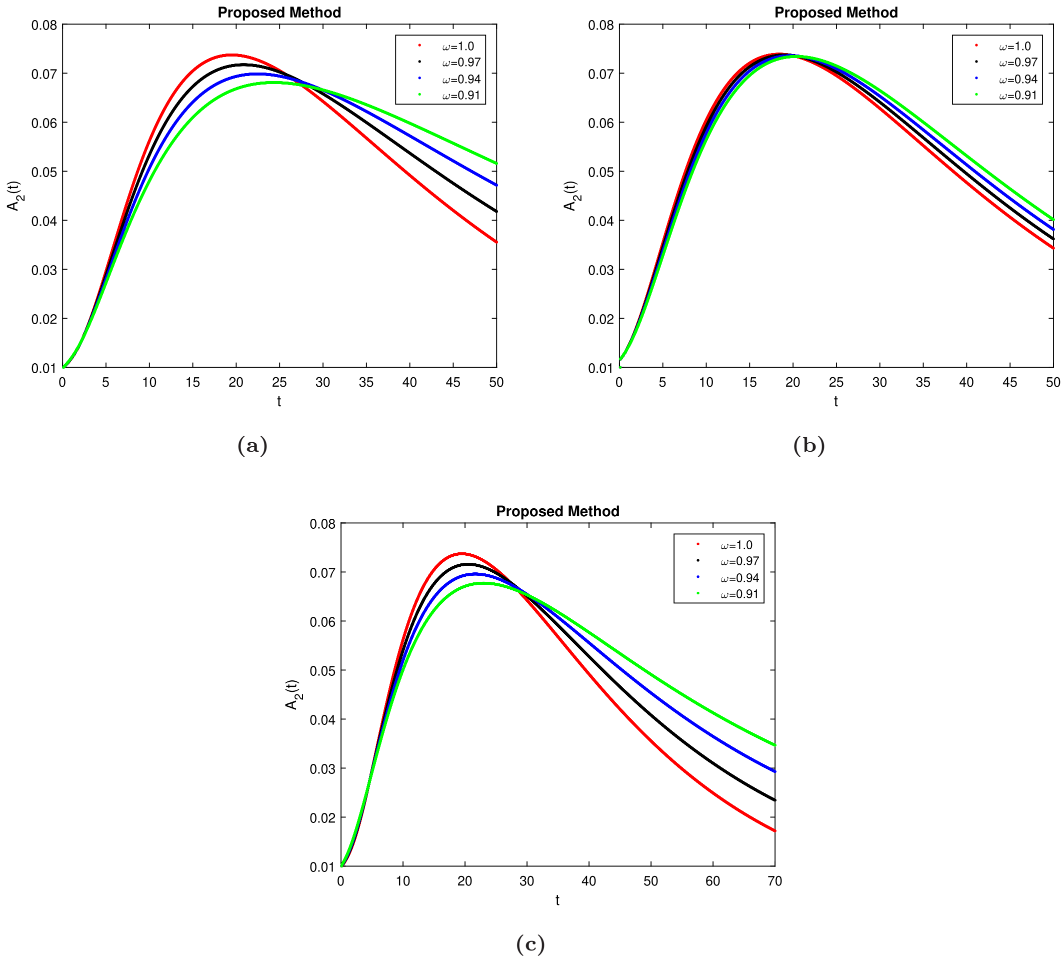


Fig. 5 Simulation of $A_2(t)$ with fractal fraction operator **(a)** power-law kernel, **(b)** exponential-law kernel and **(c)** Mittag-Leffler kernel.

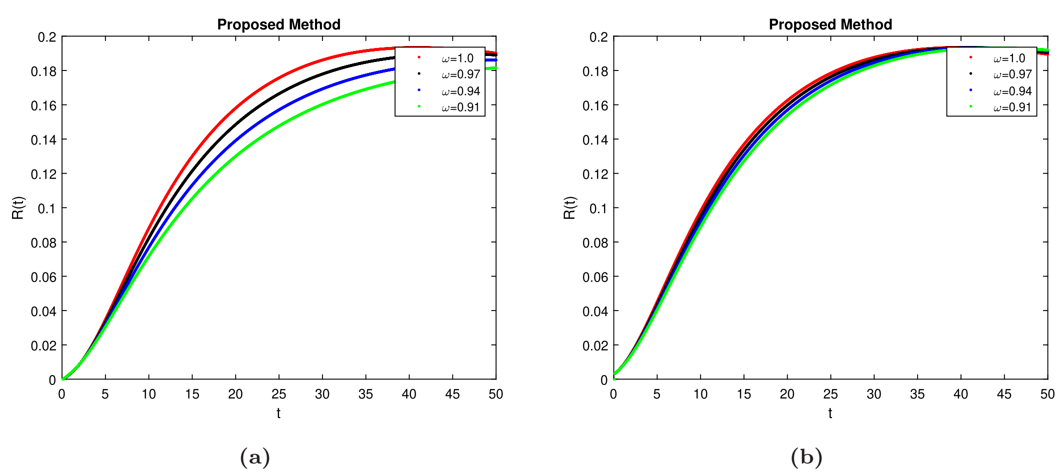
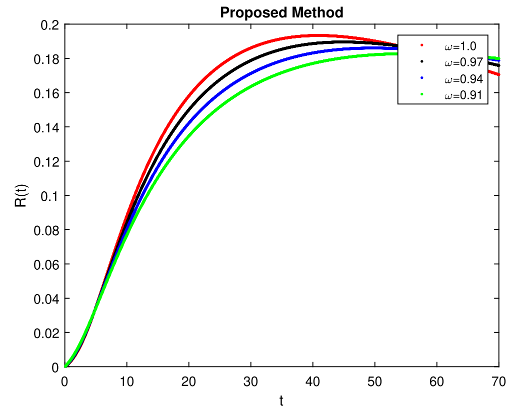


Fig. 6 Simulation of $R(t)$ with fractal fraction operator **(a)** power-law kernel, **(b)** exponential-law kernel and **(c)** Mittag-Leffler kernel.



(c)

Fig. 6 (*Continued*)

kernel, both fractional-order and quarantine grow. In Fig. 6, in the case of the power-law, initially, fractional-order derivative and recovered community grow while later both reduce. Exponential and Mittag-Leffler kernel, fractional-order derivative, and recovered communities are proportional to expansion. $(\overline{\psi_1})$ helps to maintain the low value for $(\overline{\varepsilon})$ per infectivity contact rate by removing the possibly infected population from the mass. Here, the value of R_0 is 0.9055, which is less than one. So, the infection will die out in due course. We observed that, in every case, it is converging to its equilibrium point. We can note the effect of different kernels on the proposed model, which have memory effected to control the parameters and blessings for humans to overcome the unpleasant impact of disease in society.

8. CONCLUSIONS

In this paper, we looked deeply into the COVID-19 model for Bhilwara of the state of Rajasthan of India. Initially, we discovered its equilibrium points and then talked about the stability of the model. We used fractal–fractional differential and integral operators with different kernels like power, exponential and the Mittag-Leffler function for analysis. To examine the actual action of COVID-19 in the community, we execute the process of simulation. We noted that gained outcomes are very productive, particularly for future analysis. Thus, to overcome the disease, lockdown and social distancing, and proper treatment can play a substantial role in society.

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