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A new decision analysis based on 2-tuple linguistic q-rung picture fuzzy ITARA–VIKOR method

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Abstract

The primary goal of the current work is to investigate and improve the ITARA (Indifference Threshold-Based Attribute Ratio Analysis) and VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) techniques using 2-tuple linguistic q-rung picture fuzzy sets (2TLq-RPFSs) as a backdrop. The terminology of VIKOR method to define the compromise solution is based on the key ideas of regret measure and group utility measure. The VIKOR technique uses ascending order to evaluate the options according to their potential for the highest group benefit and the least potential for individual opponent regret. The ITARA method is expanded for 2TLq-RPFSs. To create the innovative approach known as 2TLq-RPF-ITARA–VIKOR for 2TLq-RPFSs, we combine the ITARA method with VIKOR. Our motivation for using the concept of 2TLq-RPFSs is the capacity of q-RPFSs in order to support a broad range of information and the prominent properties of 2-tuple linguistic term sets to handle qualitative data. The proposed methodology accommodates the salient features of ITARA and VIKOR techniques. Moreover, the integration of two methods covers the weaknesses of each other. The expanded approach can contain significantly more data to represent information by increasing the powers of q. Additionally, utilization of linguistic factors provides the benefit to present the data with the aid of qualitative characteristics. In order to support the expanded approach, an example of choosing the best audit company is handled using the suggested methodology. In the end, a comparison analysis is conducted to validate the established approach. Therefore, we draw the conclusion that the methodology is general, simple to understand, easy to apply, and capable of capturing the linguistic multi-criteria group decision-making issues that are uncertain.

Keywords Decision-making · q-Rung picture fuzzy set · 2-Tuple linguistic term set · VIKOR.

1 Introduction

Due to the growing complexity in decision-making (DM) processes, experts seek to present their perspectives using hazy and ambiguous data. Zadeh (1965) laid the groundwork for an incredible theory, called fuzzy set (FS) theory (defined by a membership function μ) in order to better reflect the opinions of experts when faced with ambiguous and unclear information. Later, researchers developed some extensions of FSs that are very helpful for addressing multi-criteria group decision-making (MCGDM) issues. For example, Atanassov (1986) offered the concept of intuitionistic FS (IFS), Yager (2013a, b) introduced the idea of Pythagorean FS (PyFS), Senapati and Yager (2020) created the notion of Fermatean FS (FFS), and Yager Yager (2016) proposed the idea of q-rung orthopair FS (q-ROFS) by introducing a negative membership grade (ν) having the conditions, $\mu + \nu \leq 1$, $\mu^2 + \nu^2 \leq 1$, $\mu^3 + \nu^3 \leq 1$, and $\mu^q + \nu^q \leq 1$ ($q \geq 1$), respectively. Picture FS (PFS) was invented by Cuong (2014) by taking into account a neutral membership grade (γ) to obtain more accurate evaluation data. With the condition $\mu + \nu + \gamma \leq 1$, a PFS can accept three membership degrees: positive membership (PM), neutral membership (NM), and negative membership (N_eM). Gündoğdu FK, Kahraman C, (2019) proposed the idea of

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spherical FS (SFS) ($\mu^2+\nu^2+\gamma^2 \leq 1$), and studied the SF-TOPSIS technique. Mahmood et al. (2019) demonstrated a medical diagnosis problem under the background of SFSs. The idea of the q-rung PFS (q-RPFS), in which Yager's q-ROFS is used ($\mu^q+\nu^q+\gamma^q \leq 1, q \geq 1$), was first proposed by Li et al. (2018), to discuss an example of the selection of enterprise resource planning system. Liu et al. (2020) proposed some q-RPF Yager aggregation operators (AOs) and discussed two MCDM problems, including the selection of emerging technology enterprises and the selection of suitable companies for investment.

To assess the choices in DM situations, specialists often use language phrases like "good", "bad", "excellent", and "poor". Zadeh (1975) presented the idea of linguistic variables as a way to more effectively communicate human perspectives in DM. He proposed the concept of linguistic variables and emphasized that linguistic words may effectively describe linguistic information. Thanks to linguistic factors, experts can express their ideas more precisely without sacrificing any information. The 2-tuple linguistic term set (2TLTS) was established by Herrera and Martínez (2000a, b) to better convey judgement and assessment information in MCGDM situations. To more effectively convey the assessment information in DM problems, 2TLTS is a worthwhile idea. The researchers focused their attention on it because of these characteristics. Numerous researchers have given the 2TLTS a lot of consideration, and it has emerged a crucial strategy to address the linguistic DM issues. Xu (2012) worked on linguistic decision-making. Researchers have used this idea, such as Zhang (2014) presented the notion of linguistic IFSs. Garg (2018) introduced the concept of linguistic PyFSs (LPyFSs) and presented certain AOs for LPyFSs. In addition, Garg solved a real-world problem to depict the applicability and usefulness of the proposed AOs. Lin et al. (2020) introduced the idea of linguistic q-ROFS (Lq-ROFS). More information can be found by reading the Deng et al. (2018); Lin et al. (2018). The linguistic SF AOs were put forth by Jin et al. (2019) and used to address a problem. The 2-tuple intuitionistic fuzzy (2TIF) linguistic AOs were proposed by Wei (2019). Numerous AOs have been developed by researchers in linguistic environments. Lq-ROFSs served as the backdrop for the work done on Einstein AOs by Akram et al. (2021). Akram et al. (2023) developed an expanded version of MULTIMOORA (Multi-Objective Optimization Ratio Analysis plus Full Multiplicative Form) for 2-tuple linguistic PyFSs (2TLPyFSs). An MCGDM approach was expanded inside a framework, and its applicability was used to show by Akram et al. (2022). Akram and Niaz (2022) created an integrated version of COCOSO (Combined Compromise Solution) and CRITIC (Criteria Importance Through Inter-criteria Correlation) under the 2TL Fermatean FSs and practiced their extended strategy on a problem related to the Drip Irrigation System Analysis. Khan et al. (2023) improved the MULTIMOORA methodology for choosing an appropriate mining strategy in a 2T spherical fuzzy linguistic environment. Naz et al. (2022) presented a modified version of evaluation based on distance from average solution (EDAS) method under the background of 2TL-T-SFSs (2TL-T-SFSs). Recently, Akram et al. (2023) presented the concept of 2TLq-RPFSs and used it to find the solution of a MCGDM problem. For further study, the readers are referred to Akram et al. (2022, 2023); He et al. (2019a); Liu et al. (2022); Naz et al. (2023, 2022).

This article will address the following major research questions:

1. What is the significance of the presented method, or why do we study it?
2. Why did we integrate the ITARA–VIKOR method for 2TLq-RPFSs or why do we need this extended method?
3. How to implement the proposed strategy to manage the real-life decision-making issues using uncertain linguistic information?

1.1 Literature review

Scholars and researchers have developed a range of decision-making (DM) approaches throughout the years to address challenging real-world issues, such as TOPSIS Hwanga and Yoon (1981), VIKOR Opricovic (1998), ELECTREE Benayoun et al. (1966), etc. The primary goal of these techniques is to identify solutions of DM issues. Alternatives, criteria, expertise, and the choice of beginning data are the most crucial elements in these situations. These methods were initially developed to address issues with crisp data. The lack of ability to address the ambiguity and imprecision of real-world situations is a prevalent weakness among these systems. Bellman and Zadeh (1970) proposed a DM approach based on a fuzzy environment to address this problem. Later, a variety of DM strategies were put forth to deal with data uncertainties in FS theory and its related expansions. Opricovic (1998) introduced the VIKOR approach for the multiple-criteria optimization of challenging situations. This multi-attribute decision-analysis approach methodology concentrates on the compromise solution (CS), the compromise ranking list, and the range of strategy weight. It is based on the multi-attribute ordering index, which gauges how closely potential courses of action or substitutes resemble the ideal course of action. The following form of L_p metric (Eq. 1) was used as the foundation for the VIKOR method:

$$L_{p,k} = \left\{ \sum_{j=1}^{n} \left[W_j \left(\frac{f_j^{'} - f_{kj}}{f_j^{'} - f_j^{''}} \right) \right]^p \right\}^{\frac{1}{p}}, \quad 1 \leq p \leq \infty; \quad k = 1, 2, 3, \ldots, n. \tag{1}$$

Opricovic (2009) study on the development of a reservoir system for storing surface flows of the Mlava river (located in Serbia) and its tributaries for regional water supply. The author used the VIKOR approach to arrive at a compromise solution (CS) in the planning of water resources. Opricovic and Tzeng (2004) compared the VIKOR and TOPSIS methods. An expanded version of the VIKOR methodology was developed by Opricovic and Tzeng (2007) by showing its comparisons with three outranking approaches to evaluate alternative hydropower systems on the Drina River (which forms a large portion of the border between Bosnia, Herzegovina, and Serbia). Chang and Hsu (2011) suggested an enhanced model of the VIKOR approach to assess the Tseng-Wen reservoir watershed in southern Taiwan with regard to the spatial variability of its environmental characteristics. El-Santawy (2012) used the VIKOR technique to overcome the assessment issue associated with personnel training. Sanayei et al. (2010) demonstrated a hierarchy-based MCDM fuzzy VIKOR method to demonstrate the suitable suppliers in the supply chain system. Lihong et al. (2008) presented an improved VIKOR algorithm to evaluate the thermal power enterprise coal supplier and utilized the Shannon Entropy combined Analytic Hierarchy Process to compute the weights. Shemshadi et al. (2011) used the entropy measure information concept to adapt the VIKOR technique in a fuzzy environment and put it into practise to assess the suppliers. The expanded VIKOR approach was proposed by Yang and Pang (2019) by employing the hesitant interval valued PyF information and solved a real-world problem related to the metro project risk assessment to illustrate the practicality of their introduced methodology. The authors considered five alternatives on the basis of four criteria: namely, technical risk, policy risk, environmental risk and financial risk. In order to accommodate the dual information of human thought, Shumaiza et al. (2019) introduced the bipolar fuzzy VIKOR approach, in which they employed trapezoidal bipolar fuzzy numbers for the evaluation of available alternatives. In their study, the utilization and significance of their work was discussed by solving two numerical examples, namely, the selection of waste treatment method and the site selection to plant a thermal power station. An interval 2TL-VIKOR approach was given by Liu et al. (2013) and used to address a problem in which an organization needs to select a suitable material for an automative component, instrument panel. Personnel selection is a very important activity in the human resource management of any organization. The interval 2TL-VIKOR approach for personnel selection in a tertiary care hospital was put out by Liu et al. (2015). Wei et al. (2020) proposed an MCGDM VIKOR approach in a 2TL neutrosophic setting and discussed a numerical example for the selection of a green supplier for a company. He et al. (2019b) created the Pythagorean 2TL-VIKOR technique and investigated a construction project management issue. Building green airports can be considered among the most promising routes to sustainable development of ecosystems and human health. An expanded version of the VIKOR method was introduced by Xiong et al. (2022) by considering an example of green air port selection. Mi et al. (2019) merged the terminology of the VIKOR method with the "best worst method" in the q-ROF environment. From the studies (Khan et al. 2021; Arya and Kumar 2021; Gül 2021), more developments of the VIKOR approach and their applications can be studied. The TOPSIS and VIKOR methodologies for q-RPFSs were examined by Akram and Shumaiza (2021) by implementing them on two real life problems. The authors practised the methods they presented to select the right housing society for investment and the right industrial robot. In a study conducted by Yang et al. (2022), an integrated version of CRITIC and VIKOR methods was created in a q-ROF setting and its application was studied by applying the "Internet of Things" to "sustainable supply chain management".

1.2 Motivations and research gaps

The motivations and research gaps for the suggested work can be summed up as follows:

- The FS theory, developed by Zadeh (1965), is a valuable idea for representing the ambiguities and uncertainties that arise in human viewpoints. Researchers and scholars have developed several useful FS extensions throughout time, including IFSs, PyFSs, q-ROFSs, PFSs, SFSs, q-RPFSs, etc. The idea of linguistic variables was presented by Zadeh (1975). Eventually, a number of extensions to the fuzzy linguistic method were developed, including LIFSs, LPyFSs, 2TL sets, 2TLIFSs, 2TLPyFSs, 2TLq-RPFSs, etc. 2TLq-RPFSs support the essential features of 2TLTSs and q-RPFSs. A 2TLq-RPFS is designed to address the deficiencies in the current sets by taking into account the degree of PM, NM, and N_eM. The 2TLq-RPFSs greatly expand the applicability of PM, NM, and N_eM and provide experts with a freedom for how to communicate their decision-making knowledge by letting them choose the values of the parameter q. We choose this model for dynamic decision making because it is widely used, covers a larger area, has a strong framework, and offers modeling benefits.
- Hatefi (2019) introduced the ITARA approach and demonstrated its clear advantage over other weighting techniques. From an initial decision matrix, the ITARA technique immediately determines the relevance of semi-objective quantitative criteria. This straightforward and logical approach gives the criteria weight. Additionally, it creatively makes use of the idea of the indifference threshold. The ITARA approach for probabilistic hesitant

FSs was created by Liu et al. (2022) and merged with other methodologies. Simic et al. (2023) investigated the sustainable route selection of petroleum transportation using the ITARA–EDAS approach based on type-2 neutrosophic numbers. According to Lo et al. (2021), the ITARA and TOPSIS approaches were combined and used to handle a DM issue. Researchers coupled the ITARA approach with several MCDM methodologies that may be explored in Ulutaş et al. (2020); Simic et al. (2022). Additionally, the ITARA technique has been used to assess the significance of criteria in a variety of MCDM issues, including the evaluation of energy poverty (Hasheminasab et al. 2023), material selection (Sofuoğlu 2019), load response potential for distribution networks (Du et al. 2020), rehabilitation of water distribution networks (Elshaboury et al. 2020), and supplier evaluation (Chang et al. 2021). As a result, the ITARA approach has not yet been extended to the 2TLq-RPF environment. As a consequence, this study offers the 2TLq-RPF-ITARA approach to identify the criterion weights in our current MCGDM problem in order to fill the gaps in previous studies.

- In order to pick the best emergency alternative for the fire that occurred in China, Ju and Wang (2013) developed an enhanced version of the VIKOR approach using 2-tuple linguistic data. Liao et al. (2014) introduced a unique VIKOR approach for hesitant fuzzy linguistic term sets. To demonstrate the benefits of the suggested research, the writers gave some examples. Liu et al. (2014) addressed a situation involving site selection in waste management by using a linguistic variant of the VIKOR technique. To address MCDM concerns, many upgraded and expanded versions of the linguistic VIKOR technique, such as supplier selection (You et al. 2015; Wen et al. 2020) and smart healthcare (Gou et al. 2021), have been presented. The VIKOR method technique has exceptional potential for making the best option possible while taking all relevant factors into account. The VIKOR technique is also keen to provide a compromise solution with a policy that maximizes collective utility and minimizes individual regret of the opponent. We intend to extend the VIKOR approach in our work because of the VIKOR method aforementioned qualities. Moreover, there is currently no extension of the VIKOR technique for 2TLq-RPFS. As a result, we made the decision to build its extension for 2TLq-RPFSs.
- Our mathematical approach, 2TLq-RPF-ITARA–VIKOR, is created in response to the conceptual framework of the VIKOR method, the veracity of its findings, and the wider context of 2TLq-RPFSs. It combines the adaptable nature of 2TLq-RPFSs with the findings related to the methodology of ITARA and VIKOR methods in order to solve the MCGDM issues for the primary model.
- To ensure that all departments are using a documented method of documenting transactions, an audit is the examination or inspection of several books of accounts by an auditor, followed by a physical check of inventories. Via various audit organizations or firms, different sectors (private and government) carry out their audit to verify the correctness of financial accounts. From this point on, selecting the finest audit firm is crucial for every organization. That might be considered an important MCGDM problem. Therefore, we decided to use the method we shall discuss to take it into consideration.

The ITARA and VIKOR methods have not been utilized for solving any problem related to the selection of the best firm. Also, no study has combined these two remarkable decision-making techniques into a single framework for the 2-tuple linguistic q-rung picture fuzzy environment. In order to fill the gaps in the literature, this study suggests a method based on the 2TLq-RPF-ITARA–VIKOR for choosing the best audit firm.

1.3 Contributions

In order to arrive at a final choice, this study focuses on creating a more straightforward and effective MCGDM methodology called the 2TLq-RPF-ITARA–VIKOR method. The ITARA technique is used in the proposed methodology to determine the values of the criterion weights. The developed method applies the 2TLq-RPF weighted averaging (2TLq-RPFWA) operator to aggregate the decision outcomes. This study's key contributions are stated as follows:

- The first contribution focuses on the creation of an expanded VIKOR method to address the unclear information that arises in practical issues. As a result, we are able to effectively construct an improved VIKOR method to handle the 2TLq-RPF information.
- The second contribution is concerned with improving the ITARA method for determining the significance of the criteria. In order to calculate the weights of the criteria, we therefore designed an expanded ITARA approach.
- The third contribution is devoted to providing a 2TLq-RPF decision environment with an integrated ITARA–VIKOR structure that incorporates the key characteristics of 2TLq-RPFS. In order to address the MCGDM difficulties, we finally created the 2TLq-RPF-ITARA–VIKOR approach.
- The feasibility of the proposed 2TLq-RPF-ITARA–VIKOR methodology is demonstrated through a numerical example involving the selection of the best audit company to conduct the audit. The final results are also checked through a parameter analysis.

- The complete procedure of the presented method is elaborated via a diagram.
- The comparative study through different techniques, namely, the 2TL-T-SF-EDAS method, the q-RPF-VIKOR method, and the q-ROF-CRITIC-VIKOR method, is conducted to verify the validity of the proposed strategy.

1.4 Novelty of the presented work

In this article, we build an expanded VIKOR technique using the idea of 2TLq-RPFSs. Researchers have only used the ITARA approach to a small number of MCGDM issues, and it has not yet been expanded to 2TLq-RPFSs. To establish the criterion weights, we use the ITARA concept. In addition, we propose the ITARA strategy for the 2TLq-RPF setting. This research is the first to offer a comprehensive framework for the MCGDM using the ITARA and VIKOR methods within the 2TLq-RPF framework. Through a diagram, we demonstrate the key steps of the developed strategy. Additionally, we take into account a problem that involves choosing the best company to audit a certain institution and use the 2TLq-RPF-ITARA–VIKOR approach to resolve it. We have highlighted the consequences of our suggested strategy to explain its importance, dominance, and authenticity in comparison to other decision-making techniques already in use. In the last years, various extended versions of the VIKOR method were established by the researchers, such as the hesitant fuzzy linguistic VIKOR method (Liao et al. 2014), 2-tuple linguistic VIKOR method (Ju and Wang 2013), linguistic VIKOR method (You et al. 2015; Gou et al. 2021; Wen et al. 2020), q-ROF-CRITIC-VIKOR method (Yang et al. 2022), q-RPF-VIKOR method (Akram and Shumaiza 2021), etc. The aforementioned methodologies of VIKOR contributed in numerous fields, but none of them can manage the 2TLq-RPF data. The novel elements of the work that is being presented are as follows: (1) The 2TLq-RPFSs structure is flexible and can handle a variety of data. Hence, in our study, we decide to use this structure. (2) Even so, several scholars have examined and published expanded versions of the ITARA and VIKOR approaches. The literature does not, however, contain any study that combines these two techniques. In order to address real-world issues, we are inclined to provide the 2TLq-RPF-ITARA–VIKOR technique. (3) In order to solve an issue, we apply the extended technique, which further transforms the expert assessments into 2TLq-RPF numbers. We described the detailed steps and the required computations for the DM process.

1.5 Paper structure

This article presentation may be summed up as follows: The fundamental ideas pertaining to 2TL term sets and q-RPFSs are summarized in Sect. 2. The 2TLq-RPF-ITARA–VIKOR technique and a thorough flowchart for the MCGDM difficulties are shown in Sect. 3. In Sect. 4, the suggested technique is applied to an example involving the choice of the best firm for an audit. Along with the analysis of the suggested approach, Sect. 5 compares the created procedure with a few other widely used current methodologies. Sect. 6 presents the discussion and the implications of the study. The conclusion is presented in Sect. 7 at the end.

2 Preliminaries

This section goes through a few fundamental 2TLq-RPFSs ideas to aid the readers in understanding the suggested technique.

Definition 1 Akram et al. (2023) A 2TLq-RPFS A $(q \geq 1)$ is defined as:

$$A = \{\big(g, ((\hbar_\alpha(g), E(g)), (\hbar_\beta(g), F(g)), (\hbar_\gamma(g), G(g)))\big) : g \in K\}, \tag{2}$$

where $(\hbar_\alpha(g), E(g))$, $(\hbar_\beta(g), F(g))$ and $(\hbar_\gamma(g), G(g))$ are the degrees of PM, NM, and N_eM, respectively. A 2TLq-RPFS should satisfy the following conditions:

$$\hbar_\alpha(g), \hbar_\beta(g), \hbar_\gamma(g) \in A, \quad E(g), F(g), G(g) \in [-0.5, 0.5), \tag{3}$$

$$\begin{aligned} &0 \leq \Lambda^{-1}(\hbar_\alpha(g), E(g)) \leq \Theta + 1, \\ &0 \leq \Lambda^{-1}(\hbar_\beta(g), F(g)) \leq \Theta + 1, \\ &0 \leq \Lambda^{-1}(\hbar_\gamma(g), G(g)) \leq \Theta + 1, \end{aligned} \tag{4}$$

$$0 \leq \Big(\Lambda^{-1}(\hbar_\alpha(g), E(g))\Big)^q + \Big(\Lambda^{-1}(\hbar_\beta(g), F(g))\Big)^q + \Big(\Lambda^{-1}(\hbar_\gamma(g), G(g))\Big)^q \leq (\Theta + 1)^q. \tag{5}$$

For simplicity, we call $A = \big((\hbar_\alpha, E), (\hbar_\beta, F), (\hbar_\gamma, G)\big)$, a 2TL$q$-RPF number (2TL$q$-RPFN) with $0 \leq \Lambda^{-1}(\hbar_\alpha, E) \leq \Theta+1$, $0 \leq \Lambda^{-1}(\hbar_\beta, F) \leq \Theta+1$, $0 \leq \Lambda^{-1}(\hbar_\gamma, G) \leq \Theta+1$, $0 \leq \big(\Lambda^{-1}(\hbar_\alpha, E)\big)^q + \big(\Lambda^{-1}(\hbar_\beta, F)\big)^q + \big(\Lambda^{-1}(\hbar_\gamma, G)\big)^q \leq (\Theta + 1)^q$.

Definition 2 Akram et al. (2023) Let a, b, c denote the values obtained after performing an operation on the LTs' indices in a LTS S with $a, b, c \in [0, \Theta]$. In addition, suppose that

$$\alpha = \text{round(a)}, \quad \beta = \text{round(b)}, \quad \gamma = \text{round(c)}, \quad \text{and} \tag{6}$$

$$E = a - \alpha, \quad F = b - \beta, \quad G = c - \gamma, \tag{7}$$

where, $E, F, G \in [-0.5, 0.5)$, then E, F, G are the values of symbolic translation.

Definition 3 Akram et al. (2023) Let $S = \{\hbar_k : k = 0, 1, 2, \ldots, \Theta\}$ represents a LTS and $a, b, c \in [0, \Theta]$. Then the function $\Lambda : [0, \Theta] \rightarrow S \times [-0.5, 0.5)$ to acquire the 2TL information (equivalent to a, b, c) can be defined as:

$$\Lambda(a)=\begin{cases}\hbar_\alpha, & \alpha=\text{round(a)}\\ E=a-\alpha, & E\in[-0.5,0.5),\end{cases}\tag{8}$$

$$\Lambda(b)=\begin{cases}\hbar_\beta, & \beta=\text{round(b)}\\ F=b-\beta, & F\in[-0.5,0.5),\end{cases}\tag{9}$$

$$\Lambda(c)=\begin{cases}\hbar_\gamma, & \gamma=\text{round(c)}\\ G=c-\gamma, & G\in[-0.5,0.5),\end{cases}\tag{10}$$

Definition 4 Akram et al. (2023) Let $S=\{\hbar_k : k=0,1,2,\ldots,\Theta\}$ represents a LTS and consider a 2TLq-RPFN

$$A=\left((\hbar_\alpha,E),(\hbar_\beta,F),(\hbar_\gamma,G)\right),\tag{11}$$

then, each 2TLq-RPFN is restored to its equivalent numerical value $a,b,c\in[0,\Theta]$, using the function $\Lambda^{-1}: S\times[-0.5,0.5)\to[0,\Theta]$, where

$$\Lambda^{-1}(\hbar_\alpha,E)=\alpha+E=a,\tag{12}$$

$$\Lambda^{-1}(\hbar_\beta,F)=\beta+F=b,\tag{13}$$

$$\Lambda^{-1}(\hbar_\gamma,G)=\gamma+G=c.\tag{14}$$

Definition 5 Akram et al. (2023) Let $A=\left((\hbar_\alpha,E),(\hbar_\beta,F),(\hbar_\gamma,G)\right)$, $A_1=((\hbar_{\alpha_1},E_1),(\hbar_{\beta_1},F_1),(\hbar_{\gamma_1},G_1))$ and $A_2=((\hbar_{\alpha_2},E_2),(\hbar_{\beta_2},F_2),(\hbar_{\gamma_2},G_2))$ are three 2TLq-RPFNs with $q\geq 1$ and $\rho>0$. Then

1. $$A_1\bigoplus A_2=\left(\begin{array}{c}\Lambda\left(\Theta\left(1-\left(1-\left(\frac{\Lambda^{-1}(\hbar_{\alpha_1},E_1)}{\Theta}\right)^q\right)\left(1-\left(\frac{\Lambda^{-1}(\hbar_{\alpha_2},E_2)}{\Theta}\right)^q\right)\right)^{\frac{1}{q}}\right),\\ \Lambda\left(\Theta\left(\frac{\Lambda^{-1}\left(\hbar_{\beta_1},F_1\right)}{\Theta}\right)\left(\frac{\Lambda^{-1}\left(\hbar_{\beta_2},F_2\right)}{\Theta}\right)\right),\\ \Lambda\left(\Theta\left(\frac{\Lambda^{-1}\left(\hbar_{\gamma_1},G_1\right)}{\Theta}\right)\left(\frac{\Lambda^{-1}\left(\hbar_{\gamma_2},G_2\right)}{\Theta}\right)\right)\end{array}\right).$$

2. $$A_1\bigotimes A_2=\left(\begin{array}{c}\Lambda\left(\Theta\left(\frac{\Lambda^{-1}\left(\hbar_{\alpha_1},E_1\right)}{\Theta}\right)\left(\frac{\Lambda^{-1}\left(\hbar_{\alpha_2},E_2\right)}{\Theta}\right)\right),\\ \Lambda\left(\Theta\left(1-\left(1-\left(\frac{\Lambda^{-1}(\hbar_{\beta_1},F_1)}{\Theta}\right)^q\right)\left(1-\left(\frac{\Lambda^{-1}(\hbar_{\beta_2},F_2)}{\Theta}\right)^q\right)\right)^{\frac{1}{q}}\right),\\ \Lambda\left(\Theta\left(1-\left(1-\left(\frac{\Lambda^{-1}(\hbar_{\gamma_1},G_1)}{\Theta}\right)^q\right)\left(1-\left(\frac{\Lambda^{-1}(\hbar_{\gamma_2},G_2)}{\Theta}\right)^q\right)\right)^{\frac{1}{q}}\right)\end{array}\right).$$

3. ρA
$$=\left(\begin{array}{c}\Lambda\left(\Theta\left(1-\left(1-\left(\frac{\Lambda^{-1}(\hbar_\alpha,E)}{\Theta}\right)^q\right)^\rho\right)^{\frac{1}{q}}\right),\\ \Lambda\left(\Theta\left(\frac{\Lambda^{-1}(\hbar_\beta,F)}{\Theta}\right)^\rho\right),\Lambda\left(\Theta\left(\frac{\Lambda^{-1}(\hbar_\gamma,G)}{\Theta}\right)^\rho\right)\end{array}\right).$$

4. A^ρ
$$=\left(\begin{array}{c}\Lambda\left(\Theta\left(\frac{\Lambda^{-1}(\hbar_\alpha,E)}{\Theta}\right)^\rho\right),\\ \Lambda\left(\Theta\left(1-\left(1-\left(\frac{\Lambda^{-1}(\hbar_\beta,F)}{\Theta}\right)^q\right)^\rho\right)^{\frac{1}{q}}\right),\\ \Lambda\left(\Theta\left(1-\left(1-\left(\frac{\Lambda^{-1}(\hbar_\gamma,G)}{\Theta}\right)^q\right)^\rho\right)^{\frac{1}{q}}\right)\end{array}\right).$$

3 2TL*q*-RPF ITARA–VIKOR method

In order to overcome MCGDM difficulties, this section will build the VIKOR technique for the 2TLq-RPF environment.

Finding a CS with the most benefit to the group and the least amount of personal regret is our goal. In addition, the weights for the criteria are derived using the ITARA approach. The provided 2TLq-RPF-ITARA–VIKOR approach is capable of handling the difficult MCGDM problems.

To pick the best choice from a list of m available options or alternatives $A = \{A_1, A_2, A_3, \ldots, A_m\}$ which are to be examined on the basis of n criteria $C = \{C_1, C_2, C_3, \ldots, C_n\}$, a panel of l decision experts $\mathcal{E} = \{\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \ldots, \mathcal{E}_l\}$ is invited for the judgement of alternatives. In the MCGDM problem, the normalized weights of the experts are represented by the weight vector $\mathbf{W} = (\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_3, \ldots, \mathbf{W}_l)^T$. The steps of the presented method are also shown graphically in Fig. 1. The suggested strategy's steps are categorized as follows:

Step 1: Identification of linguistic terms. To describe how alternative scenarios perform during the DM process, it is crucial to use the appropriate linguistic phrases. In MCGDM challenges, decision experts typically want to use linguistic terminology to assess the options or alternatives that are offered. For the MCGDM technique, the experts in this method first determine the linguistic phrases and their related 2TLq-RPFNs.

Step 2: Formation of individual decision matrices. In this step, every expert constructs the evaluation matrix and presents the evaluation information in terms of 2TLq-RPFNs. Let $\mathbb{D} = (\mathbb{M}^k_{ij})_{m\times n}$ be the evaluation matrices presented by the experts'. The judgement matrix of the expert $\mathcal{E}^k$ can be represented as follows:

$$\mathbb{D} = (\mathbb{M}^k_{ij})_{m\times n} \begin{bmatrix} c^k_{11} & c^k_{12} & \ldots & c^k_{1n} \\ c^k_{21} & c^k_{22} & \ldots & c^k_{2n} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ c^k_{m1} & c^k_{m2} & \ldots & c^k_{mn} \end{bmatrix}. \tag{15}$$

Each term of the evaluation matrix $\mathbb{D} = (\mathbb{M}^k_{ij})_{m\times n}$ $(k = 1, 2, \ldots, l)$ represents a 2TLq-RPFN as $c^k_{ij} = ((\hbar^k_{\alpha_{ij}}, E^k_{ij}), (\hbar^k_{\beta_{ij}}, F^k_{ij}), (\hbar^k_{\gamma_{ij}}, G^k_{ij}))$, where $(\hbar^k_{\alpha_{ij}}, E^k_{ij})$, $(\hbar^k_{\beta_{ij}}, F^k_{ij})$ and $(\hbar^k_{\gamma_{ij}}, G^k_{ij})$ are the degrees of membership, abstinence, and non-membership, respectively.

Step 3: Construction of aggregated matrix. The individual evaluation matrices of the experts need to be cumulated to construct a combined aggregated evaluation matrix $\mathbb{G} = (\mathbb{M}_{ij})_{m\times n}$. For this purpose, the 2TLq-RPFWA operator is utilized with the experts' weights, which is presented in (2).

$$c_{ij} = 2TLq - \mathrm{RPFWA}_{\mathbf{W}}(c^1_{ij}, c^2_{ij}, \ldots, c^l_{ij}) =$$

$$\left\{ \begin{array}{c} \Lambda\left(\Theta\left(1 - \prod_{k=1}^{l}\left(1 - \left(\frac{\Lambda^{-1}(\hbar_{\alpha_k}, E_k)}{\Theta}\right)^q\right)^{\mathbf{W}_k}\right)^{\frac{1}{q}}\right), \\ \Lambda\left(\Theta\prod_{k=1}^{l}\left(\frac{\Lambda^{-1}(\hbar_{\beta_k}, F_k)}{\Theta}\right)^{\mathbf{W}_k}\right), \\ \Lambda\left(\Theta\prod_{k=1}^{l}\left(\frac{\Lambda^{-1}(\hbar_{\gamma_k}, G_k)}{\Theta}\right)^{\mathbf{W}_k}\right) \end{array} \right\}. \tag{16}$$

Step 4: Computation of criteria weights using ITARA technique. The ITARA approach is based on the idea of indifference threshold and dispersion logic is a new weighting approach presented by Hatefi (2019). We present the ITARA technique to compute the criterion weights. The following key steps are highlighted:

Step 1* : Determination of the order assessment matrix. The entries of the order assessment matrix $\mathcal{O}$ can be determined by listing the elements of the columns of the combined aggregated evaluation matrix $\mathbb{G}$ in increasing order. An entry c_{ij} of the matrix $\mathcal{O}$ is such that $S(c_{ij}) \leq S(c_{i+1,j})$, where $S(c_{ij})$ and $S(c_{i+1,j})$ represent the scores of c_{ij} and $c_{i+1,j}$, respectively.

Step 2* : Calculation of adjacent distance matrix. In this step, the adjacent distance matrix is constructed by computing the adjacent distance between c_{ij} and $c_{i+1,j}$, which can be calculated as given in Eq. 17:

$$d_{ij} = \Lambda\left(\frac{\Theta}{2}\left(\begin{array}{c}\left|\left(\frac{\Lambda^{-1}(\hbar_{\alpha_{ij}}, E_{ij})}{\Theta}\right)^q - \left(\frac{\Lambda^{-1}(\hbar_{\alpha_{i+1j}}, E_{i+1j})}{\Theta}\right)^q\right| + \left|\left(\frac{\Lambda^{-1}(\hbar_{\beta_{ij}}, F_{ij})}{\Theta}\right)^q - \left(\frac{\Lambda^{-1}(\hbar_{\beta_{i+1j}}, F_{i+1j})}{\Theta}\right)^q\right| + \\ \left|\left(\frac{\Lambda^{-1}(\hbar_{\gamma_{ij}}, G_{ij})}{\Theta}\right)^q - \left(\frac{\Lambda^{-1}(\hbar_{\gamma_{i+1j}}, G_{i+1j})}{\Theta}\right)^q\right|\end{array}\right)\right). \tag{17}$$

Step 3* : Computation of difference matrix. The entries n_{ij} of the difference matrix $\mathcal{N}$ can be computed by finding the difference between d_{ij} and $\mathcal{IT}_j$ given as follows:

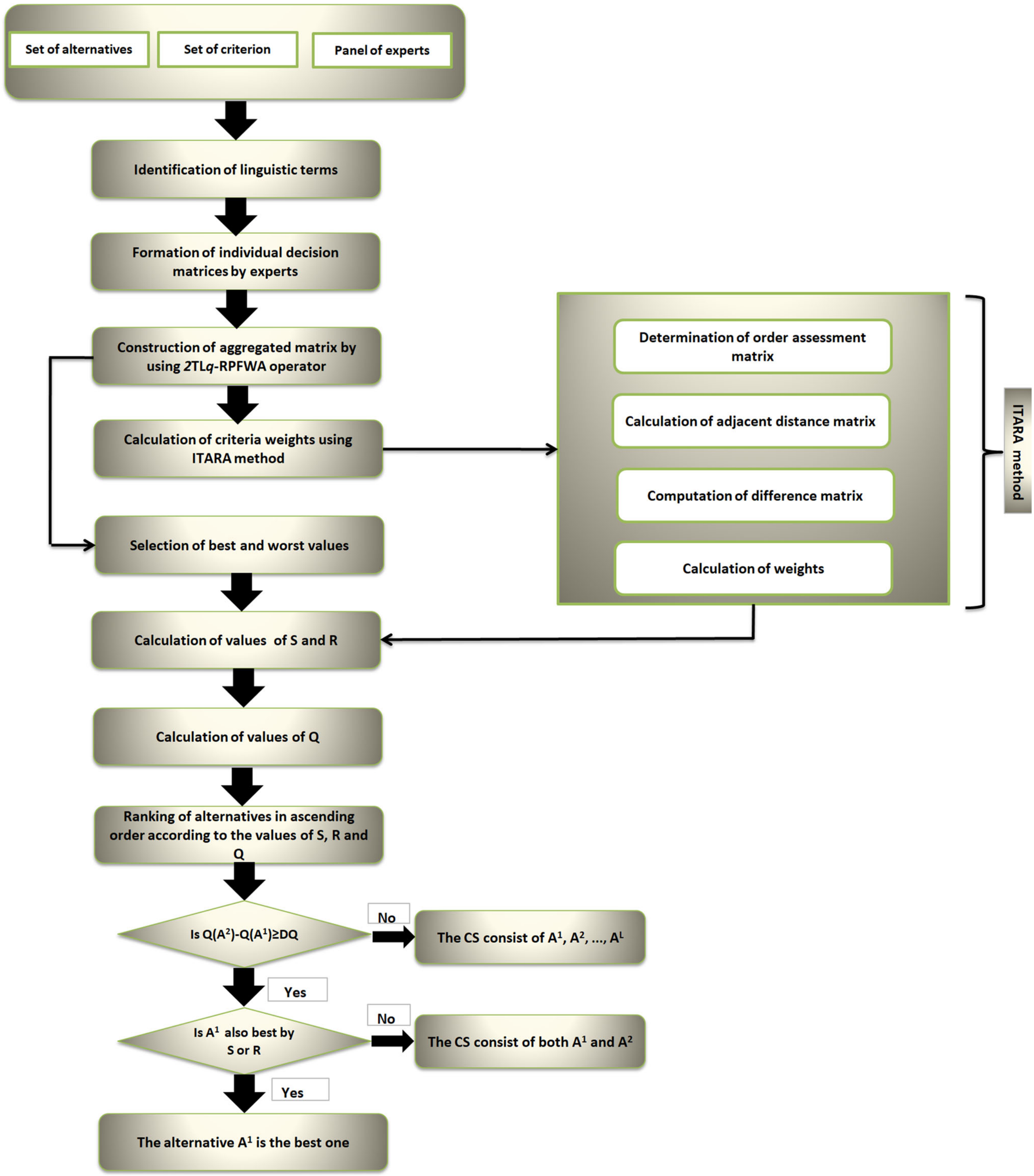

Fig. 1 Steps of 2TLq-RPF-ITARA–VIKOR method

$$n_{ij} = \begin{cases} \Lambda^{-1}(d_{ij}) - \xi_j, & \text{if } \Lambda^{-1}(d_{ij}) > \Lambda^{-1}(\mathcal{IT}_j); \\ 0, & \text{if } \Lambda^{-1}(d_{ij}) \leq \Lambda^{-1}(\mathcal{IT}_j), \end{cases} \tag{18}$$

where $\xi_j (j = 1, 2, 3, \cdots, n)$ are the indifference thresholds of the criteria that can be set by the experts.

Step 4* : Computation of weights. The criterion weights are determined in this stage using Eq. 19:

$$\mathcal{W}_j = \left(\frac{\left(\frac{\Lambda^{-1}(\psi_j, H_j)}{\Theta} \right)}{\sum_{j=1}^{n} \left(\frac{\Lambda^{-1}(\psi_j, H_j)}{\Theta} \right)} \right), \quad \text{where}$$

$$\psi_j = \left(\sum_{i=1}^{m-1} n_{ij}^{p} \right)^{\frac{1}{p}} \; (1 < p < \infty), \;\; j = 1, 2, 3, \cdots, n. \tag{19}$$

Step 5: Selection of best and worst values. The components of the 2TLq-RPF aggregated matrix $\mathbb{G}$ based on the score values, are compared to point out the best and worst values for each criteria, which can be estimated as follows:

$$S(c_{ij}) = \Lambda \left\{ \Theta + \left(\frac{\Lambda^{-1}(\hbar_{\alpha_{ij}}, E_{ij})}{\Theta} \right)^q - \left(\frac{\Lambda^{-1}(\hbar_{\beta_{ij}}, F_{ij})}{\Theta} \right)^q - \left(\frac{\Lambda^{-1}(\hbar_{\gamma_{ij}}, G_{ij})}{\Theta} \right)^q \right\}. \tag{20}$$

The best values $c_j^{'} = ((\hbar_{\alpha_j}^{'}, E_j^{'}), (\hbar_{\beta_j}^{'}, F_j^{'}), (\hbar_{\gamma_j}^{'}, G_j^{'}))$ and the worst values $c_j^{''} = ((\hbar_{\alpha_j}^{''}, E_j^{''}), (\hbar_{\beta_j}^{''}, F_j^{''}), (\hbar_{\gamma_j}^{''}, G_j^{''}))$, for each criteria is obtained on the basis of the following conditions:

$$c_j^{'} = \begin{cases} \max_{1 \leq i \leq m} c_{ij}, & \text{if } C_j \in BT; \\ \min_{1 \leq i \leq m} c_{ij}, & \text{if } C_j \in CT, \end{cases} \tag{21}$$

$$c_j^{''} = \begin{cases} \max_{1 \leq i \leq m} c_{ij}, & \text{if } C_j \in CT; \\ \min_{1 \leq i \leq m} c_{ij}, & \text{if } C_j \in BT, \end{cases} \tag{22}$$

where BT and CT represent the benefit type and cost type criteria, respectively.

Step 6: Calculation of $\mathbb{S}$ and $\mathbb{R}$. In this step, the values of group utility measure (UM) $\mathbb{S}$ and individual regret measure (IRM) $\mathbb{R}$ are computed based on the following equations:

$$\mathbb{S}_i = \sum_{j=1}^{n} \mathcal{W}_j \left(\frac{\Lambda^{-1} d(c_j^{'} - c_{ij})}{\Lambda^{-1} d(c_j^{'} - c_j^{''})} \right), \tag{23}$$

$$\mathbb{R}_i = \max_j \mathcal{W}_j \left(\frac{\Lambda^{-1} d(c_j^{'} - c_{ij})}{\Lambda^{-1} d(c_j^{'} - c_{ij}^{''})} \right), \tag{24}$$

where $d(c_j^{'} - c_{ij})$ is the distance between the best value $c_j^{'}$ and the preference value c_{ij}. Similarly, $d(c_j^{'} - c_j^{''})$ computes the distance between the best value, $c_j^{'}$, and the worst value, $c_j^{''}$.

Step 7: Computation of the values of VIKOR index $\mathbb{Q}$. Based on the values of $\mathbb{S}$ and $\mathbb{R}$, the values of $\mathbb{Q}$ are determined as presented in Eq. 25:

$$\mathbb{Q}_i = \left(v \left(\frac{\frac{\Lambda^{-1}(\mathbb{S}_i - \mathbb{S}^{'})}{\Theta}}{\frac{\Lambda^{-1}(\mathbb{S}^{'} - \mathbb{S}^{''})}{\Theta}} \right) + (1 - v) \left(\frac{\frac{\Lambda^{-1}(\mathbb{R}_i - \mathbb{R}^{'})}{\Theta}}{\frac{\Lambda^{-1}(\mathbb{R}^{'} - \mathbb{R}^{''})}{\Theta}} \right) \right), \tag{25}$$

where $\mathbb{S}^{'} = \max_i \mathbb{S}_i$, $\mathbb{S}^{''} = \min_i \mathbb{S}_i$, $\mathbb{R}^{'} = \max_i \mathbb{R}_i$ and $\mathbb{R}^{''} = \min_i \mathbb{R}_i$. The majority of attributes weight strategies are known as v, and their value is an element of the close unit interval.

Step 8: Ranking of alternatives. $\mathbb{S}$, $\mathbb{R}$, and $\mathbb{Q}$ values are written in ascending order to sort the alternatives. In this stage, three ranking lists are created, which are then used to reach a CS.

Step 9: Finding of CS. The top-rated alternative, or CS, as determined by the $\mathbb{Q}$ measurement is presented at this stage. The best-ranked option is the top-ranked option A^1 with regard to $\mathbb{Q}$ if and only if the following criteria are met:

- $\mathbb{Q}_1$: Acceptable advantage.

  $$\mathbb{Q}(A^2) - \mathbb{Q}(A^1) \geq D\mathbb{Q}, \tag{26}$$

  where $D\mathbb{Q} = \frac{1}{m-1}$ and m represent the total number of available alternatives. The second positioned alternative is denoted by the notation A^2 in the ranking list of $\mathbb{Q}$.
- $\mathbb{Q}_2$: Acceptable stability in decision-making. This condition states that the best-ranked option by $\mathbb{Q}$ must also be the best-ranked alternative by $\mathbb{S}$ or $\mathbb{R}$.

The optimal solution cannot be reached if any of the aforementioned requirements are not met, and instead, a collection of CSs are found in the following cases:

- $\mathbb{Q}_1$: The alternatives A^1 and A^2 are part of a CS if only the condition $\mathbb{Q}_2$ is false.
- $\mathbb{Q}_2$: The CS consists of the options A^1, A^2, ..., and A^L if the condition $\mathbb{Q}_1$ is not met, where A^L is acquired from the relationship $\mathbb{Q}(A^L) - \mathbb{Q}(A^1) < D\mathbb{Q}$ for maximum L.

4 Application

This section gives an illustration of how to choose the finest audit company to conduct an institution's audit. It also provides a study of the impact of parameter q on outcomes.

4.1 Description of the problem

A thorough, objective, and documented procedure is used to collect audit data, which is then objectively evaluated to determine the extent to which the audit criteria are satisfied. An audit is a dynamic and complex process with several interconnected tasks. International Standards on Auditing outline an auditor's duties and responsibilities for each stage of the audit. Every time a company conducts an audit, an audit team is present. They consist of a client, a lead auditor, an auditor, an auditor in training, a technical specialist, an observer, and a guide. Auditors check the accuracy and legal compliance of the financial records kept by companies and other organizations. They may be conducting a financial audit of their own or another organization's financial records, as well as acting in an advisory position by making recommendations for risk and cost reduction measures. By conducting a fair examination of processes, systems, and operations, audit firms are essential in confirming a company's financial statements and determining whether internal controls are in place and functioning effectively to minimize risks and ensure that organizational goals and objectives are met. Before extending credit or supplying funding for a firm, prospective lenders and investors frequently want externally audited financial accounts. If it turns out that an auditor missed significant misstatements, it will be detrimental to the company and the profession as a whole. The main duties of an auditor are: (1) assembling, confirming, and evaluating spreadsheet data, (2) analyzing the organization's financial records and internal control procedures; (3) assessing the financial risk levels inside organizations and ensuring the accuracy and dependability of financial data and reporting; (4) determining which processes need to be changed and where they should be changed, as well as generating reports, commentaries, and financial statements, (5) sharing information with the management team, including outcomes and suggestions; (6) ensuring that rules, processes, laws, and regulations are appropriately followed and adhered to while conducting wage evaluations. Currently, different audit and accounting firms are working in Pakistan, such as A.F. Ferguson, KPMG (Klynveld Peat Marwick Goerdeler), EY (Ernst and Young) Ford Rhodes, Yousuf Adil Chartered Accountants, Grant Thornton Anjum Rahman, Riaz Ahmad and Company, etc. It is very vital for various businesses to carry out their audits through a fair examination process, operations, and confirmation to ensure their financial records audit, and to find out whether internal controls are in place and working properly. As a result, audits are crucial to reducing risks and guaranteeing that corporate goals and objectives are reached. The choice of the finest audit company might therefore be seen as a significant issue. In this example, an institution must choose the finest audit company to conduct its audit. The institution invites a panel consisting of four experts $\{\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4\}$, which are required to evaluate the seven audit companies or firms $\{\mathbf{L}, \mathbf{N}, \mathbf{P}, \mathbf{R}, \mathbf{T}, \mathbf{V}, \mathbf{X}\}$ on the basis of five criteria $\{FQ, GL, PE, PR, RM\}$. The five criteria are presented in Fig. 2, and their brief descriptions are summarized as follows:

- FQ : *The firm's quality risk assessment procedure.* The risk evaluation of quality arrangements is already a common practise among firms. Firms must adopt a more integrated strategy and a customized approach as risk assessment becomes a requirement. A quality management system that is fit for the specific company and the services it offers. The gains in audit quality that come about as a result of organizations setting quality objectives, identifying and evaluating quality risks, and putting those quality risk answers into action are quite significant. Firms need to take this into account in their acceptance and continuation procedures, as well as the pertinent ethical standards.
- GL : *Governance and leadership.* In order to establish and maintain a climate and culture conducive to the execution of high-quality auditing, clear governance and leadership that strongly emphasizes the significance of quality and anticipate the behaviors of the audit engagement team are essential. High audit quality must be fostered throughout the organization by senior leadership, which models the proper beliefs, ethics, and behaviors.
- PE : *Performance evaluation and correction.* An efficient feedback loop is required for a quality management system to track the effectiveness of the audit activity. To address the quality issues that the company has identified, effective monitoring and remediation must be adequately developed.
- PR : *Planning for resources and managing people.* For audits to be performed well, people are essential. The best talents must be attracted and kept by audit companies. In order to service their current clients and support their expansion ambitions in terms of both client numbers and industry knowledge, companies must have an adequate number of employees who are also well-trained and experienced.
- RM : *Resources-Methodology and technology.* All organizations must make considerable and ongoing investments in their auditing procedures and technology. Numerous initiatives are mainly focused on data analytics, audit support technologies, and standardization

Fig. 2 Qualities of a good audit firm

> and uniformity. Project management and work-flow management systems are also widely used. Delivering high-quality audits requires a regularly applied process and the use of technology, understood and delivered by adequately qualified individuals.

All five criteria are of the benefit type. The weight vector $\mathbf{W} = (0.35, 0.25, 0.25, 0.15)^T$ is allocated to the experts. Suppose, $S = \{\hbar_0, \hbar_1, \hbar_2, \hbar_3, \hbar_4, \hbar_5, \hbar_6, \hbar_7, \hbar_8\}$ be the set of nine linguistic terms. At first, the experts' panel form the linguistic terms (see Fig. 3) to describe the alternatives in the form of 2TLq-RPFNs. After that, the experts' are requested to make an evaluation to judge the most optimal alternative in the form of 2TLq-RPFNs.

4.2 Procedural steps

The details of the proposed 2TLq-RPF-ITARA–VIKOR technique to determine the best firm for audit are presented as follows:
Step 1. Fig. 3 depicts the experts' seven-point linguistic term scale and the corresponding 2TLq-RPFNs.
Step 2. The four experts' $\mathcal{E}_k$, $(k = 1, 2, 3, 4)$ individual judgement matrices, are provided in Table 1.
Step 3. The linguistic assessments of the experts are combined by utilizing the 2TLq-RPFWA operator, and the resulting aggregated matrix in the form of 2TLq-RPFNs is summarized in Table 2.
Step 4. The criteria weights based on the ITARA technique are computed in this step.
Step 1*. The values of the order assessment matrix are shown in Table 3.
Step 2*. The adjacent distance matrix is presented in Table 4.
Step 3*. The difference matrix is given in Table 5, and the threshold values set by the experts are (0.5, 0.6, 0.3, 0.45, 0.35).
Step 4*. Finally, the criteria weights are calculated and shown in Table 5.
Step 5. This step selects the best and worst values for each criterion (see Table 6) based on the aggregated matrix score values calculated in *Step 3*.

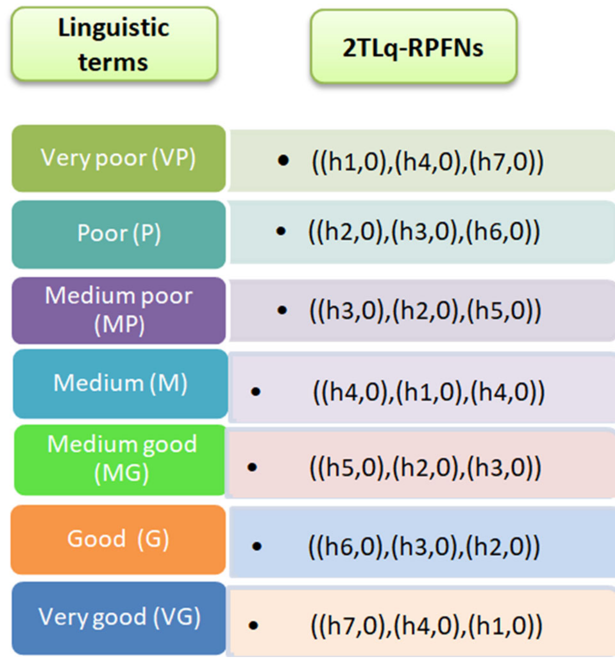

Fig. 3 LTs and their corresponding 2TLq-RPFNs

Step 6. The values of group UM $\mathbb{S}$, IRM $\mathbb{R}$ and their corresponding alternative ranks are given in Table 7.
Step 7. Table 7 consists of the values of $\mathbb{Q}$ $(\upsilon = 0.5)$ along with alternative ranks.
Step 8. The ranking lists of alternatives by $\mathbb{S}$, $\mathbb{R}$, and $\mathbb{Q}$ are shown in Table 7.
Step 9. The alternative **L**, which is ranked highest and satisfies both requirements of the VIKOR technique, is as follows:

- $\mathbb{Q}(\mathbf{N}) - \mathbb{Q}(\mathbf{L}) = 0.1772 - 0 = 0.1772 \geq \dfrac{1}{7-1} = 0.1667$.
- **L** is the best alternative by both $\mathbb{S}$ and $\mathbb{R}$.

The ranking results of alternatives based on $\mathbb{Q}$ by utilizing different values of υ are listed in Table 8 and graphically shown in Fig. 4. The alternatives rankings slightly differ, but the best alternative remains consistent for all values of υ.

4.3 Parameter influence

We solve this numerical example using five different parameter values (q), i.e., $q = 2$, $q = 3$, $q = 4$, $q = 5$, and $q = 6$, in order to see the impact of changing values of the parameter q on the decision-making results. Table 9 provides the values of the ranking metric $\mathbb{Q}$, the accompanying ranking, and the best answer. All of these calculations are performed using $\upsilon = 0.5$. Table 9 shows that although rankings may vary somewhat, **L** is always the best option when all possible values of q are taken into account. The authenticity of the results

Table 1 Linguistic assessments presented by $\mathcal{E}_1$, $\mathcal{E}_2$, $\mathcal{E}_3$ and $\mathcal{E}_4$

Experts	Alternatives	FQ	GL	PE	PR	RM
$\mathcal{E}_1$	**L**	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$	$((\hbar_3,0),(\hbar_2,0),(\hbar_5,0))$	$((\hbar_7,0),(\hbar_4,0),(\hbar_1,0))$	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_5,0),(\hbar_2,0),(\hbar_3,0))$
	N	$((\hbar_5,0),(\hbar_2,0),(\hbar_3,0))$	$((\hbar_3,0),(\hbar_2,0),(\hbar_5,0))$	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$
	P	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$	$((\hbar_5,0),(\hbar_2,0),(\hbar_3,0))$	$((\hbar_5,0),(\hbar_2,0),(\hbar_3,0))$	$((\hbar_1,0),(\hbar_4,0),(\hbar_7,0))$
	R	$((\hbar_3,0),(\hbar_2,0),(\hbar_5,0))$	$((\hbar_7,0),(\hbar_4,0),(\hbar_1,0))$	$((\hbar_5,0),(\hbar_2,0),(\hbar_3,0))$	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$	$((\hbar_3,0),(\hbar_2,0),(\hbar_5,0))$
	T	$((\hbar_7,0),(\hbar_4,0),(\hbar_1,0))$	$((\hbar_1,0),(\hbar_4,0),(\hbar_7,0))$	$((\hbar_5,0),(\hbar_2,0),(\hbar_3,0))$	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$	$((\hbar_3,0),(\hbar_2,0),(\hbar_5,0))$
	V	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_2,0),(\hbar_3,0),(\hbar_6,0))$	$((\hbar_2,0),(\hbar_3,0),(\hbar_6,0))$	$((\hbar_7,0),(\hbar_4,0),(\hbar_1,0))$	$((\hbar_2,0),(\hbar_3,0),(\hbar_6,0))$
	X	$((\hbar_1,0),(\hbar_4,0),(\hbar_7,0))$	$((\hbar_3,0),(\hbar_2,0),(\hbar_5,0))$	$((\hbar_7,0),(\hbar_4,0),(\hbar_1,0))$	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_1,0),(\hbar_4,0),(\hbar_7,0))$
$\mathcal{E}_2$	**L**	$((\hbar_5,0),(\hbar_2,0),(\hbar_3,0))$	$((\hbar_3,0),(\hbar_2,0),(\hbar_5,0))$	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_5,0),(\hbar_2,0),(\hbar_3,0))$	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$
	N	$((\hbar_5,0),(\hbar_2,0),(\hbar_3,0))$	$((\hbar_2,0),(\hbar_3,0),(\hbar_6,0))$	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$
	P	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$	$((\hbar_5,0),(\hbar_2,0),(\hbar_3,0))$	$((\hbar_7,0),(\hbar_4,0),(\hbar_1,0))$	$((\hbar_2,0),(\hbar_3,0),(\hbar_6,0))$
	R	$((\hbar_3,0),(\hbar_2,0),(\hbar_5,0))$	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$	$((\hbar_3,0),(\hbar_2,0),(\hbar_5,0))$	$((\hbar_3,0),(\hbar_2,0),(\hbar_5,0))$
	T	$((\hbar_7,0),(\hbar_4,0),(\hbar_1,0))$	$((\hbar_3,0),(\hbar_2,0),(\hbar_5,0))$	$((\hbar_5,0),(\hbar_2,0),(\hbar_3,0))$	$((\hbar_3,0),(\hbar_2,0),(\hbar_5,0))$	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$
	V	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$	$((\hbar_2,0),(\hbar_3,0),(\hbar_6,0))$	$((\hbar_1,0),(\hbar_4,0),(\hbar_7,0))$	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_2,0),(\hbar_3,0),(\hbar_6,0))$
	X	$((\hbar_1,0),(\hbar_4,0),(\hbar_7,0))$	$((\hbar_2,0),(\hbar_3,0),(\hbar_6,0))$	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_5,0),(\hbar_2,0),(\hbar_3,0))$	$((\hbar_2,0),(\hbar_3,0),(\hbar_6,0))$
$\mathcal{E}_3$	**L**	$((\hbar_3,0),(\hbar_2,0),(\hbar_5,0))$	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$	$((\hbar_7,0),(\hbar_4,0),(\hbar_1,0))$	$((\hbar_5,0),(\hbar_2,0),(\hbar_3,0))$	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$
	N	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$	$((\hbar_3,0),(\hbar_2,0),(\hbar_5,0))$	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$
	P	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$	$((\hbar_5,0),(\hbar_2,0),(\hbar_3,0))$	$((\hbar_1,0),(\hbar_4,0),(\hbar_7,0))$
	R	$((\hbar_3,0),(\hbar_2,0),(\hbar_5,0))$	$((\hbar_7,0),(\hbar_4,0),(\hbar_1,0))$	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$	$((\hbar_3,0),(\hbar_2,0),(\hbar_5,0))$	$((\hbar_3,0),(\hbar_2,0),(\hbar_5,0))$
	T	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_1,0),(\hbar_4,0),(\hbar_7,0))$	$((\hbar_5,0),(\hbar_2,0),(\hbar_3,0))$	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$	$((\hbar_2,0),(\hbar_3,0),(\hbar_6,0))$
	V	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_2,0),(\hbar_3,0),(\hbar_6,0))$	$((\hbar_3,0),(\hbar_2,0),(\hbar_5,0))$	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_1,0),(\hbar_4,0),(\hbar_7,0))$
	X	$((\hbar_2,0),(\hbar_3,0),(\hbar_6,0))$	$((\hbar_2,0),(\hbar_3,0),(\hbar_6,0))$	$((\hbar_7,0),(\hbar_4,0),(\hbar_1,0))$	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_2,0),(\hbar_3,0),(\hbar_6,0))$
$\mathcal{E}_4$	**L**	$((\hbar_5,0),(\hbar_2,0),(\hbar_3,0))$	$((\hbar_2,0),(\hbar_3,0),(\hbar_6,0))$	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_7,0),(\hbar_4,0),(\hbar_1,0))$
	N	$((\hbar_5,0),(\hbar_2,0),(\hbar_3,0))$	$((\hbar_3,0),(\hbar_2,0),(\hbar_5,0))$	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_7,0),(\hbar_4,0),(\hbar_1,0))$	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$
	P	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$	$((\hbar_5,0),(\hbar_2,0),(\hbar_3,0))$	$((\hbar_5,0),(\hbar_2,0),(\hbar_3,0))$	$((\hbar_5,0),(\hbar_2,0),(\hbar_3,0))$	$((\hbar_1,0),(\hbar_4,0),(\hbar_7,0))$
	R	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_5,0),(\hbar_2,0),(\hbar_3,0))$	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$	$((\hbar_2,0),(\hbar_3,0),(\hbar_6,0))$
	T	$((\hbar_7,0),(\hbar_4,0),(\hbar_1,0))$	$((\hbar_1,0),(\hbar_4,0),(\hbar_7,0))$	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$	$((\hbar_3,0),(\hbar_2,0),(\hbar_5,0))$	$((\hbar_3,0),(\hbar_2,0),(\hbar_5,0))$
	V	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_2,0),(\hbar_3,0),(\hbar_6,0))$	$((\hbar_2,0),(\hbar_3,0),(\hbar_6,0))$	$((\hbar_7,0),(\hbar_4,0),(\hbar_1,0))$	$((\hbar_2,0),(\hbar_3,0),(\hbar_6,0))$
	X	$((\hbar_2,0),(\hbar_3,0),(\hbar_6,0))$	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$	$((\hbar_6,0),(\hbar_3,0),(\hbar_2,0))$	$((\hbar_4,0),(\hbar_1,0),(\hbar_4,0))$	$((\hbar_2,0),(\hbar_3,0),(\hbar_6,0))$

Table 2 Aggregated matrix in the form of 2TLq-RPFNs

Alternatives	FQ	GL	PE	PR	RM
L	$((\hbar_4, 0.2774), (\hbar_2, -0.4308), (\hbar_4, -0.2303))$	$((\hbar_3, 0.1846), (\hbar_2, -0.2127), (\hbar_5, -0.1402))$	$((\hbar_7, -0.3107), (\hbar_4, -0.4348), (\hbar_1, 0.3195))$	$((\hbar_6, -0.4363), (\hbar_2, 0.4495), (\hbar_2, 0.4495))$	$((\hbar_6, -0.4110), (\hbar_2, 0.0651), (\hbar_2, 0.4704))$
N	$((\hbar_5, -0.2123), (\hbar_2, -0.3182), (\hbar_3, 0.2237))$	$((\hbar_3, -0.2089), (\hbar_2, 0.2134), (\hbar_5, 0.2332))$	$((\hbar_6, 0.0000), (\hbar_3, 0.0000), (\hbar_2, 0.0000))$	$((\hbar_6, 0.2050), (\hbar_3, 0.1323), (\hbar_2, -0.1975))$	$((\hbar_4, 0.0000), (\hbar_1, 0.0000), (\hbar_4, 0.0000))$
P	$((\hbar_5, -0.3041), (\hbar_1, 0.3161), (\hbar_3, 0.3636))$	$((\hbar_4, 0.1799), (\hbar_1, 0.1096), (\hbar_4, -0.1689))$	$((\hbar_5, -0.2123), (\hbar_2, -0.3182), (\hbar_3, 0.2237))$	$((\hbar_6, -0.2305), (\hbar_2, 0.3784), (\hbar_2, 0.2795))$	$((\hbar_1, 0.3281), (\hbar_4, -0.2776), (\hbar_7, -0.2646))$
R	$((\hbar_3, 0.1799), (\hbar_2, -0.1975), (\hbar_5, -0.1646))$	$((\hbar_7, -0.3107), (\hbar_4, -0.4348), (\hbar_1, 0.3195))$	$((\hbar_5, -0.4466), (\hbar_1, 0.4142), (\hbar_3, 0.4641))$	$((\hbar_4, -0.4477), (\hbar_1, 0.4142), (\hbar_4, 0.4721))$	$((\hbar_3, -0.1229), (\hbar_2, 0.1254), (\hbar_5, 0.1386))$
T	$((\hbar_7, -0.1833), (\hbar_4, -0.2776), (\hbar_1, 0.1892))$	$((\hbar_2, -0.2385), (\hbar_3, 0.3636), (\hbar_6, 0.4353))$	$((\hbar_5, -0.1249), (\hbar_2, -0.1975), (\hbar_3, 0.1323))$	$((\hbar_4, -0.3512), (\hbar_1, 0.3195), (\hbar_4, 0.3734))$	$((\hbar_3, 0.1099), (\hbar_2, -0.1388), (\hbar_5, -0.0508))$
V	$((\hbar_6, -0.3466), (\hbar_2, 0.2795), (\hbar_2, 0.3784))$	$((\hbar_2, 0.0000), (\hbar_3, 0.0000), (\hbar_6, 0.0000))$	$((\hbar_2, 0.1379), (\hbar_3, -0.0870), (\hbar_6, -0.0421))$	$((\hbar_7, -0.4041), (\hbar_3, 0.4641), (\hbar_1, 0.4142))$	$((\hbar_2, -0.1934), (\hbar_3, 0.2237), (\hbar_6, 0.2357))$
X	$((\hbar_1, 0.4892), (\hbar_4, -0.4348), (\hbar_7, -0.4186))$	$((\hbar_3, -0.2223), (\hbar_2, 0.2076), (\hbar_5, 0.2969))$	$((\hbar_7, -0.3107), (\hbar_4, -0.4348), (\hbar_1, 0.3195))$	$((\hbar_6, -0.4304), (\hbar_2, 0.2990), (\hbar_2, 0.4559))$	$((\hbar_2, -0.2776), (\hbar_3, 0.3178), (\hbar_6, 0.3326))$

Table 3 Order assessment matrix in the form of 2TLq-RPFNs

Alternatives	FQ	GL	PE	PR	RM
L	$((\hbar_1, 0.4892), (\hbar_4, -0.4348), (\hbar_7, -0.4186))$	$((\hbar_2, -0.2385), (\hbar_3, 0.3636), (\hbar_6, 0.4353))$	$((\hbar_2, 0.1379), (\hbar_3, -0.0870), (\hbar_6, -0.0421))$	$((\hbar_4, -0.4477), (\hbar_1, 0.4142), (\hbar_4, 0.4721))$	$((\hbar_1, 0.3281), (\hbar_4, -0.2776), (\hbar_7, -0.2646))$
N	$((\hbar_3, 0.1799), (\hbar_2, -0.1975), (\hbar_4, -0.1646))$	$((\hbar_3, -0.2223), (\hbar_3, 0.0000), (\hbar_6, 0.0000))$	$((\hbar_5, -0.4466), (\hbar_1, 0.4142), (\hbar_3, 0.4641))$	$((\hbar_4, -0.3512), (\hbar_1, 0.3195), (\hbar_4, 0.3734))$	$((\hbar_2, -0.2776), (\hbar_3, 0.3178), (\hbar_6, 0.3326))$
P	$((\hbar_4, 0.2774), (\hbar_2, -0.4308), (\hbar_4, -0.2303))$	$((\hbar_3, -0.22223), (\hbar_2, 0.2076), (\hbar_5, 0.2969))$	$((\hbar_5, -0.2123), (\hbar_2, -0.3182), (\hbar_3, 0.2237))$	$((\hbar_6, -0.4363), (\hbar_2, 0.4495), (\hbar_2, 0.4495))$	$((\hbar_2, -0.1934), (\hbar_3, 0.2237), (\hbar_6, 0.2357))$
R	$((\hbar_5, -0.3041), (\hbar_1, 0.3116), (\hbar_3, 0.3636))$	$((\hbar_3, -0.2089), (\hbar_2, 0.2134), (\hbar_5, 0.2332))$	$((\hbar_5, -0.1249), (\hbar_2, -0.1975), (\hbar_3, 0.1323))$	$((\hbar_6, -0.4304), (\hbar_2, 0.2990), (\hbar_2, 0.4559))$	$((\hbar_3, -0.1229), (\hbar_2, 0.1254), (\hbar_5, 0.1386))$
T	$((\hbar_5, -0.2123), (\hbar_2, -0.3182), (\hbar_3, 0.2237))$	$((\hbar_3, 0.1846), (\hbar_2, -0.2127), (\hbar_5, -0.1402))$	$((\hbar_6, 0.0000), (\hbar_3, 0.0000), (\hbar_2, 0.0000))$	$((\hbar_6, -0.2305), (\hbar_2, 0.3784), (\hbar_2, 0.2795))$	$((\hbar_3, 0.1099), (\hbar_2, -0.1388), (\hbar_5, -0.0508))$
V	$((\hbar_6, -0.3466), (\hbar_2, 0.2795), (\hbar_2, 0.3784))$	$((\hbar_4, 0.1799), (\hbar_1, 0.1096), (\hbar_4, -0.1689))$	$((\hbar_7, -0.3107), (\hbar_4, -0.4348), (\hbar_1, 0.3195))$	$((\hbar_6, 0.2050), (\hbar_3, 0.1323), (\hbar_2, -0.1975))$	$((\hbar_4, 0.0000), (\hbar_1, 0.0000), (\hbar_4, 0.0000))$
X	$((\hbar_7, -0.1833), (\hbar_4, -0.2776), (\hbar_2, 0.1892))$	$((\hbar_7, -0.3107), (\hbar_4, -0.4348), (\hbar_1, 0.3195))$	$((\hbar_7, -0.3107), (\hbar_4, -0.4348), (\hbar_1, 0.3195))$	$((\hbar_7, -0.4041), (\hbar_3, 0.4641), (\hbar_1, 0.4142))$	$((\hbar_6, -0.4110), (\hbar_2, 0.0651), (\hbar_2, 0.4704))$

Table 4 Adjacent distance matrix

Alternatives	FQ	GL	PE	PR	RM
L	$(\hbar_2, 0.3306)$	$(\hbar_1, -0.4610)$	$(\hbar_3, -0.1159)$	$(\hbar_0, 0.1142)$	$(\hbar_1, -0.4178)$
N	$(\hbar_1, 0.1338)$	$(\hbar_1, -0.0134)$	$(\hbar_0, 0.2891)$	$(\hbar_2, 0.1892)$	$(\hbar_0, 0.1332)$
P	$(\hbar_0, 0.4614)$	$(\hbar_0, 0.0482)$	$(\hbar_0, 0.1154)$	$(\hbar_0, 0.0507)$	$(\hbar_1, 0.4605)$
R	$(\hbar_0, 0.1805)$	$(\hbar_0, 0.4891)$	$(\hbar_1, 0.4872)$	$(\hbar_0, 0.2171)$	$(\hbar_0, 0.2724)$
T	$(\hbar_1, 0.0089)$	$(\hbar_1, 0.1396)$	$(\hbar_1, -0.0802)$	$(\hbar_1, -0.2927)$	$(\hbar_1, 0.0804)$
V	$(\hbar_2, -0.2869)$	$(\hbar_3, 0.2307)$	$(\hbar_0, 0.0000)$	$(\hbar_1, -0.4724)$	$(\hbar_2, -0.2251)$

Table 5 Difference matrix

Alternatives	FQ	GL	PE	PR	RM
L	$(\hbar_2, -0.1694)$	$(\hbar_0, 0.0000)$	$(\hbar_3, -0.4159)$	$(\hbar_0, 0.0000)$	$(\hbar_0, 0.2322)$
N	$(\hbar_1, -0.3662)$	$(\hbar_0, 0.3866)$	$(\hbar_0, 0.0000)$	$(\hbar_2, -0.2608)$	$(\hbar_0, 0.0000)$
P	$(\hbar_0, 0.0000)$	$(\hbar_0, 0.0000)$	$(\hbar_0, 0.0000)$	$(\hbar_0, 0.0000)$	$(\hbar_1, 0.1105)$
R	$(\hbar_0, 0.0000)$	$(\hbar_0, 0.0000)$	$(\hbar_1, 0.1872)$	$(\hbar_0, 0.0000)$	$(\hbar_0, 0.0000)$
T	$(\hbar_1, -0.4911)$	$(\hbar_1, -0.4604)$	$(\hbar_1, -0.3802)$	$(\hbar_0, 0.2573)$	$(\hbar_1, -0.2696)$
V	$(\hbar_1, 0.2131)$	$(\hbar_3, -0.3693)$	$(\hbar_0, 0.0000)$	$(\hbar_0, 0.0000)$	$(\hbar_1, 0.4249)$
Sum	$(\hbar_4, 0.1864)$	$(\hbar_4, -0.4431)$	$(\hbar_4, 0.3911)$	$(\hbar_2, -0.0035)$	$(\hbar_3, 0.4980)$
Weights	0.2375	0.2018	0.2491	0.1133	0.1984

Table 6 Best and worst values in the form of 2TLq-RPFNs

	FQ	GL	PE
f^+	$((\hbar_7, -0.1833), (\hbar_4, -0.2776), (\hbar_2, 0.1892))$	$((\hbar_7, -0.3107), (\hbar_4, -0.4348), (\hbar_1, 0.3195))$	$((\hbar_7, -0.3107), (\hbar_4, -0.4348), (\hbar_1, 0.3195))$
f^-	$((\hbar_1, 0.4892), (\hbar_4, -0.4348), (\hbar_7, -0.4186))$	$((\hbar_2, -0.2385), (\hbar_3, 0.3636), (\hbar_6, 0.4353))$	$((\hbar_2, 0.1379), (\hbar_3, -0.0870), (\hbar_6, -0.0421))$

	PR	RM
f^+	$((\hbar_7, -0.4041), (\hbar_3, 0.4641), (\hbar_1, 0.4142))$	$((\hbar_6, -0.4110), (\hbar_2, 0.0651), (\hbar_2, 0.4704))$
f^-	$((\hbar_4, -0.4477), (\hbar_1, 0.4142), (\hbar_4, 0.4721))$	$((\hbar_1, 0.3281), (\hbar_4, -0.2776), (\hbar_7, -0.2646))$

Table 7 $\mathbb{S}$, $\mathbb{R}$ and $\mathbb{Q}$

Alternatives	$\mathbb{S}$	$\mathbb{R}$	$\mathbb{S}$ rank	$\mathbb{R}$ rank	$\mathbb{Q}$	$\mathbb{Q}$ rank
L	0.3469	0.1610	1	1	0.0000	1
N	0.4253	0.1718	2	2	0.1772	2
P	0.6174	0.1984	5	4	0.6121	5
R	0.5544	0.1875	4	3	0.4571	3
T	0.5385	0.2018	3	5	0.5147	4
V	0.6852	0.2491	7	7	1.0000	7
X	0.6307	0.2375	6	6	0.8536	6

is unaffected by the value of the parameter q. In essence, the adjustable q's aim is to increase the scope of applications for the suggested technique since it maintains accuracy across the existing sets boundaries. The ranking positions obtained by taking different values of q are also shown in Fig. 5.

5 Comparative analysis

This section includes some comparison studies of the supplied methodology with the previous strategies in order to show the viability, emphasize the importance, and highlight the superiority of the offered techniques over the existing methods. We stressed the main steps of each of the exist-

Table 8 Rankings for different values of v

Alternatives	$v = 0.1$	$v = 0.2$	$v = 0.3$	$v = 0.4$	$v = 0.5$	$v = 0.6$	$v = 0.7$	$v = 0.8$	$v = 0.9$
L	1	1	1	1	1	1	1	1	1
N	2	2	2	2	2	2	2	2	2
P	4	5	5	5	5	5	5	5	5
R	3	3	3	3	3	3	3	3	4
T	5	4	4	4	4	4	4	4	3
V	7	7	7	7	7	7	7	7	7
X	6	6	6	6	6	6	6	6	6

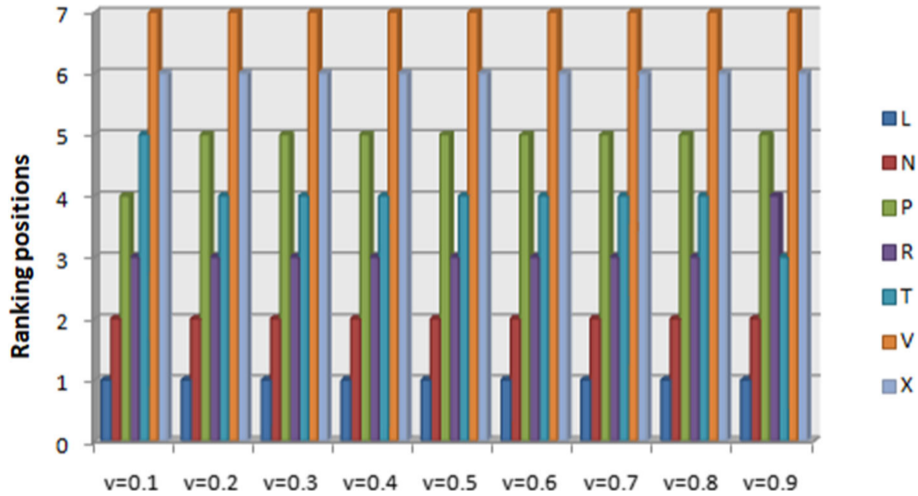

Fig. 4 Ranking positions using different values of v

Table 9 Parameter effect

Values of parameter q	Ranking	Best alternative
$q = 2$	**L** > **N** > **R** > **T** > **P** > **X** > **V**	**L**
$q = 3$	**L** > **N** > **T** > **R** > **V** > **P** > **X**	**L**
$q = 4$	**L** > **N** > **T** > **R** > **P** > **X** > **V**	**L**
$q = 5$	**L** > **R** > **N** > **P** > **X** > **T** > **V**	**L**
$q = 6$	**L** > **R** > **N** > **P** > **T** > **X** > **V**	**L**

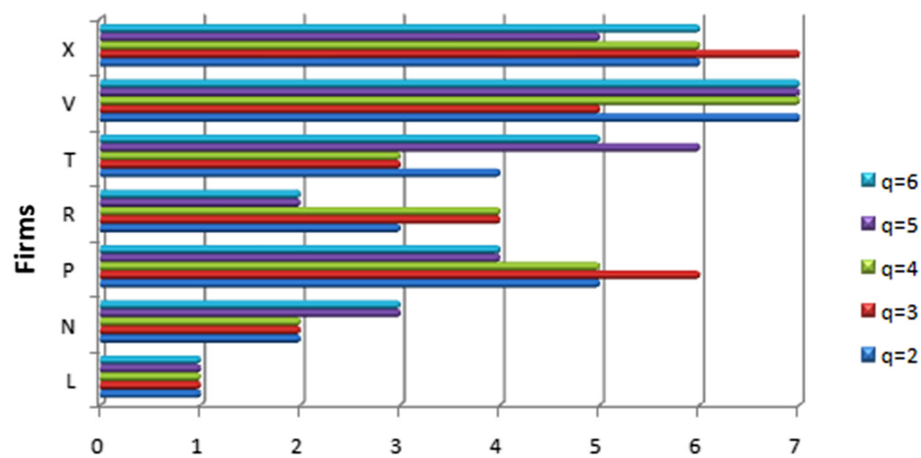

Fig. 5 Ranking positions using different values of q

ing approaches we utilize to solve the numerical issue. We employ the 2TL-T-SF-EDAS method (Naz et al. 2022), the q-RPF-VIKOR method (Akram and Shumaiza 2021), and the q-ROF-CRITIC-VIKOR approach (Yang et al. 2022).

- First, we apply the 2TL-T-SF-EDAS method to the presented application. We do not need to normalize the initial matrices because all criteria are of the benefit type. Now apply the 2TL-T-spherical fuzzy Maclaurin symmetric mean (2TL T-SFWMSM) operator Naz et al. (2022) ($q = 2$ and $t = 3$) to form a combined matrix (see Table 10). The aggregated values scores are shown in Table 11. Further, Table 12 shows the appraisal score (AS) values with scores. The values of the positive distance from average solution (PDAS) and negative distance from average solution (NDAS) matrices of the alternatives are depicted in Table 13. Finally, the assessing values of positive weighted distance (SP), negative weighted distance (SN), normalized SP (NSP), normalized SN (NSN), integrative AS (IAS), also the order of the choices are presented in Table 14. From Table 14, we see that **L** is the best audit firm.

- We used the q-RPF-VIKOR approach ($q = 2$) proposed by Akram and Shumaiza (2021) to solve the given numer-

Table 10 Aggregated matrix using 2TL T-SFWMSM in the form of 2TLT-SFNs

Alternatives	FQ	GL	PE
L	$((\hbar_7, -0.2231), (\hbar_1, -0.1346), (\hbar_2, 0.0760))$	$((\hbar_6, 0.2893), (\hbar_1, 0.0128), (\hbar_3, -0.2827))$	$((\hbar_8, -0.3789), (\hbar_2, -0.1118), (\hbar_1, -0.2571))$
N	$((\hbar_7, 0.0216), (\hbar_1, -0.0954), (\hbar_2, -0.3044))$	$((\hbar_6, 0.1201), (\hbar_1, 0.1602), (\hbar_3, -0.0764))$	$((\hbar_7, 0.4478), (\hbar_2, -0.4610), (\hbar_1, 0.0098))$
P	$((\hbar_7, -0.0826), (\hbar_1, -0.1692), (\hbar_2, -0.1184))$	$((\hbar_7, -0.2033), (\hbar_1, -0.3965), (\hbar_2, 0.0236))$	$((\hbar_7, 0.0216), (\hbar_1, -0.0954), (\hbar_2, -0.3044))$
R	$((\hbar_6, 0.3480), (\hbar_1, -0.0533), (\hbar_3, -0.3649))$	$((\hbar_8, -0.3789), (\hbar_2, -0.1118), (\hbar_1, -0.2571))$	$((\hbar_7, -0.0756), (\hbar_1, -0.2164), (\hbar_2, -0.1578))$
T	$((\hbar_8, -0.3325), (\hbar_2, -0.0266), (\hbar_1, -0.3466))$	$((\hbar_5, 0.1630), (\hbar_2, -0.1184), (\hbar_4, 0.0546))$	$((\hbar_7, 0.0563), (\hbar_1, -0.0533), (\hbar_2, -0.3599))$
V	$((\hbar_7, 0.2725), (\hbar_1, 0.3466), (\hbar_1, 0.3429))$	$((\hbar_6, -0.3224), (\hbar_2, -0.4610), (\hbar_3, 0.4475))$	$((\hbar_6, -0.4039), (\hbar_2, -0.4123), (\hbar_4, -0.4363))$
X	$((\hbar_5, 0.1689), (\hbar_2, -0.1118), (\hbar_4, 0.0376))$	$((\hbar_6, 0.0394), (\hbar_1, 0.2502), (\hbar_3, 0.0371))$	$((\hbar_8, -0.3789), (\hbar_2, -0.1118), (\hbar_1, -0.2571))$

Alternatives	PR	RM
L	$((\hbar_7, 0.2820), (\hbar_1, 0.2919), (\hbar_1, 0.2991))$	$((\hbar_7, 0.2054), (\hbar_1, 0.2916), (\hbar_1, 0.4741))$
N	$((\hbar_7, 0.4942), (\hbar_2, -0.3599), (\hbar_1, -0.0533))$	$((\hbar_7, -0.2645), (\hbar_1, -0.4997), (\hbar_2, 0.1024))$
P	$((\hbar_7, 0.2780), (\hbar_1, 0.3429), (\hbar_1, 0.3466))$	$((\hbar_5, 0.0181), (\hbar_2, -0.0266), (\hbar_4, 0.1834))$
R	$((\hbar_6, 0.4999), (\hbar_1, -0.2092), (\hbar_2, 0.4348))$	$((\hbar_6, 0.1727), (\hbar_1, 0.1079), (\hbar_3, -0.1471))$
T	$((\hbar_7, -0.4579), (\hbar_1, -0.2571), (\hbar_2, 0.3763))$	$((\hbar_6, 0.2372), (\hbar_1, 0.0669), (\hbar_3, -0.2097))$
V	$((\hbar_8, -0.4062), (\hbar_2, -0.1655), (\hbar_1, -0.2092))$	$((\hbar_5, 0.4462), (\hbar_2, -0.3044), (\hbar_4, -0.2834))$
X	$((\hbar_7, 0.2509), (\hbar_1, 0.2943), (\hbar_1, 0.3687))$	$((\hbar_5, 0.4065), (\hbar_2, -0.2589), (\hbar_4, -0.2130))$

Table 11 Scores of Table 10

Alternatives	FQ	GL	PE	PR	RM
L	$(\hbar_7, -0.3990)$	$(\hbar_6, 0.0107)$	$(\hbar_8, -0.4044)$	$(\hbar_7, 0.2087)$	$(\hbar_7, 0.1091)$
N	$(\hbar_7, -0.0983)$	$(\hbar_6, -0.1932)$	$(\hbar_7, 0.4031)$	$(\hbar_7, 0.4542)$	$(\hbar_7, -0.4408)$
P	$(\hbar_7, -0.2306)$	$(\hbar_7, -0.3686)$	$(\hbar_7, -0.0983)$	$(\hbar_7, 0.1972)$	$(\hbar_4, 0.4800)$
R	$(\hbar_6, 0.0846)$	$(\hbar_8, -0.4044)$	$(\hbar_7, -0.2154)$	$(\hbar_6, 0.27)$	$(\hbar_6, -0.1273)$
T	$(\hbar_8, -0.3523)$	$(\hbar_5, -0.3615)$	$(\hbar_7, -0.0562)$	$(\hbar_6, 0.3220)$	$(\hbar_6, -0.0552)$
V	$(\hbar_7, 0.1929)$	$(\hbar_5, 0.2719)$	$(\hbar_5, 0.1635)$	$(\hbar_8, -0.4350)$	$(\hbar_5, -0.0095)$
X	$(\hbar_5, -0.3490)$	$(\hbar_6, -0.2969)$	$(\hbar_8, -0.4044)$	$(\hbar_7, 0.1689)$	$(\hbar_5, -0.0694)$

Table 12 AS matrix with scores

Alternatives	AS values	Scores
AS_1	$((\hbar_7, -0.0391), (\hbar_1, 0.1749), (\hbar_2, -0.1974))$	$(\hbar_7, -0.1747)$
AS_2	$((\hbar_7, -0.4926), (\hbar_1, 0.2529), (\hbar_2, 0.4346))$	$(\hbar_6, 0.2762)$
AS_3	$((\hbar_7, 0.2217), (\hbar_1, 0.2869), (\hbar_1, 0.3929))$	$(\hbar_7, 0.1383)$
AS_4	$((\hbar_7, 0.2257), (\hbar_1, 0.1499), (\hbar_1, 0.3968))$	$(\hbar_7, 0.1412)$
AS_5	$((\hbar_6, 0.2146), (\hbar_1, 0.2377), (\hbar_3, -0.1703))$	$(\hbar_6, -0.0866)$

Table 13 PDAS and NDAS

Alternatives	PDAS					NDAS				
L	0	0	0.0641	0.0095	0.2022	0.0329	0.0423	0	0	0
N	0.0112	0	0.0371	0.0438	0.1092	0	0.0748	0	0	0
P	0	0.0566	0	0.0078	0	0.0082	0	0.0331	0	0.2424
R	0	0.2102	0	0	0	0.1085	0	0.0495	0.122	0.0069
T	0.1205	0	0	0	0.0053	0	0.2609	0.0272	0.1147	0
V	0.0539	0	0	0.0593	0	0	0.16	0.2766	0	0.1561
X	0	0	0.0641	0.0039	0	0.3186	0.0913	0	0	0.1662

Table 14 Assessing values of SP, SN, NSP, NSN and IAS

Alternatives	SP	SN	NSP	NSN	IAS	Ranks
L	0.0572	0.0163	1	0.8767	0.9384	1
N	0.0385	0.0151	0.6731	0.8858	0.7795	2
P	0.0123	0.0583	0.215	0.559	0.387	5
R	0.0424	0.0533	0.7413	0.5968	0.6691	3
T	0.0297	0.0724	0.5192	0.4523	0.4858	4
V	0.0195	0.1322	0.3409	0	0.1705	6
X	0.0164	0.1271	0.2867	0.0386	0.1627	7

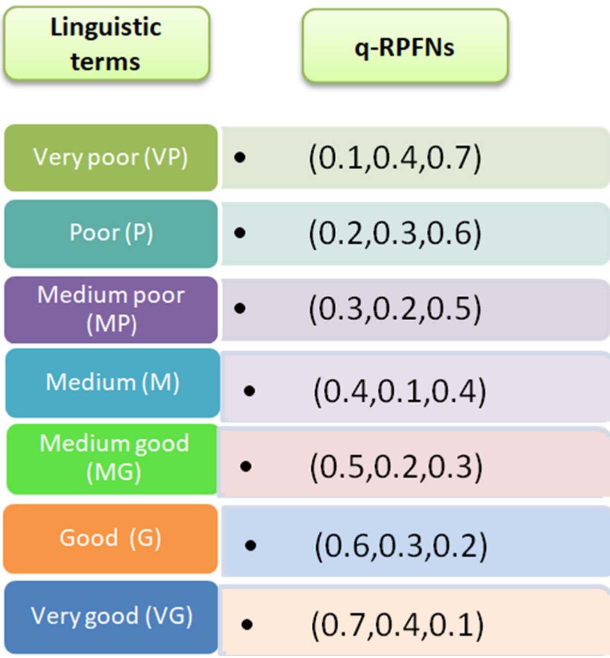

Fig. 6 LTs and their corresponding q-RPFNs

ical example. First, the 2TLq-RPFNs are transformed into q-RPFNs as given in Fig. 6. Here, the same procedures as those in Akram and Shumaiza (2021) are followed. The expert evaluations in the form of q-RPFNs are shown in Table 15. The aggregated matrix is shown in Tables 16 and 17 calculates the score values for each row. Table 18 provides a summary of the projection values of each alternative corresponding to each criteria. Table 19 also shows the entropy values, divergence values, and criteria weights. The best and worst values for each criterion are shown in Table 20. The UM ($\mathbb{S}$), IRM ($\mathbb{R}$), VIKOR index ($\mathbb{Q}$), and their corresponding alternatives ranks are all shown in Table 21. The best possible option is **L**, as is obvious.

- Now, on the suggested application, we apply the q-ROF-CRITIC-VIKOR technique ($q = 2$) introduced by Yang et al. (2022). We followed the identical procedures as those listed in Yang et al. (2022). All calculations are performed without taking into account the neutral membership grade. Table 22 depicts the expert evaluations in the form of q-ROFNs, and the aggregated matrix is displayed in Table 23. Table 24 displays the scores of Table 23. The CRITIC approach is used to calculate the criteria weights, which are provided as follows:

$$\mathcal{W}_1 = 0.2032, \quad \mathcal{W}_2 = 0.2135, \quad \mathcal{W}_3 = 0.1844, \quad \mathcal{W}_4 = 0.2378, \quad \mathcal{W}_5 = 0.1611.$$

Table 25 displays the best values for each criterion, (f^+), and the worst values, (f^-). Table 26 combines the UM ($\mathbb{S}$), IRM ($\mathbb{R}$), VIKOR index ($\mathbb{Q}$), and their corresponding alternatives ranks. We can see that using this approach, **L** is the best option.

6 Discussion and implications of the proposed work

We observe the following main differences between the proposed method and the existing ones:

1. In Sect. 5, the necessary steps and computations are performed using the existing methodologies, namely the 2TL-T-SF-EDAS method, the q-RPF-VIKOR method, and the q-ROF-CRITIC-VIKOR method. The ranking results as well as the best alternative obtained by employing the presented and existing methodologies are listed in Table 27 and graphically displayed in Fig. 7. Although the complete ranking lists are not the same, the best alternative is the same using all approaches.
2. While handling the problem by applying the 2TL-T-SF-EDAS method, we see that the results of five firms (**L**, **N**, **P**, **R**, **T**) are identical with our proposed approach. Only the ranking positions of two firms (**V** and **X**) are different. The best alternative is **L**, which shows the authenticity and reliability of our suggested methodology. Although the frameworks presented in this research and the 2TL-T-SF-EDAS method are applicable to

Table 15 Assessments in the form of q-RPFNs

Experts	Alternatives	FQ	GL	PE	PR	RM
$\mathcal{E}_1$	**L**	(0.4, 0.1, 0.4)	(0.3, 0.2, 0.5)	(0.7, 0.4, 0.1)	(0.6, 0.3, 0.2)	(0.5, 0.2, 0.3)
	N	(0.5, 0.2, 0.3)	(0.3, 0.2, 0.5)	(0.6, 0.3, 0.2)	(0.6, 0.3, 0.2)	(0.4, 0.1, 0.4)
	P	(0.4, 0.1, 0.4)	(0.4, 0.1, 0.4)	(0.5, 0.2, 0.3)	(0.5, 0.2, 0.3)	(0.1, 0.4, 0.7)
	R	(0.3, 0.2, 0.5)	(0.7, 0.4, 0.1)	(0.5, 0.2, 0.3)	(0.4, 0.1, 0.4)	(0.3, 0.2, 0.5)
	T	(0.7, 0.4, 0.1)	(0.1, 0.4, 0.7)	(0.5, 0.2, 0.3)	(0.4, 0.1, 0.4)	(0.3, 0.2, 0.5)
	V	(0.6, 0.3, 0.2)	(0.2, 0.3, 0.6)	(0.2, 0.3, 0.6)	(0.7, 0.4, 0.1)	(0.2, 0.3, 0.6)
	X	(0.1, 0.4, 0.7)	(0.3, 0.2, 0.5)	(0.7, 0.4, 0.1)	(0.6, 0.3, 0.2)	(0.1, 0.4, 0.7)
$\mathcal{E}_2$	**L**	(0.5, 0.2, 0.3)	(0.3, 0.2, 0.5)	(0.6, 0.3, 0.2)	(0.5, 0.2, 0.3)	(0.4, 0.1, 0.4)
	N	(0.5, 0.2, 0.3)	(0.2, 0.3, 0.6)	(0.6, 0.3, 0.2)	(0.6, 0.3, 0.2)	(0.4, 0.1, 0.4)
	P	(0.6, 0.3, 0.2)	(0.4, 0.1, 0.4)	(0.5, 0.2, 0.3)	(0.7, 0.4, 0.1)	(0.2, 0.3, 0.6)
	R	(0.3, 0.2, 0.5)	(0.6, 0.3, 0.2)	(0.4, 0.1, 0.4)	(0.3, 0.2, 0.5)	(0.3, 0.2, 0.5)
	T	(0.7, 0.4, 0.1)	(0.3, 0.2, 0.5)	(0.5, 0.2, 0.3)	(0.3, 0.2, 0.5)	(0.4, 0.1, 0.4)
	V	(0.4, 0.1, 0.4)	(0.2, 0.3, 0.6)	(0.1, 0.4, 0.7)	(0.6, 0.3, 0.2)	(0.2, 0.3, 0.6)
	X	(0.1, 0.4, 0.7)	(0.2, 0.3, 0.6)	(0.6, 0.3, 0.2)	(0.5, 0.2, 0.3)	(0.2, 0.3, 0.6)
$\mathcal{E}_3$	**L**	(0.3, 0.2, 0.5)	(0.4, 0.1, 0.4)	(0.7, 0.4, 0.1)	(0.5, 0.2, 0.3)	(0.6, 0.3, 0.2)
	N	(0.4, 0.1, 0.4)	(0.3, 0.2, 0.5)	(0.6, 0.3, 0.2)	(0.6, 0.3, 0.2)	(0.4, 0.1, 0.4)
	P	(0.4, 0.1, 0.4)	(0.4, 0.1, 0.4)	(0.4, 0.1, 0.4)	(0.5, 0.3, 0.3)	(0.1, 0.4, 0.7)
	R	(0.3, 0.2, 0.5)	(0.7, 0.4, 0.1)	(0.4, 0.1, 0.4)	(0.3, 0.3, 0.5)	(0.3, 0.2, 0.5)
	T	(0.6, 0.3, 0.2)	(0.1, 0.4, 0.7)	(0.5, 0.2, 0.3)	(0.4, 0.1, 0.4)	(0.2, 0.3, 0.6)
	V	(0.6, 0.3, 0.2)	(0.2, 0.3, 0.6)	(0.3, 0.2, 0.5)	(0.6, 0.3, 0.2)	(0.1, 0.4, 0.7)
	X	(0.2, 0.3, 0.6)	(0.2, 0.3, 0.6)	(0.7, 0.4, 0.1)	(0.6, 0.3, 0.2)	(0.2, 0.3, 0.6)
$\mathcal{E}_4$	**L**	(0.5, 0.2, 0.3)	(0.2, 0.3, 0.6)	(0.6, 0.3, 0.2)	(0.6, 0.3, 0.2)	(0.7, 0.4, 0.1)
	N	(0.5, 0.2, 0.3)	(0.3, 0.2, 0.5)	(0.6, 0.3, 0.2)	(0.7, 0.4, 0.2)	(0.4, 0.1, 0.4)
	P	(0.4, 0.1, 0.4)	(0.5, 0.2, 0.3)	(0.5, 0.2, 0.3)	(0.5, 0.2, 0.3)	(0.1, 0.4, 0.7)
	R	(0.4, 0.1, 0.4)	(0.6, 0.3, 0.2)	(0.5, 0.2, 0.3)	(0.4, 0.1, 0.4)	(0.2, 0.3, 0.6)
	T	(0.7, 0.4, 0.1)	(0.1, 0.4, 0.7)	(0.4, 0.1, 0.4)	(0.3, 0.2, 0.5)	(0.3, 0.2, 0.5)
	V	(0.6, 0.3, 0.2)	(0.2, 0.3, 0.6)	(0.2, 0.3, 0.6)	(0.7, 0.4, 0.1)	(0.2, 0.3, 0.6)
	X	(0.2, 0.3, 0.6)	(0.4, 0.1, 0.4)	(0.6, 0.3, 0.2)	(0.4, 0.1, 0.4)	(0.2, 0.3, 0.6)

Table 16 Aggregated matrix in the form of q-RPFNs

Alternatives	FQ	GL	PE	PR	RM
L	(0.4254, 0.1569, 0.3770)	(0.3176, 0.1787, 0.4860)	(0.6645, 0.3565, 0.1320)	(0.5542, 0.2449, 0.2449)	(0.5478, 0.2065, 0.2470)
N	(0.4780, 0.1682, 0.3224)	(0.2788, 0.2213, 0.5233)	(0.6000, 0.3000, 0.2000)	(0.6176, 0.3132, 0.2000)	(0.4000, 0.1000, 0.4000)
P	(0.4639, 0.1316, 0.3364)	(0.4173, 0.1110, 0.3831)	(0.4780, 0.1682, 0.3224)	(0.5647, 0.2632, 0.2280)	(0.1326, 0.3722, 0.6735)
R	(0.3176, 0.1803, 0.4835)	(0.6645, 0.3565, 0.1320)	(0.4542, 0.1414, 0.3464)	(0.3545, 0.1565, 0.4472)	(0.2875, 0.2125, 0.5139)
T	(0.6784, 0.3722, 0.1189)	(0.1750, 0.3364, 0.6435)	(0.4870, 0.1803, 0.3132)	(0.3642, 0.1320, 0.4373)	(0.3099, 0.1861, 0.4949)
V	(0.5612, 0.2280, 0.2378)	(0.2000, 0.3000, 0.6000)	(0.2132, 0.2913, 0.5958)	(0.6547, 0.3464, 0.1414)	(0.1805, 0.3224, 0.6236)
X	(0.1487, 0.3565, 0.6581)	(0.2766, 0.2208, 0.5297)	(0.6645, 0.3565, 0.1320)	(0.5535, 0.2299, 0.2456)	(0.1721, 0.3318, 0.6333)

handle the MCGDM issues, In the 2TL-T-SF-EDAS approach, the authors utilize the criteria weights directly, whereas our proposed strategy is based on the ITARA technique to determine the weights of the criterion. In most of the DM problems, the weights are not known in advance. Our proposed method provides a newly extended ITARA method for 2TLq-RPFSs.

3. When we solve the problem by utilizing the q-RPF-VIKOR method, the ranking results of five firms (**L**, **N**, **R**, **T**, **X**) remain similar to our presented approach. The positions of firms **V** and **P** are changed by using the q-RPF-VIKOR method. In this method, the entropy measure is used to determine the criterion weights. However, the best audit firm remains consistent, which is **L**, show-

ing the authenticity of our proposed method. This method is based on q-RPFSs and is incapable of dealing with linguistic data. Our proposed approach, on the other hand, can capture linguistic assessments within the boundaries of 2TLq-RPFSs.

4. While solving the problem by employing the q-ROF-CRITIC-VIKOR method, the ranking results of only two firms **L**, and **N** remain consistent with our presented method. The positions of remaining five firms (**P**, **R**, **T**, **V**, **X**) are changed. The q-ROF-CRITIC-VIKOR method is based on the CRITIC approach to compute the criteria weights. Furthermore, this method considers only two membership degrees, i.e., the degree of positive membership and the degree of negative membership, which may produce different results due to the absence of a neutral membership grade. However, the best audit firm remains consistent, which is **L**. The q-ROF-CRITIC-VIKOR method only handles the two membership degrees (positive membership and negative membership), while in most of the MCGDM issues, the experts also show a neutral kind of opinion. Therefore, the main deficiency in this methodology is the lack of neutral membership grade and its inability to manage the assessments with the help of linguistic terms.
5. Data ambiguity and uncertainty may be handled using the existing FS structures and their related additions. Despite these striking characteristics, the 2TLq-RPFS is a more generic structure that can take into account both linguistic data and uncertainty at the same time. The 2TLq-RPFS concept may be converted into the 2TLq-ROFS by setting the neutral part equal to zero. By setting $q = 1$ and ignoring neutral component, it may be transformed into 2TL-IFS. Moreover, it may be transformed into 2TL-PyFS by setting $q = 2$ and the neutral component equal to zero. The wide range of 2TLq-RPFSs enables the decision-makers to express their views more freely, improving the decision's correctness.
6. Our suggested MCGDM technique is flexible and adept at making decisions; it is not just restricted to 2TLq-RPFNs, since it also performs equally well with q-ROF and q-RPF data. Consequently, the methodology we offer is a reliable and adaptable strategy that can precisely manage both conventional and linguistically ambiguous information.

Table 17 Scores of Table 16

Alternatives	FQ	GL	PE	PR	RM
L	0.3381	0.2776	0.4323	0.3957	0.3988
N	0.3654	0.2516	0.4100	0.4144	0.3300
P	0.3616	0.3384	0.3654	0.3992	0.1418
R	0.2782	0.4323	0.3554	0.3004	0.2578
T	0.4359	0.1678	0.3689	0.3080	0.2722
V	0.4021	0.1967	0.2019	0.4295	0.1799
X	0.1540	0.2491	0.4323	0.3977	0.1728

Table 18 Projection values

Alternatives	FQ	GL	PE	PR	RM
L	0.1448	0.1451	0.1685	0.1496	0.2275
N	0.1565	0.1315	0.1598	0.1567	0.1882
P	0.1548	0.1768	0.1424	0.1509	0.0809
R	0.1191	0.2259	0.1385	0.1136	0.1470
T	0.1867	0.0877	0.1438	0.1165	0.1553
V	0.1722	0.1028	0.0787	0.1624	0.1026
X	0.0659	0.1302	0.1685	0.1504	0.0986

Table 19 Entropy values, divergence values and weights

Alternatives	FQ	GL	PE	PR	RM
Entropy values	0.9804	0.9775	0.9885	0.9958	0.9701
Divergence values	0.0196	0.0225	0.0115	0.0042	0.0299
Weights	0.2235	0.2566	0.1311	0.0479	0.3409

6.1 Managerial implications

Given the aforementioned findings, the suggested approach has managerial implications for improving the choice of the best firm for audit. Via various audit organizations or firms, many sectors (private and government) carry out their audit to verify the correctness of financial accounts. The audit firms examine the company's financial records and internal control procedures, determine which processes need to be changed and where they should be changed, generate reports, commentaries, and financial statements, and share information with the management team, including results and suggestions, ensuring that rules, processes, and the law are followed. We discussed how to select the top auditing firm from a list of seven possibilities using a set of criteria. The factors taken into account include: FQ : The firm's quality risk assessment procedure; GL : Governance and leadership; PE : Performance evaluation and correction; PR : Planning for resources and managing people; RM : Resources-Methodology and technology. The seven firms are assessed using the aforementioned criteria. The proposed strategy is capable to address the challenging MCGDM issues and is able to demonstrate the linguistic evaluations within the boundaries of 2TLq-RPFSs.

Table 20 Best and worst values in the form of q-RPFNs

	FQ	GL	PE	PR	RM
f^+	(0.6784, 0.3722, 0.1189)	(0.6645, 0.3565, 0.1320)	(0.6645, 0.3565, 0.1320)	(0.6547, 0.3464, 0.1414)	(0.5478, 0.2065, 0.2470)
f^-	(0.1487, 0.3565, 0.6581)	(0.1750, 0.3364, 0.6435)	(0.2132, 0.2913, 0.5958)	(0.3545, 0.1565, 0.4472)	(0.1326, 0.3722, 0.6735)

Table 21 $\mathbb{S}$, $\mathbb{R}$ and $\mathbb{Q}$ based on q-RPF VIKOR method

Alternatives	$\mathbb{S}$	$\mathbb{S}$ rank	$\mathbb{R}$	$\mathbb{R}$ rank	$\mathbb{Q}$	$\mathbb{Q}$ rank
L	0.3261	1	0.1868	1	0	1
N	0.4545	2	0.2030	2	0.2034	2
P	0.6704	5	0.3409	7	0.9045	7
R	0.4835	3	0.2057	3	0.2462	3
T	0.5535	4	0.2566	4	0.4936	4
V	0.7286	6	0.2955	5	0.8256	5
X	0.7517	7	0.3039	6	0.8799	6

Table 22 Assessments in the form of q-ROFNs

Experts	Alternatives	FQ	GL	PE	PR	RM
$\mathcal{E}_1$	**L**	(0.4, 0.4)	(0.3, 0.5)	(0.7, 0.1)	(0.6, 0.2)	(0.5, 0.3)
	N	(0.5, 0.3)	(0.3, 0.5)	(0.6, 0.2)	(0.6, 0.2)	(0.4, 0.4)
	P	(0.4, 0.4)	(0.4, 0.4)	(0.5, 0.3)	(0.5, 0.3)	(0.1, 0.7)
	R	(0.3, 0.5)	(0.7, 0.1)	(0.5, 0.3)	(0.4, 0.4)	(0.3, 0.5)
	T	(0.7, 0.1)	(0.1, 0.7)	(0.5, 0.3)	(0.4, 0.4)	(0.3, 0.5)
	V	(0.6, 0.2)	(0.2, 0.6)	(0.2, 0.6)	(0.7, 0.1)	(0.2, 0.6)
	X	(0.1, 0.7)	(0.3, 0.5)	(0.7, 0.1)	(0.6, 0.2)	(0.1, 0.7)
$\mathcal{E}_2$	**L**	(0.5, 0.3)	(0.3, 0.5)	(0.6, 0.2)	(0.5, 0.3)	(0.4, 0.4)
	N	(0.5, 0.3)	(0.2, 0.6)	(0.6, 0.2)	(0.6, 0.2)	(0.4, 0.4)
	P	(0.6, 0.2)	(0.4, 0.4)	(0.5, 0.3)	(0.7, 0.1)	(0.2, 0.6)
	R	(0.3, 0.5)	(0.6, 0.2)	(0.4, 0.4)	(0.3, 0.5)	(0.3, 0.5)
	T	(0.7, 0.1)	(0.3, 0.5)	(0.5, 0.3)	(0.3, 0.5)	(0.4, 0.4)
	V	(0.4, 0.4)	(0.2, 0.6)	(0.1, 0.7)	(0.6, 0.2)	(0.2, 0.6)
	X	(0.1, 0.7)	(0.2, 0.6)	(0.6, 0.2)	(0.5, 0.3)	(0.2, 0.6)
$\mathcal{E}_3$	**L**	(0.3, 0.5)	(0.4, 0.4)	(0.7, 0.1)	(0.5, 0.3)	(0.6, 0.2)
	N	(0.4, 0.4)	(0.3, 0.5)	(0.6, 0.2)	(0.6, 0.2)	(0.4, 0.4)
	P	(0.4, 0.4)	(0.4, 0.4)	(0.4, 0.4)	(0.5, 0.3)	(0.1, 0.7)
	R	(0.3, 0.5)	(0.7, 0.1)	(0.4, 0.4)	(0.3, 0.5)	(0.3, 0.5)
	T	(0.6, 0.2)	(0.1, 0.7)	(0.5, 0.3)	(0.4, 0.4)	(0.2, 0.6)
	V	(0.6, 0.2)	(0.2, 0.6)	(0.3, 0.5)	(0.6, 0.2)	(0.1, 0.7)
	X	(0.2, 0.6)	(0.2, 0.6)	(0.7, 0.1)	(0.6, 0.2)	(0.2, 0.6)
$\mathcal{E}_4$	**L**	(0.5, 0.3)	(0.2, 0.6)	(0.6, 0.2)	(0.6, 0.2)	(0.7, 0.1)
	N	(0.5, 0.3)	(0.3, 0.5)	(0.6, 0.2)	(0.7, 0.2)	(0.4, 0.4)
	P	(0.4, 0.4)	(0.5, 0.3)	(0.5, 0.3)	(0.5, 0.3)	(0.1, 0.7)
	R	(0.4, 0.4)	(0.6, 0.2)	(0.5, 0.3)	(0.4, 0.4)	(0.2, 0.6)
	T	(0.7, 0.1)	(0.1, 0.7)	(0.4, 0.4)	(0.3, 0.5)	(0.3, 0.5)
	V	(0.6, 0.2)	(0.2, 0.6)	(0.2, 0.6)	(0.7, 0.1)	(0.2, 0.6)
	X	(0.2, 0.6)	(0.4, 0.4)	(0.6, 0.2)	(0.4, 0.4)	(0.2, 0.6)

Table 23 Aggregated matrix in the form of q-ROFNs

Alternatives	FQ	GL	PE	PR	RM
L	(0.4254, 0.3770)	(0.3176, 0.4860)	(0.6645, 0.1320)	(0.5542, 0.2449)	(0.5478, 0.2470)
N	(0.4780, 0.3224)	(0.2788, 0.5233)	(0.6000, 0.2000)	(0.6176, 0.2000)	(0.4000, 0.4000)
P	(0.4639, 0.3364)	(0.4173, 0.3831)	(0.4780, 0.3224)	(0.5647, 0.2280)	(0.1326, 0.6735)
R	(0.3176, 0.4835)	(0.6645, 0.1320)	(0.4542, 0.3464)	(0.3545, 0.4472)	(0.2875, 0.5139)
T	(0.6784, 0.1189)	(0.1750, 0.6435)	(0.4870, 0.3132)	(0.3642, 0.4373)	(0.3099, 0.4949)
V	(0.5612, 0.2378)	(0.2000, 0.6000)	(0.2132, 0.5958)	(0.6547, 0.1414)	(0.1805, 0.6236)
X	(0.1487, 0.6581)	(0.2766, 0.5297)	(0.6645, 0.1320)	(0.5535, 0.2456)	(0.1721, 0.6333)

6.2 Theoretical implications

First, the established method is used in the 2TLq-RPF context, allowing experts to use LTs to express their conclusions and evaluations. As a consequence, the provided approach successfully addresses the practical MCGDM problem connected to 2TLq-RPFS. The flexibility of the devised strategy for obtaining information on real LTs is then shown. Our 2TLq-RPF-ITARA–VIKOR approach, a recommended MCGDM tactic, can give a CS focusing on the policy to maximize group utility and reduce individual regret. The main advantage of the recommended approach is its ability to handle the confusing data that is usually present in practical scenarios. The experts in real-world decision-making challenges are likely to improperly assign weights to the criteria due to time restrictions, a lack of knowledge, or other variables. No one is presumably aware of the weight assigned to each criterion. The suggested methodology calculates the criterion weights using the ITARA approach as a result. The benefits of the suggested ways may be effectively used by other networks or sectors to make important decisions on the MCGDM in other domains, thus they are not just restricted to the problem under consideration. By carefully analyzing the contributing elements and potential alternatives, researchers may use these methodologies to more effectively address ambiguous information in business, medicine, engineering, etc. Several practical MCGDM problems that crop up across many fields may be addressed by the proposed effort. As shown in the scenario under consideration, professionals can apply the accepted methodology to select the best solutions.

Table 24 Scores of Table 23

Alternatives	FQ	GL	PE	PR	RM
L	0.5194	0.4323	0.7121	0.6236	0.6195
N	0.5623	0.4019	0.6600	0.6707	0.5000
P	0.5510	0.5137	0.5623	0.6335	0.2820
R	0.4335	0.7121	0.5432	0.4628	0.4093
T	0.7230	0.3083	0.5695	0.4707	0.4256
V	0.6292	0.3400	0.3452	0.7043	0.3219
X	0.2945	0.3980	0.7121	0.6230	0.3143

7 Conclusion and future directions

It has been recognized that the 2TLTSs are suitable structures for illustrating linguistic evaluation values using LTs. Due to the large space of acceptable triplets, the 2TLq-RPFS is a more comprehensive and generalized structure. Additionally, the 2TLq-RPFSs offer notable characteristics to represent the experts' preference ratings using LTs and within the bounds of q-RPFSs. For the purpose of choosing the best audit firm from a group of seven audit firms, this study suggested a 2TLq-RPF-based ITARA–VIKOR model. In addition to this, parameter analysis is done to verify the stability of the suggested technique. The importance and reliability of the developed approach have been assessed and demonstrated by comparison with other procedures. The 2TLq-RPF-ITARA–VIKOR technique is tested and validated against three other methods: the 2TLT-SF-EDAS method, the q-RPF-VIKOR method, and the q-ROF-CRITIC-VIKOR method. Findings show that **L** is the top audit firm. Also, while contrasting the proposed strategy with earlier methods, the best alternative is consistently reached. As a result, we draw the conclusion that the technique is generic, simple to comprehend, practical to use, and capable of capturing the linguistic MCGDM difficulties. To be clear, it is important to mention that the following key points outline the primary contributions of the proposed work: (1) proposed a novel strategy for dealing with MCGDM problems in a linguistic setting; (2) presented an efficient approach when the criteria weights are unknown in advance; (3) established an improved and robust ITARA–VIKOR method to the 2TLq-RPF context; and (4) proposed the MCGDM problem of best audit firm selection. Our future research directions are summarized as follows:

- We are planning to extend our future research to other MCGDM methods, such as CODAS, ELECTREE, and PROMETHEE for 2TLq-RPFSs.

Table 25 Best and worst values in the form of q-ROFNs

	FQ	GL	PE	PR	RM
f^+	(0.6784, 0.1189)	(0.6645, 0.1320)	(0.6645, 0.1320)	(0.6547, 0.1414)	(0.5478, 0.2470)
f^-	(0.1487, 0.6581)	(0.1750, 0.6435)	(0.2132, 0.5958)	(0.3545, 0.4472)	(0.1326, 0.6735)

Table 26 $\mathbb{S}$, $\mathbb{R}$ and $\mathbb{Q}$ based on q-ROF-CRITIC-VIKOR method

Alternatives	$\mathbb{S}$	$\mathbb{S}$ rank	$\mathbb{R}$	$\mathbb{R}$ rank	$\mathbb{Q}$	$\mathbb{Q}$ rank
L	0.4218	1	0.18	1	0	1
N	0.4496	2	0.1923	2	0.1201	2
P	0.6428	4	0.1803	3	0.4304	3
R	0.6604	5	0.273	7	0.9629	6
T	0.6795	7	0.2667	6	0.9661	7
V	0.6288	3	0.2122	4	0.5747	4
X	0.6669	6	0.2076	5	0.6239	5

Table 27 Ranking results using proposed and existing techniques by taking $q = 2$

Techniques	Ranking orders	Best alternative
2TLq-RPF ITARA–VIKOR method (proposed)	**L** > **N** > **R** > **T** > **P** > **X** > **V**	**L**
2TL T-SF-EDAS method Naz et al. (2022)	**L** > **N** > **R** > **T** > **P** > **V** > **X**	**L**
q-RPF VIKOR method Akram and Shumaiza (2021)	**L** > **N** > **R** > **T** > **V** > **X** > **P**	**L**
q-ROF CRITIC-VIKOR method Yang et al. (2022)	**L** > **N** > **P** > **V** > **X** > **R** > **T**	**L**

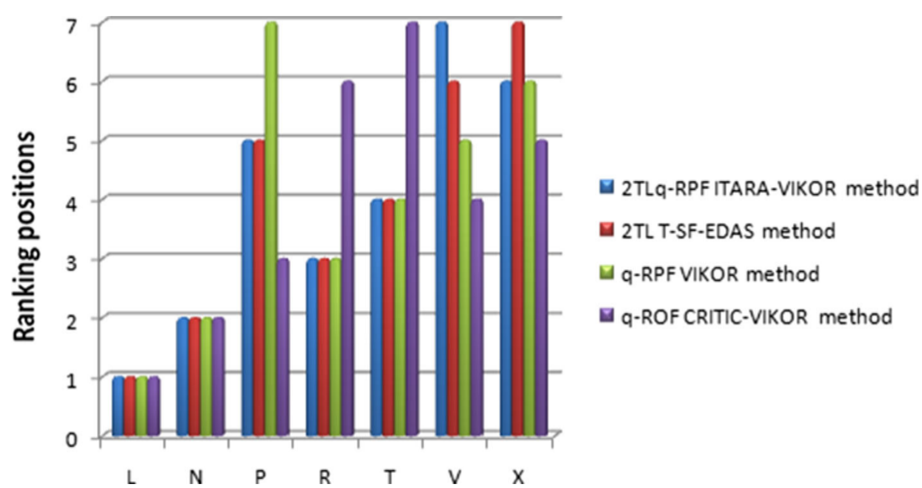

Fig. 7 Ranking positions using proposed and existing methods

- Furthermore, we aim to expand the VIKOR approach by integrating it with other methodologies to compute criteria weights, such as CRITIC, DEMATEL, and SWARA.
- The suggested research can be expanded by increasing the number of experts', alternatives, and criteria. The presented model effectiveness can be verified by applying it to different domains. We will extend our work to other study fields, such as supply chain management and renewable energy resources, to investigate the applicability of the current work.

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Declarations

Conflict of interest The authors declare no conflict of interest.

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