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A generalized dusty Brinkman type fluid of MHD free convection two phase flow between parallel plates

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ABSTRACT

Fractional order derivative operators are regarded to be very complex mathematical tools for implementing more realistic solutions in a wide range of physics and engineering problems. Due to significance applications of fractional derivatives, the aim of this article is analyzed the combinational effect of the magnetic field and heat transfer on Magneto hydrodynamic, free convective, two-phase flow of generalized Brinkman type dusty fluid among two parallel plates. The buoyant force causes the flow, which is aided by natural convection to transfer heat. Furthermore, the left plate travels at a constant speed and the right plate remains fixed, and all spherically dust particles are homogeneously dispersed among the fluid. The problem is modeled mathematically via set of partial differential equations. The Caputo-Fabrizio fractional derivative is employed to generalize the derived PDE's. A combination of Laplace and finite sine Fourier transformations is utilized to get the closed form solution of the problem. The impacts of many factors on temperature, Brinkman fluid, and dust particle velocity have also been examined, including a Grashof number, magnetic parameter, Reynold number, Peclet number and Dusty fluid parameter. Mathcad-15 is used to plot the graphical findings for the dusty fluid, Brinkman fluid, and temperature profiles. For various embedded parameters, the Brinkman fluid and dusty fluid behave similarly. It is concluded that the fractional Brinkman type dusty fluid is more realistic feature as compared to the classical one. The rate of heat transfer can be enhance with the passage of time.

1. Introduction

Fractional calculus is a combination of relatively obscure scientific results relating to non-integer level expansions of integration and differentiation. These findings have been collected over centuries in numerous disciplines of mathematics, but they have received limited attention in physics and other-oriented science subjects until recently. G.W. Leibniz worked on fractional calculus from (1695–1697). After that, another famous mathematician L. Euler studied different models in fractional calculus [1]. Due to extensive uses of fractional calculus in physical models and other nature are very famous in the research area. By engaging the Caputo fractional (CF) operator together with Fick's and Fourier's Laws, Khan et al. [2] concluded that the fractional procedure is more realistic and practical than the conventional approach since they offer several outputs that may enhance the best fit of real-world data. Sheikh et al. [3] investigated the flow of nanofluid utilizing Fourier's and Fick's generalized laws. Rehman et al. [4], and Ali et al. [5] mentioned in their work that the fractional differential equation is addressed from the standpoint of conformable derivatives. Considering a fractional-based model, Khan et al. [6]

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concluded that the drilling fluid velocity is regulated by the addition of hybrid nanoparticles as opposed to mono nanofluids. Furthermore, the percentage of heat transmission was improved by up to 11.149 percent by incorporating various nanoparticle forms into water. Jiang et al. [7] evaluated the flow of nanofluid via a stretched surface using Fourier's and Fick's law. Sheikh et al. [8] studied the flow of non-Newtonian fluids through a comparative examination of fractional derivatives. Using fractional Caputo-Fabrizio derivatives, Shah et al. [9] studied the irregular convective flow of viscous liquids. Shao et al. [10] examined the MHD convective fluids using Laplace transform with the Fourier-Sine transform. Baleanu et al. [11] published a paper on mathematical analysis of the liver using the CF fractional derivative. Many others are study different fluid models for fluid problems as [12–14].

Over the past few years, heat transfer in nanomaterial flows has received a lot of attention. Traditional exploitable materials have very poor thermal transport properties, including kerosine oil, engine oil, air, ethylene oil, mineral, gasoline oil, and water (because of its lower heat conductance). These materials' ability to conduct heat can be improved by adding nanoparticles to it. Because they exhibit different thermophysical behaviors, such as viscosity, thermal conductivity, heat transfer coefficient, and thermal diffusion coefficient, nanoliquids perform better in terms of thermal conductivity than base liquids. A substance known as a nanofluid is created by suspending solid nanoparticles (1–100 nm) in a base liquid [15,16]. Nanotechnology have remarkable applications in electronics cooling, heat exchangers, solar collectors, food processing micro-manufacturing, chemical industries, heat pumps and nuclear applications etc. Firstly Choi [17] analyzed the different processes of nanoparticles. He discussed that the nanoparticles and base liquid are associated and changed the heat characteristics of nanomaterials. Rehman et al. [18] reported the numerical simulation of the physical properties of magnetized suspended nanoparticles in a rotatory frame. Ahmed et al. [19], Voller et al. [20], and Falcini [21] discussed many irregular models of high-temperature dispersal and compare fractional derivative and non-linear diffusivity treatment. More recently Bhatti et al. [22] reported the Application of thermal energy storage through swimming of gyrotactic microorganisms in MHD Williamson nanofluid flow between rotating circular plates embedded in porous medium. Alrabaiah et al. [23] reported the generalized electro-osmotic Brinkman-type nanofluid as application cleansing of contaminated water.

The groundbreaking work of Brinkman [24] culminated in a structure that may be applied to huge, permeable areas. He used the Navier-Stoke equation to explore the behavior of fluid flow which is generated by dynamic viscosity on the outside of a dense group of tiny spheres. Ali et al. [25] investigated a model for a new exact solution for the development of a Brinkman fluid generated by an unbounded circulatory plate through the Laplace transform. Ali et al. [26] examined the combination of Brinkman fluid with unsteady magnetohydrodynamics and the impact of thermal radiation on heat transfer. As the radiations and Newtonian heating parameters were increased, then the sharing of fluid velocity and temperature was considerably enhanced. Khan et al. [23] studied the stream of unstable magnetohydrodynamic for Brinkman flow flowing through an elastic plate with shear force increasing at the bottom plate. The results were obtained by Fourier and finite Fourier transform methods. Khan et al. [27] also investigated that, how heat dissipation, thermal radiation chemical reactions, and magnetohydrodynamics affected Brinkman fluid over a perpendicular plate. The development of double-diffusive disease was investigated by ubey and Dubey et al. [28]. Saqib et al. [29] reported the Brinkman type dust flow as application of blood flow.

The fluid of dust particles in the fluid has a wide range of mechanical uses, including transportation, manufacturing of steel and cement, flying ash from thermal plants, and the detrimental impact of air conditioners. Two-phase flows with solid particles distributed in nanofluids and hybrid nanofluids have a huge range of industrial uses. Many scholars recently investigated fluid flow with embedded dust particles. Due to their two-phase structure, dusty fluid models have piqued researchers interest. This occurrence happens in fluid flows with solid particle dispersion. The deliberation of modeling, analyzing, and solving, the flow of dusty particles has been accelerated. Saffman [30] is the one who started the research on dusty fluid laminar flow. Dusty gas flow in the boundary layer area was studied by Chakrabarti [31]. Datta and Mishra [32] explored the flow of dusty fluid over a semi-infinite plate. The hydromagnetic flow of dusty fluid across a stretching surface in the presence of suction velocity was examined by Vajravelu and Nayfeh [33]. The computational solution which is both for two-dimensional boundary layer flow along with heat transfer of dusty fluid over the extending area was obtained by Gireesha et al. [34]. Gireesha et al. [35] provided a mathematical solution for MHD fluid and heat transport of dusty flow over a linearly extending sheet. Shukla and Stenflo [36] studied the dust fluid motions in strongly coupled dusty plasmas. It is demonstrated that strongly coupled dusty plasmas can exhibit vortex-like dust fluid motions (or zonal winds) when subjected to large amplitude shear waves.

Based on the above motivation, the main purpose of the work is to find the exact solution to generalized Brinkman-type dusty fluid by utilizing the newly developed C-F time fractional derivative. The flow is bounded between two comparable plates. While the dusty fluid particles are dispersed equally in the Brinkman fluid. The fluid and dust particle equation is modeled separately which makes a two-phase flow model. The C-F time derivative is elaborate to make the system more generally. The exact solution is found by jointly using Laplace and finite sine Fourier transform. The Nusselt number and the collision of different embedded factors are discussed graphically by using Mathcad-15 software.

2. Mathematical formulation

Let us suppose the incompressible, unsteady flow of Brinkman type dusty fluid between two vertical and comparable plates. A dust particle is dispersed equally in the entire fluid. As illustrated in geometrical Fig. 1, the distance among two vertical and comparable plates is presented by d. The motion of fluid is taken on the x-direction. The uniform magnetic field and the flow field's direction are transverse to each other. The right plate remains stationary, and the left plate moves at a consistent velocity. $H(t)u_0$. The temperature of the left plate is $T_d + (T_w - T_d)At$ while T_d represents the temperature of the right plate.

Velocity and temperature are related to each other as we have [27,37]:

$$\rho \frac{\partial u(y,t)}{\partial t} + \rho\beta u(y,t) = \mu \frac{\partial^2 u(y,t)}{\partial y^2} + K_0 N_0\big(v(y,t) - u(y,t)\big) + \rho g \beta_T \big(T(y,t) - T_d\big) - \sigma B_0^2 u(y,t), \tag{1}$$

$$m\frac{\partial v(y,t)}{\partial t} = K_0\big(u(y,t) - v(y,t)\big), \tag{2}$$

$$\rho c_p \frac{\partial T(y,t)}{\partial t} = k\frac{\partial^2 T(y,t)}{\partial y^2}. \tag{3}$$

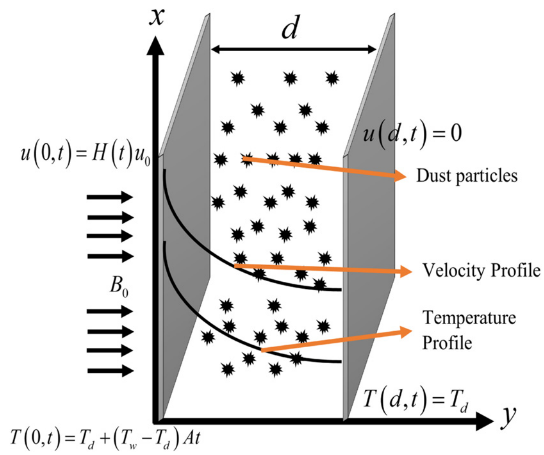

Fig. 1. Problem geometric view.

Using the conditions:

$$\left.\begin{array}{ll} u(y,0)=v(y,0)=0, & u(0,t)=H(t)u_0 \\ T(y,0)=T_d, & u(d,t)=0 \\ T(0,t)=T_d+(T_w-T_d)At, & T(d,t)=T_d \end{array}\right\}. \tag{4}$$

Since $H(T)$ represent the Heaviside step function while A is the constant with measurement T^{-1}. β represent the Brinkman fluid parameter and $K_\circ$ be the Stock resistance coefficient. Similarly, the following parameters represent C_p, ν, K, p, g, σ, $N_\circ$, B_T $B_\circ$, and A special heat capacity, kinematic viscosity, thermal conductivity, fluid density, gravitational acceleration, electrical conductivity, the density of dust particle magnetic field coefficient of thermal expansion, and variable temperature respectively.

According to the dimensions of variables:

$$\left.y^*=\frac{y}{d},\quad u^*=\frac{u}{u_0},\quad v^*=\frac{v}{u_0},\quad t^*=\frac{u_0}{d}t,\quad \theta^*=\frac{T-T_d}{T_w-T_d}\right\}. \tag{5}$$

Using Eq. (5) in Eq. (1), Eq. (2), Eq. (3) and Eq. (4) we conclude dimensionless form as:

$$\mathrm{Re}\,\frac{\partial u(y,t)}{\partial t}=\frac{\partial^2 u(y,t)}{\partial y^2}-Hu(y,t)+K\big(v(y,t)-u(y,t)\big)+Gr\theta(y,t), \tag{6}$$

$$\frac{\partial v(y,t)}{\partial t}=P_m\big\{u(y,t)-v(y,t)\big\}, \tag{7}$$

$$\frac{\partial \theta(y,t)}{\partial t}=Pe^{-1}\frac{\partial^2\theta(y,t)}{\partial y^2}, \tag{8}$$

With dimensionless physical conditions are:

$$\left.\begin{array}{ll} u(y,0)=v(y,0)=0, & T(y,0)=0 \\ u(0,t)=H(t), & u(1,t)=0 \\ T(0,t)=t, & T(1,t)=0 \end{array}\right\}. \tag{9}$$

$$M=\frac{\sigma B_0^2 d^2}{\rho\upsilon},\qquad Gr=\frac{g\beta_T d^2(T_w-T_d)}{u_0\upsilon},\qquad K=\frac{K_0N_0d^2}{\rho\upsilon},\qquad H=M+\lambda$$

$$Pe=\frac{\rho c_p u_0 d}{k},\qquad \lambda=\frac{\beta d^2}{\upsilon},\qquad \mathrm{Re}=\frac{u_0 d}{\upsilon},\qquad P_m=\frac{K_0 d}{mu_0},$$

where, Gr, M, K, Re, P_e, P_m, and λ are Grashof number, Magnetic parameter, Dusty fluid parameter, Reynold number, Peclet number, Particles mass parameter and Brinkman fluid parameter respectively.

Now using the CFD and fractional parameter α in equations (6), (7) and (8) respectively, we have:

$${}^{CF}D_t^{\alpha}\,\mathrm{Re}\,u(y,t)=\frac{\partial^2 u(y,t)}{\partial y^2}-Hu(y,t)+K\big(v(y,t)-u(y,t)\big)+Gr\theta(y,t), \tag{10}$$

$$\frac{1}{P_m}{}^{CF}D_t^{\alpha}v(y,t)=u(y,t)-v(y,t), \tag{11}$$

$${}^{CF}D_t^{\alpha}Pe\theta(y,t)=\frac{\partial^2\theta(y,t)}{\partial y^2}. \tag{12}$$

Here, ${}^{CF}D_t^{\alpha}$ represent CF for fractional operator and where α is fractional parameter [29]:

$$ {}^{CF}D_t^{\alpha} f(t) = \frac{N(\alpha)}{(1-\alpha)} \int_0^t \exp\left(-\frac{\alpha(t-\tau)}{(1-\alpha)}\right) f'(\tau) d(\tau), \tag{13} $$

where $N(\alpha)$ is the normalization function. Such that $N(0) = N(1) = 1$, $\alpha \in (0, 1)$.

3. Solutions of the problem

Using the finite Fourier sine transforms and joint Laplace transform we get the exact solutions of the system.

Temperature profile calculation: Using the Laplace transform on equation (12), we get:

$$ \frac{qg_0 Pe}{q+g_1} \bar{\theta}(y,q) = \frac{\partial^2 \theta(y,q)}{\partial y^2}. \tag{14} $$

Similarly, the transformed form of equation (9) is:

$$ \left.\begin{array}{ll} \bar{u}(y,q)|_{q=0} = \bar{v}(y,q)|_{q=0} = \bar{\theta}(y,q)\big|_{q=0} = 0 & \\ \bar{u}(0,q)|_{y=0} = H(q), & \bar{u}(1,q) = 0 \\ \bar{\theta}(0,q) = \dfrac{1}{q^2}, & \bar{\theta}(1,q) = 0 \end{array}\right\}. \tag{15} $$

If we multiply equation (14) by $\mathrm{Sin}(n\pi)$ on both sides and then take its integration and apply limits from 0 to d with y. Using initial and boundary conditions and then applying the finite then used the finite sine Fourier transform we get:

$$ \bar{\theta}_{FT}(\eta, q) = \frac{X_1}{q^2}\left(\frac{q+g_1}{q+X_2}\right). \tag{16} $$

Equation (16) can be written as:

$$ \bar{\theta}_{FT}(y,q) = \frac{g_1 X_1}{q^2 X_2} - \frac{X_1(g_1 - X_2)}{q X_2^2} + \frac{X_1(g_1 - X_2)}{X_2^2(X_2+q)}. \tag{17} $$

The inverse Laplace transform of equation (17) is given by:

$$ \theta_{FT}(y,t) = \frac{g_1 X_1}{X_2} t - X_3 + X_3 \exp(-X_2 t). \tag{18} $$

Here,

$$ g_0 = \frac{1}{1-\alpha}, \qquad g_1 = \frac{\alpha}{1-\alpha}, \qquad X_1 = \frac{n\pi}{Peg_0 + (n\pi)^2}, \qquad X_2 = \frac{(n\pi)^2 g_1}{Peg_0 + (n\pi)^2}, \qquad X_3 = \frac{X_1(g_1 - X_2)}{X_2^2}. $$

The inverse finite sine-Fourier transform to equation (18), we get the solution of temperature profile:

$$ \theta(y,t) = t(1-y) + 2\sum_{n=1}^{\infty} \left(X_3\big(1 + \exp(-X_2 t)\big) \sin(n\pi y)\right). \tag{19} $$

With the help of Laplace transform of equation (11) and using equation (15), we get:

$$ \frac{qg_0}{q+g_1} \bar{v}(y,q) = P_m\big(\bar{u}(y,q) - \bar{v}(y,q)\big). \tag{20} $$

As a result, we get after calculation:

$$ \bar{v}(y,q) = \left(\frac{q+g_1}{g_2(q+g_3)}\right) \bar{u}(y,q). \tag{21} $$

Here,

$$ g_2 = \frac{P_m + g_0}{P_m}, \qquad g_3 = \frac{g_1 P_m}{P_m + g_0}. $$

Solution of velocity profile: Applying the consequent transformation [38]:

$$ u(y,t) = \bar{\chi}(y,t) + (1-y)H(t). \tag{22} $$

By putting equation (22) into equation (10), we get:

$$\mathrm{Re}\left({}^{CF}D_t^{\alpha}\left(\bar{\chi}(y,t)-(1-y)H(t)\right)\right)=\frac{\partial^2}{\partial y^2}\left(\bar{\chi}(y,t)-(1-y)H(t)\right)+K\left(\frac{v(y,t)}{(\bar{\chi}(y,t)-(1-y)H(t))}-1\right)$$
$$\cdot\left(\bar{\chi}(y,t)-(1-y)H(t)\right)-H\left(\bar{\chi}(y,t)-(1-y)H(t)\right)+Gr\theta(y,t). \tag{23}$$

With transformed initial and boundary conditions are:

$$\bar{\chi}(0,t)=0, \qquad \bar{\chi}(y,0)=0, \qquad \bar{\chi}(1,t)=0. \tag{24}$$

By using Laplace transform to equations (23) and (24), we have:

$$\frac{\mathrm{Re}\, g_0 q}{q+g_1}\bar{\chi}(y,q)+\frac{\mathrm{Re}\, g_0 q}{q+g_1}(1-y)H(q)=\left\{\begin{array}{l}\dfrac{d^2}{dy^2}\bar{\chi}(y,q)+K\left\{\left(\dfrac{q+g_1}{g_2(q+g_3)}\right)-1\right\}\left(\bar{\chi}(y,q)+(1-y)H(q)\right)\\ -H\bar{\chi}(y,q)-H(1-y)H(q)+Gr\bar{\theta}(y,q)\end{array}\right\}. \tag{25}$$

Now applying finite sine Fourier transform to Eq. (25) we get:

$$\bar{\chi}_{FT}(y,q)=\left(\frac{q^2-g_4q-g_5}{q^2+X_4q+X_5}\right)(1-y)H(q)+\frac{Grg_2X_1}{q^2}\cdot\frac{(q+g_1)^2}{q+X_2}\cdot\frac{(q+g_3)}{q^2+X_4q+X_5}. \tag{26}$$

Equation (26) can be written as:

$$\bar{\chi}_{FT}(y,q)=\left\{1-\frac{X_8}{q+X_6}+\frac{X_9}{q-X_7}\right\}(1-y)H(q)-\left\{\frac{X_{10}}{q^2}-\frac{X_{11}}{q}-\frac{X_{12}}{q+X_2}-\frac{X_{13}}{q+X_6}-\frac{X_{14}}{q-X_7}\right\}, \tag{27}$$

where,

$$g_4=\frac{\mathrm{Re}\, g_0g_1g_2-Hb_1+K}{g_2(\mathrm{Re}\, g_0+H)}, \qquad g_5=\frac{Kg_1-Hg_2g_1g_3}{g_2(\mathrm{Re}\, g_0+H)}, \qquad g_6=GrX_1g_2,$$

$$b_1=g_2(g_1+g_3), \quad b_2=g_0g_1g_2, \quad b_3=g_1g_2g_3, \quad b_4=g_0g_2g_3, \quad b_5=g_1g_4, \quad b_6=g_1g_5g_6, \quad b_7=g_0g_2,$$

$$X_4=\frac{\mathrm{Re}\, g_0g_2g_3+(n\pi)^2b_1-g_1^2K-Hb_1}{\mathrm{Re}\, b_7+(n\pi)^2g_2-K-Hg_2}, \qquad X_5=\frac{(n\pi)^2b_5-b_6K-Hb_5}{\mathrm{Re}\, b_7+(n\pi)^2g_2-K-Hg_2},$$

$$X_6=\frac{X_4-\sqrt{X_4^2-4X_5}}{2}, \qquad X_7=\frac{X_4+\sqrt{X_4^2-4X_5}}{2}, \qquad X_8=\frac{g_4+g_5X_6-X_6^2}{X_6+X_7}, \qquad X_9=\frac{X_7^2+g_5X_7-g_4}{X_6+X_7},$$

$$X_{10}=\frac{g_6g_1^2g_3}{X_2X_6X_7}, \qquad X_{11}=\frac{g_6(g_1^2g_3(X_2X_7-X_2X_6+X_6X_7)-X_2X_6X_7(g_1^2+2g_1g_3))}{(X_2X_6X_7)^2},$$

$$X_{12}=\frac{g_6(g_1g_3(g_1-2X_1)+X_2^2(g_3-2g_1-1)-g_1^2X_7)}{X_2^2(X_2-X_6)(X_2-X_7)},$$

$$X_{13}=\frac{g_6(g_1g_3(g_1-2X_7)+X_7^2(g_3+2g_1-1)-g_1^2X_6)}{X_6^2(X_6-X_2)(X_6+X_7)},$$

$$X_{14}=\frac{g_6(g_1g_3(g_1+2X_7)+X_7^2(g_3+2g_1+1)+g_1^2X_7)}{X_7^2(X_2+X_7)(X_6+X_7)}.$$

The inverse Laplace transform of equation (27), as:

$$\bar{\chi}_{FT}(y,t)=\left\{\begin{array}{l}\left\{H(t)-H(t)*X_8\exp(-X_6t)+X_9H(t)*\exp(X_7t)\right\}(1-y)-X_{10}t\\ +X_{11}+X_{12}\exp(-X_2t)+X_{13}\exp(-X_6t)+X_{14}\exp(X_7t)\end{array}\right\}. \tag{28}$$

The inverse finite sine-Fourier transform of equation (28), as:

$$\bar{\chi}(y,t)=2\sum_{n=1}^{\infty}\left(\begin{array}{l}\left\{H(t)-X_8\exp(-X_6t)*H(t)+X_9\exp(X_7t)*H(t)\right\}(1-y)\\ -X_{10}t+X_{11}+X_{12}\exp(-X_2t)+X_{13}\exp(-X_6t)+X_{14}\exp(X_7t)\end{array}\right)\sin(n\pi y). \tag{29}$$

Using equation (29) in (22), we get:

$$u(y,t)=H(t)(1-y)+2\sum_{n=1}^{\infty}\left(\begin{array}{l}\left\{H(t)-X_8\exp(-X_6t)*H(t)+X_9\exp(X_7t)*H(t)\right\}(1-y)\\ -X_{10}t+X_{11}+X_{12}\exp(-X_2t)+X_{13}\exp(-X_6t)+X_{14}\exp(X_7t)\end{array}\right)\sin(n\pi y). \tag{30}$$

4. Special case: Brinkman fluid model in the non-existence of particle velocity

Eq. (6) can be converted into a single phase flow fluid model when we put $v(y,t)=0$.

$$\mathrm{Re}\,\frac{\partial u(y,t)}{\partial t}=\frac{\partial^2 u(y,t)}{\partial y^2}-K\big(u(y,t)\big)-Hu(y,t)+Gr\theta(y,t). \tag{31}$$

Used the definition of fractional derivatives ${}^{CF}D_t^{\alpha}$ on equation (31), we get;

$${}^{CF}D_t^{\alpha}\,\mathrm{Re}\,u(y,t)=\frac{\partial^2 u(y,t)}{\partial y^2}-K\big(u(y,t)\big)-Hu(y,t)+Gr\theta(y,t). \tag{32}$$

Later by using Laplace transform to Equations (32), we get:

$$\frac{\mathrm{Re}\,g_0 q}{q+g_1}\bar{\chi}(y,q)+\frac{\mathrm{Re}\,g_0 q}{q+g_1}(1-y)H(q)=\left\{\begin{array}{l}\dfrac{d^2}{dy^2}\bar{\chi}(y,q)-K\big(\bar{\chi}(y,q)+\big(1-y\big)H(q)\big)\\ -H\bar{\chi}(y,q)-H(1-y)H(q)+Gr\bar{\theta}(y,q)\end{array}\right\}. \tag{33}$$

With the finite sine Fourier transform of (33) and, we have:

$$\bar{\chi}_{FT}(y,q)=\frac{g_8}{q^2}\cdot\frac{(q+g_1)}{(q+X_2).(q+X_{26})}-(1-y)H(q).\frac{(q+g_7)}{(q+X_{26})}. \tag{34}$$

If we separate the RHS of separation of equation (34) by using partial fraction w.r.t. s, as a result:

$$\bar{\chi}_{FT}(y,q)=\left(\frac{X_{27}}{q}+\frac{X_{28}}{q^2}+\frac{X_{29}}{q+X_1}+\frac{X_{30}}{q+X_{26}}\right)-\left\{1-\frac{X_{26}-g_7}{q+X_{26}}\right\}(1-y)H(q), \tag{35}$$

where

$$X_{26}=\frac{g_1(n\pi)^2+g_1(K+H)}{\mathrm{Re}\,g_0+(n\pi)^2+K+H},\qquad g_7=\frac{g_1(K+H)}{\mathrm{Re}\,g_0+K+H},\qquad g_8=GrX_1,$$
$$X_{27}=\frac{g_8X_2-g_1g_8}{X_2^2(X_2-X_{26})}+\frac{g_8X_{26}-g_1g_8}{X_{26}^2(X_{26}-X_1)},\qquad X_{28}=\frac{g_1g_8}{X_1X_{26}},\qquad X_{29}=\frac{g_1g_8-g_8X_2}{X_2^2(X_2+X_{26})},\qquad X_{30}=\frac{g_1g_8-g_8X_{26}}{X_{26}^2(X_1+X_{26})}.$$

By applying joint Laplace and finite sine-Fourier inverse transform of equation (35) and putting in equation (22), we get;

$$u(y,t)=H(t)(1-y)+2\sum_{n=1}^{\infty}\left(\begin{array}{l}\big\{X_{27}+tX_{28}+X_{29}\exp(-tX_1)+X_{30}\exp(-tX_{26})\big\}\\ -(1-y)\big\{H(t)-(X_{26}-g_7)\exp(-X_{26}t)*H(t)\big\}\end{array}\right)\sin(n\pi y). \tag{36}$$

Nusselt number: Expressions of Nusselt number are achieved from Equations (19) by using the relation from Khan et al. [27].

$$Nu=-\left.\frac{\partial\theta(y,t)}{\partial y}\right|_{y=0}. \tag{37}$$

5. Results and discussion

The aim of this article is to study a generalized Brinkman fluid model with the unsteady flow of the viscoelastic vertical channel along with heat transfer and MHD effects. The joint Laplace transformed, and finite sine Fourier transforms are used to get the closed form of solutions. The effect of different parameters on temperature and velocity profiles of fluid and dust particles are depicted in the figures and tables.

Fig. 2 demonstrates the analysis of the Brinkman parameter and compares the Newtonian viscous fluid velocities with Brinkman-type fluid velocity. For $\lambda=0$ the system become Newtonian viscous fluid which has a high velocity as compared to the Brinkman type fluid. This indicate that the increasing value of Brinkman parameter retard the velocity profile more as compared to Newtonian viscous fluid. Because of higher values of Brinkman parameter strengthening the drag force, as a result the velocity of both fluid and dust particles and fluid retards.

Fig. 3 is investigated to study the impact of fractional parameter α on Brinkman and dusty fluid velocities. The memory effect of α is very strong on both velocities. The variation of fractional parameter α gives several solutions of dust particles and fluid velocities when the other parameters are fixed. Which is a good choice for experimental study to compared there results more accurate. The several solutions indicate that the fractional model is more realistic as compared to the classical one. Fig. 4 discussed the analysis of Gr on fluid and dust particle velocity. As a result it is cleared that when the Gr increase as the buoyancy forces increase. This rise is justified by the buoyancy forces in the fluid, which develop in response to the higher temperature and speed up the fluid's motion.

The Reynolds number is the ratio of inertial forces to viscous forces when a fluid experiences relative internal movement as a result of different fluid velocities. When Re increased the inertial force will be increased, which retard the viscous forces as a result the velocities profile of the fluid and dust particle decreases. The impact of Re is represented for both velocities in Fig. 5.

Fig. 6 gives the result as the effect of P_m on the dust particle and Brinkman-type fluid velocities. When P_m increasing the dust particles mass is increased and consequently, retardation appears in Brinkman-type fluid and dust particle velocity. Physically, the greater value of the mass particle parameter made fluid more viscous as result the retardation in both velocities appear

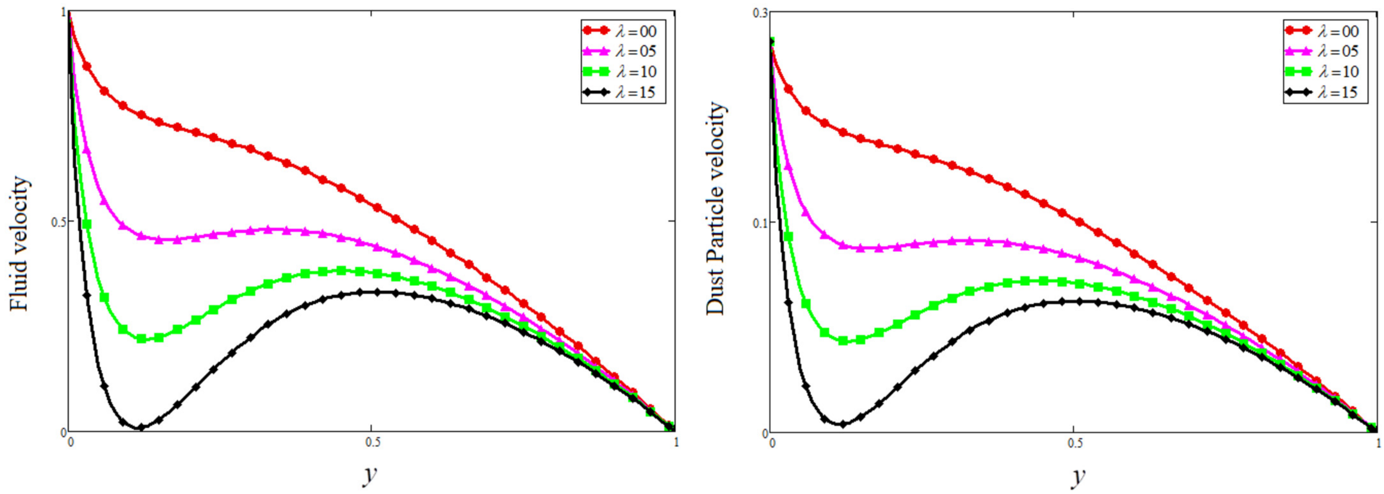

Fig. 2. Variation in fluid and dust particle velocities against Brinkman parameter.

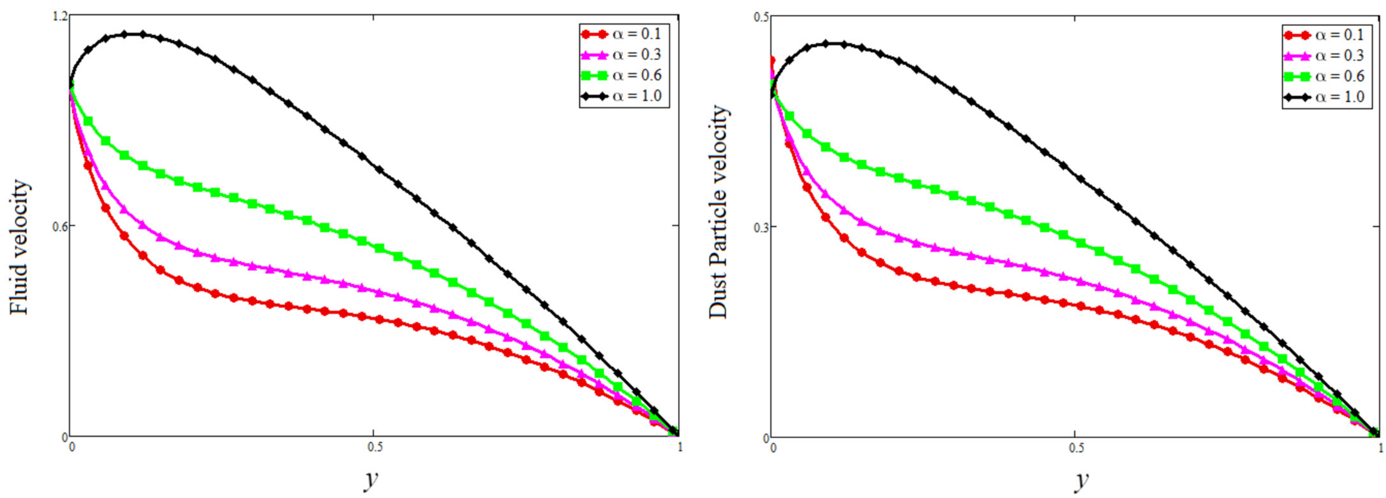

Fig. 3. Variation in fluid and dust particle velocities against fractional parameter.

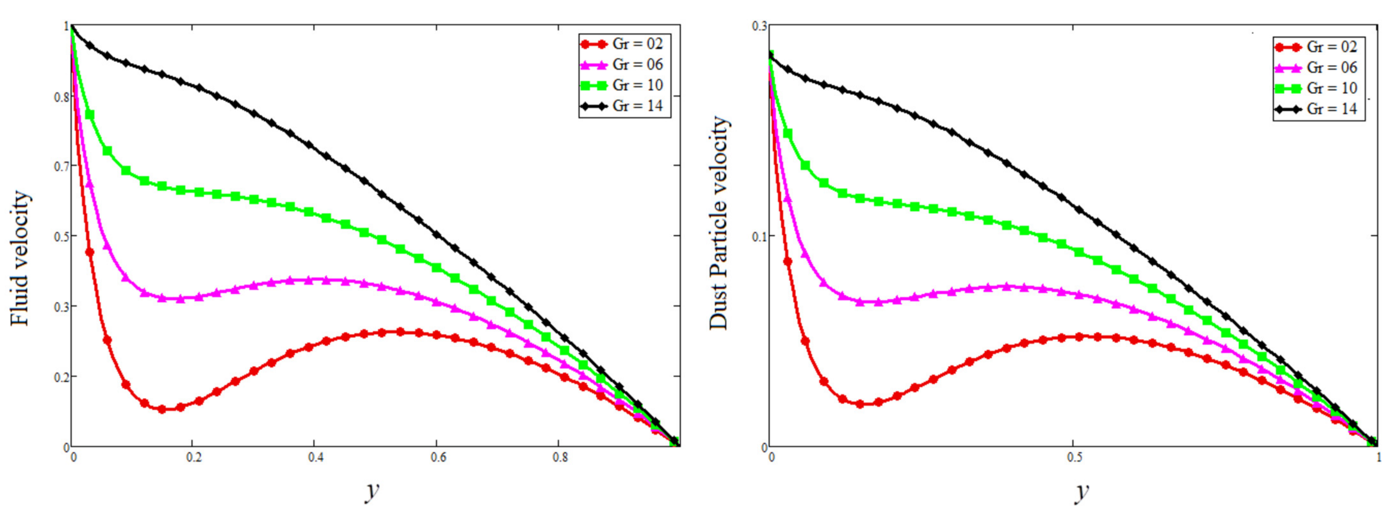

Fig. 4. Variation in fluid and dust particle velocities against Grashof number.

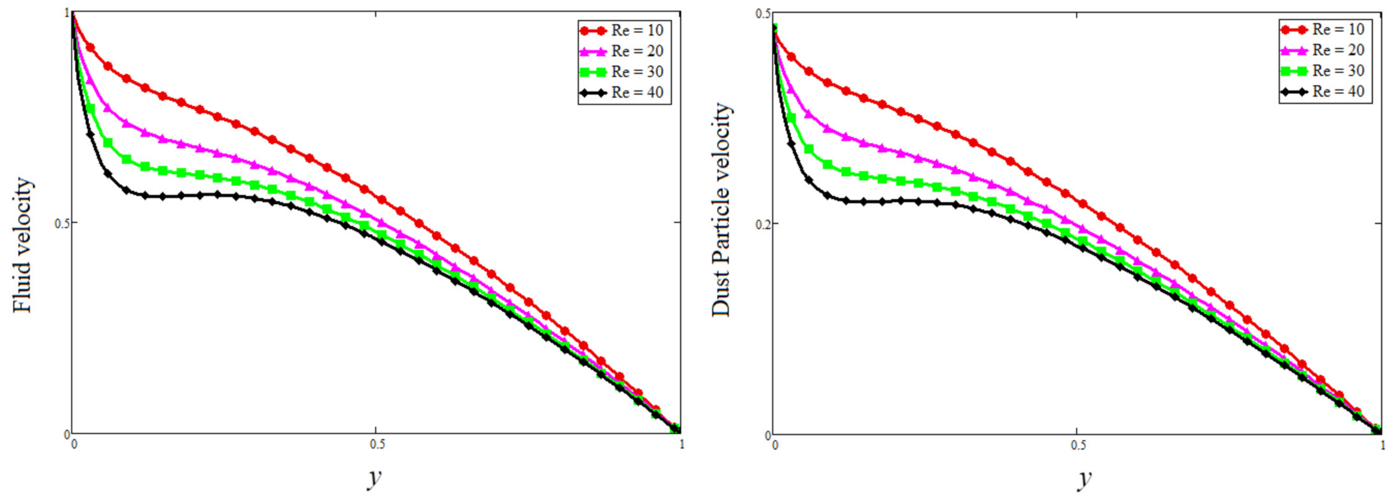

Fig. 5. Variation in fluid and dust particle velocities against Reynold number.

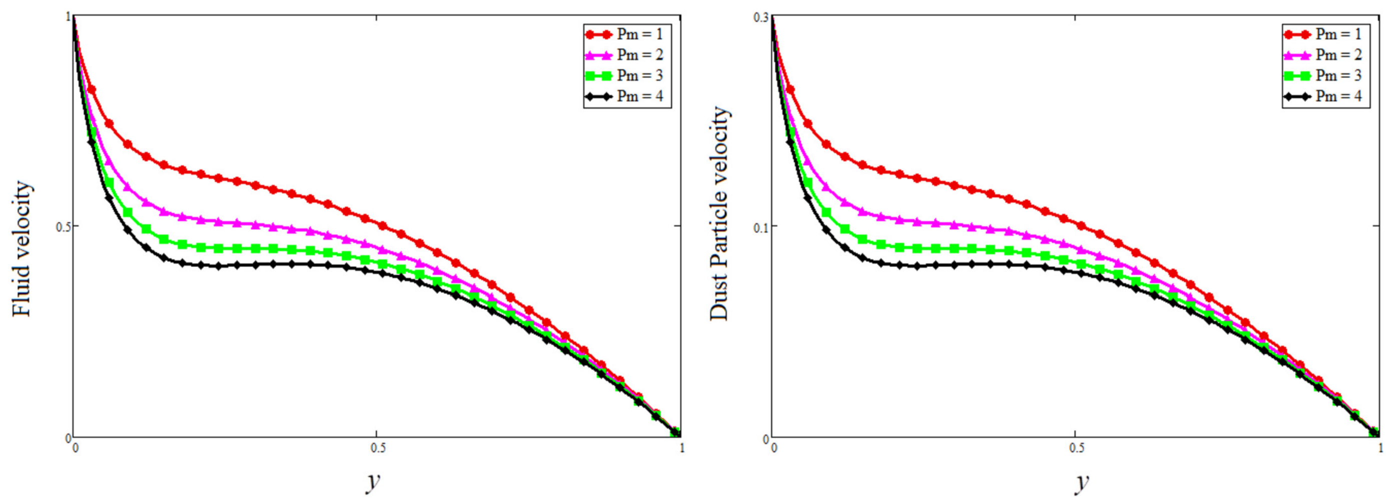

Fig. 6. Variation in fluid and dust particle velocities against Particles mass parameter.

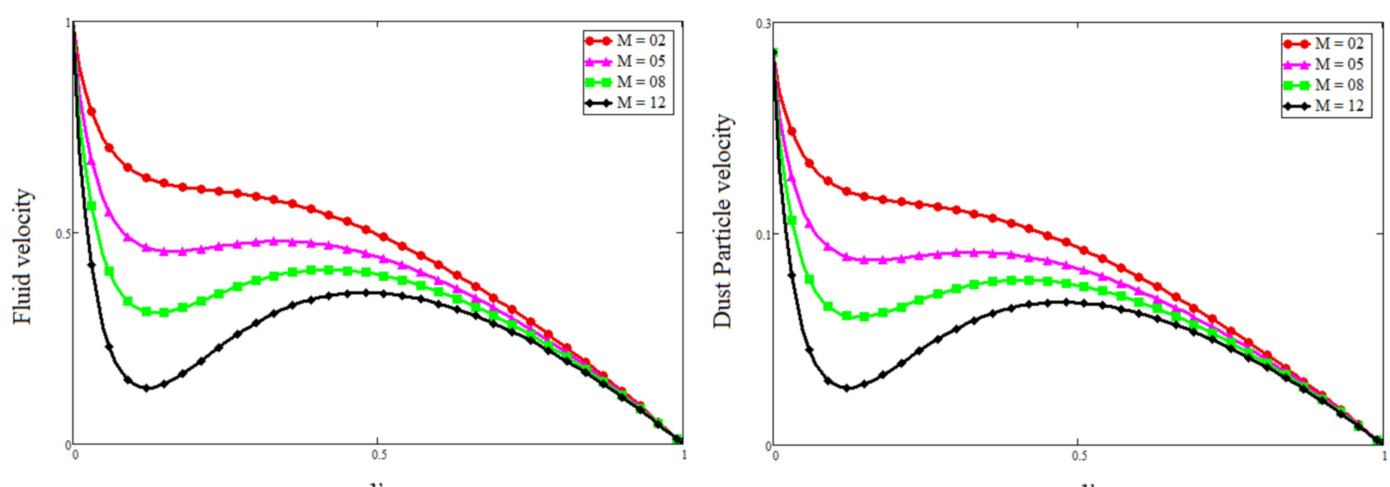

Fig. 7. Variation in fluid and dust particle velocities against magnetic parameter.

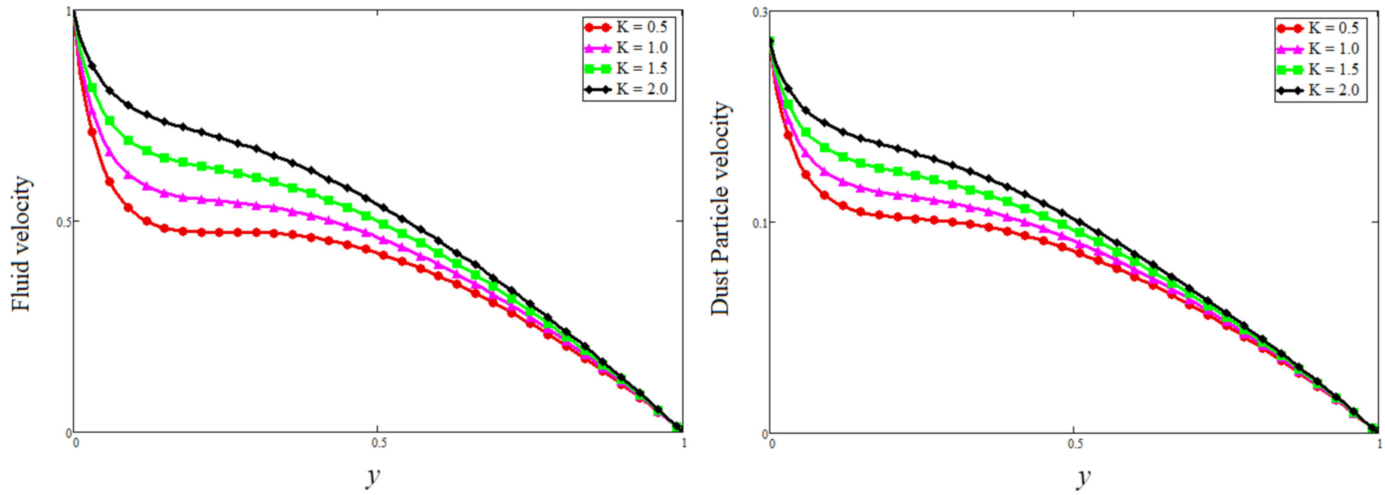

Fig. 8. Variation in fluid and dust particle velocities against Dusty fluid parameter.

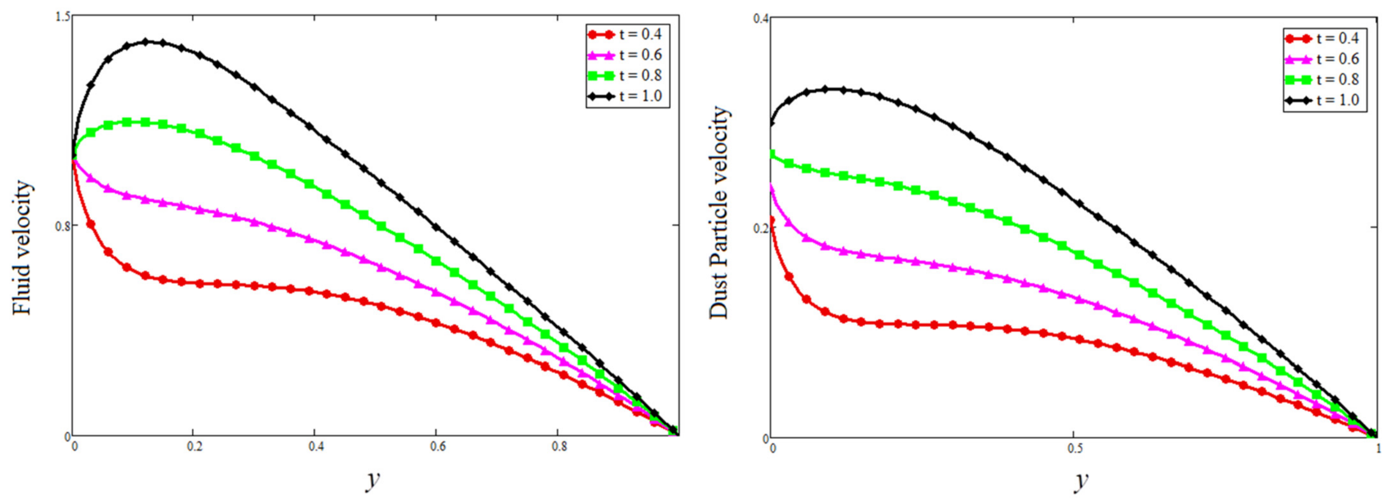

Fig. 9. Variation in fluid and dust particle velocities against time.

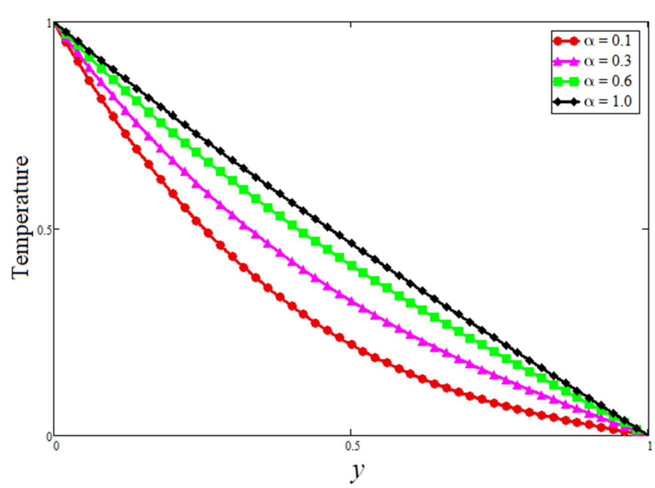

Fig. 10. Variation in temperature against fractional parameter.

The influence of M on Brinkman type fluid and dust particle velocity is shown in Fig. 7. The greater values of M improving the resistive forces to the fluid flow titled as Lorentz forces subsequently shrink the viscosity of the momentum boundary layer, which retard the Brinkman type fluid and dust particle velocity. By Stocks drag formula ($k = 6\pi r\mu$) for sphere dust particle, it is clear that the increase of dusty parameter, retard the viscous forces which result in the enhancement of fluid velocity. So, raising the number of dust particles increases the Brinkman-type fluid and dust particle velocities as shown in Fig. 8.

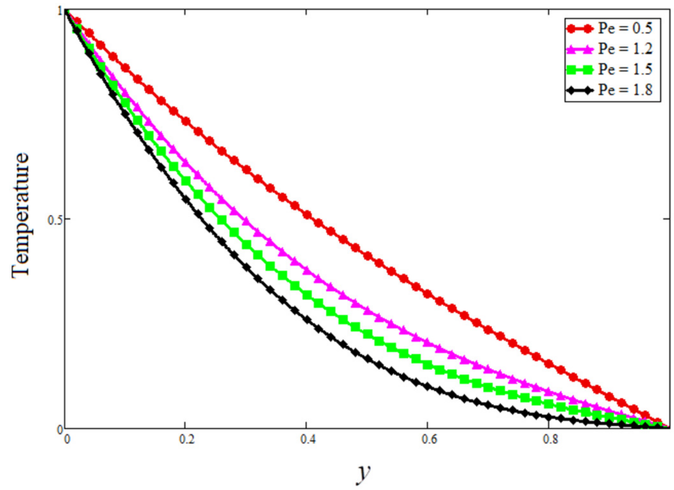

Fig. 11. Variation in temperature against Peclet number.

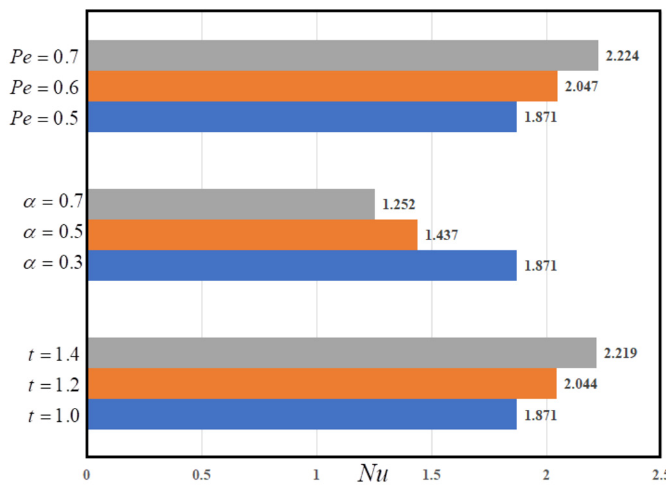

Fig. 12. Variation in *Nu* against *Pe*, α and t.

As this study is time dependent so the impact of time plays an important role in the analysis, for this purpose the impact of time t on Brinkman-type fluid and dust particle velocity is highlighted in Fig. 9. It is observed that, with the passage of time both velocities are enhanced, physically it clearly shows that the unsteady fluid is dependent on time. The impact of the fractional perimeter on the temperature profile is reported in Fig. 10. Which makes the heat profile more generalized as compared to the classical one by fixing the other parameters. By varying the fractional parameter gives more solution to the temperature profile, which makes it more realistic the temperature profile. The collision of *Pe* on temperature profile is reported in Fig. 11. As *Pe* show the ratio of viscous to thermal forces, because of the extreme predominance of viscous forces, the increase to *Pe* the increase viscous forces, which decrease the temperature profile. Heat transfer has a vital role in fluid problems, for this purpose, the rate of heat transfer is highlighted against different parameters in Fig. 12. It is clear that the heat transfer rate is increasing with the passage of time and varying the value of t, while the fractional parameter generalizes the speed of heat transfer. Which is good for an experimentalist to compare with real data.

6. Conclusions

The main theme of this paper is to investigate the generalized Brinkman type dusty fluid with free convection temperature. The flow is in between two parallel plates while the dust particles disperse equally in the entire fluid. The problem is formulated with the help of PDE along with physical boundary conditions. A fractional operator that is Caputo Fabrizio's is used to generalize the problem without a singular kernel and solved via the joint application of Laplace transform and finite sine Fourier transform. Mathcad-15 software is used to conduct parametric studies. From the current solutions, some special cases for the solutions' validity and accuracy have been discovered. The major key points from this study are summarized as follows:

- The fractional model of Brinkman type dusty fluid is more realistic feature as compared to the classical one.
- The fractional operator is the best choice for experimentalists for curve fitting data.
- Both the velocities are enhanced with the enhancement of $\alpha, t \text{and} Gr$.
- With the increasing value of K, Re, M, *Pm* and λ, both the velocities are decreasing.
- The temperature profile can be controlled by raising the value of *Pe*.
- The fractional operator generalized the heat transfer rate which make a realistic feature.

- The heat transfer rate is boosted with the greater value of Pe and t.

CRediT authorship contribution statement

Dolat Khan: Writing – original draft, Software, Formal analysis, Data curation, Conceptualization. **Subhan Ullah:** Writing – review & editing, Writing – original draft, Investigation. **Poom Kumam:** Validation, Supervision, Conceptualization. **Wiboonsak Watthayu:** Validation, Supervision, Methodology. **Zafar Ullah:** Writing – review & editing, Writing – original draft, Funding acquisition, Data curation. **Ahmed M. Galal:** Formal analysis, Methodology, Writing – original draft.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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