

Heat transfer analysis in hydromagnetic flow of couple stress fluid in presence of homogeneous and heterogeneous chemical reactions over a porous oscillatory stretchable sheet

Muhammad Naveed¹ , Muhammad Imran¹ and Sana Gul²

Abstract

The prime objective of this theoretical study is to investigate the flow, mass, and heat transfer in the time-dependent couple stress fluid over a permeable stretching oscillatory surface. The effect of applied magnetic field is also taken in the momentum equation. In addition, the energy equation is modeled by incorporating thermal radiation and heat generation. Furthermore, the concentration equation is designed in presence of homogenous and heterogeneous chemical reactions. The flow mechanism occurs because of the to and fro motion of the surface about a fixed point. For similar solution of the developed flow problem suitable similarity transformations is adopted. The analytical solution of the obtained flow equations in the form of partial differential equations (PDEs) is attained by utilizing a valuable method called Homotopy analysis method (HAM). An extensive assessment is performed to study the influence of various involved parameters namely the couple stress parameter, the relation of the parameter of the surface oscillatory frequency to its stretched rate constant, the Prandtl number, the strength of heterogeneous and homogeneous chemical reactions parameters, Schmidt number, heat production parameter and magneto-porosity parameter on the flow, concentration and temperature distributions, skin friction coefficient, and rate of heat transfer via tables and graphs and are discussed in detail. It is observed that the amplitude of velocity and skin friction coefficient shows a declining response with varying values of the couple stress parameter and it increases for higher values of the magneto-porous parameter.

Keywords

Porous oscillatory stretchable surface, magnetohydrodynamic (MHD), porous medium, couple stress fluid, homotopy analysis method, homogeneous-heterogeneous chemical reactions

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Introduction

Due to the numerous remarkable applications in many manufacturing and engineering processes, a number of engineers and researchers have considered the exploration of boundary layer stretching flows of non-Newtonian fluids with different velocities such as linear, nonlinear, exponential, and oscillatory during the last few decades. The most valuable industrial processes of motion of non-Newtonian liquids are wire drawing,

¹Department of Mathematics, Division of Science and Technology, University of Education, Lahore, Pakistan

²Department of Mathematics, Khwaja Fareed University of Engineering & Information Technology, Rahim Yar Khan, Pakistan

Corresponding author:

Muhammad Imran, Department of Mathematics, Division of Science and Technology, University of Education, Lahore 54770, Pakistan.
Email: muhammadimran6490@gmail.com



artificial fibers, hot rolling, glass blowing, aerodynamic extrusion of plastic sheets, paper production, polymer extrusion process, distillation of towers and cooling of electronic chips etc. In all of these processes, the wall stretching velocity and stretching rate have a significant impact on the final product's superiority. To the best of our information, Crane¹ was the first who scrutinized the liquid flow problem over stretchable surface by taking into account linear velocity and find out its solution in exact closed form. Because of such most importance of non-Newtonian fluids in industrial procedures, a number of researchers have expanded the Crane concept for non-Newtonian fluids in various directions over curved and flat stretching surfaces. Rasool et al.² has investigated magnetohydrodynamic Darcy-Forchhemier nanoliquid flow over nonlinear stretching sheet. Bhatti et al.³ has performed a numerical study to compute the numerical solution of activation energy and gyrotactic microorganism in magnetized nanofluid across a porous plate. Bhatti et al.⁴ has studied the magnetized flow of hybrid nanofluid across a flat elastic surface in the presence of solar energy. In another research article, Bhatti et al.⁵ performed a numerical study of hybrid nanofluid flow over a circular stretching surface with non-Darcy medium. On the other hand, in many manufacturing procedures such as metallurgy and textiles, a fascinating situation might arise when the stretching surface moves consistently with periodic velocity in a back and forth direction around its plane. Analysis of heat transfer based on modified Fourier's law in flow of Maxwell fluid over an oscillatory stretching surface was done by Sami et al.⁶ In another article, Sami and Shahzad⁷ has considered the effects of thermophoresis in third grade nanofluid across oscillatory moving surface. Naveed et al.⁸ has discussed the effects of heat transport phenomena on curvilinear flow of micropolar liquid across an oscillatory curved stretched sheet. Imran et al.⁹ has investigated the consequences of thermal and mass relaxation parameters on Eyring-Powell fluid past over a curved oscillatory wall.

Couple stress fluid is a supreme significant type of non-Newtonian liquid which was initially presented by Stokes.¹⁰ Couple stress theory is simply an extension of conservative viscous notion that supports couple stress and couples in body. Examples of couple stress liquid are colloidal liquids, lubricants, and blood which contain electro-rheological and synthetic properties. Devakar et al.¹¹ has performed an analytical research for motion of couple stress liquid with slip boundary conditions. Ali et al.¹² has analyzed the various aspects of heat transport phenomenon on couple stress liquid motion due to the oscillation of oscillatory flat stretching surface in the existence of permeable material. Sami et al.¹³ has discovered analytical results of the mixed convective flow of couple stress nanoliquid on a stretchable oscillatory surface. Alabdan et al.¹⁴ has considered

the impacts of variable viscosity for flow of couple stress nanoliquid due to oscillatory stretched sheet.

The inspection of hydromagnetic motion of an electrically conducting liquid with heat production and convective heat transport mechanism has attained a great deal of attention from various investigators, researchers and engineers due to its large number of practical utilizations in many industrial and technological operations likely in the designing purpose of cooling systems, hydromagnetic generators, storage of thermal energy, cooling of underground electric cables, gas turbine, electric motors, solar power technology, pumps, and space vehicle reentry. Sheikh and Abbas¹⁵ have considered the impacts of heat production on flow of hydromagnetic fluid across stretchable oscillatory sheet. Some researchers like Sami¹⁶ and Imran et al.¹⁷ inspected various fluid motion problems across oscillatory stretchable surfaces with heat production. Sami et al.¹⁸ has evaluated the effects of magneto-nanoparticles in non-Newtonian liquid motion across convectively oscillating moving surface. Khan et al.¹⁹ has calculated numerical simulation of hydromagnetic Power law fluid flow with convective boundary condition. The outcomes of thermal radiation on viscous fluid motion across the convectively heated oscillatory curved stretchable sheet were investigated by Abbas et al.²⁰ Naveed et al.²¹ has scrutinized the analytical results of Williamson liquid motion across convectively heated curved oscillatory Riga surface.

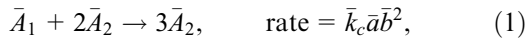
Many chemically reactive structures such as catalysis, biomedical processes, and combustion comprise both homogeneous and heterogeneous chemical reactions. The correlation between such chemical reactions is very complex. Physical examples of such processes in which homogeneous and heterogeneous chemical reactions occur include dispersion and fog deposition, food processing, ceramics manufacturing, polymers and hydrometallurgical processes. Waini et al.²² has investigated hybrid nanofluid flow with homogeneous and heterogeneous chemical reactions. Zeb et al.²³ has calculated the consequences of homogeneous heterogeneous chemical reactions on flow of non-Newtonian ferrofluid toward a stretching sheet. Effects of homogeneous and heterogeneous chemical reactions on motion of Casson nanoliquid due to curved oscillatory sheet were analyzed by Abbas et al.²⁴ Upreti et al.²⁵ has investigated the homogeneous-heterogeneous reactions effects in sisko nanofluid flow past across convectively heated stretching sheet.

A systematic examination of literature survey reflects that no one has yet studied the flow and transport mechanism (mass and heat transfer) in a couple stress fluid on an oscillatory stretching surface. Therefore, the prime motivation of the present study is to study the flow, heat, and mass transfer analysis in a non-Newtonian couple stress fluid. Furthermore, the

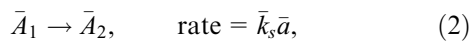
impacts of heat generation along with thermal radiation is also considered in energy equation. The consequences of both homogeneous and heterogeneous chemical reactions are also incorporated in the concentration equation. The developed flow equations in the form of nonlinear PDEs are solved analytically by employing an effective technique known as Homotopy analysis method. The influence of pertinent parameters on velocity, temperature and concentration fields are shown graphically and in tabular form and are discussed in detail.

Problem explanation

Consider the two dimensional and time-dependent boundary layer flow of an incompressible couple stress liquid across a porous stretchable oscillatory sheet (see Figure 1). The sheet oscillates about a fixed point with oscillatory velocity $\bar{w} = \bar{c}\bar{x}\sin\bar{\omega}\bar{t}$, which is considered the cause of flow mechanism. Here $(\bar{c}, \bar{\omega})$ denotes the stretching rate and oscillatory frequency of the surface respectively. A persistent magnetic field having strength B_0 is enforced along the normal direction of the oscillatory surface. Induced magnetic field impacts are excluded by choosing extremely low Reynolds numbers. Let the oscillatory wall temperature is $\bar{T}_w$ and the ambient temperature is $\bar{T}_\infty$ with $\bar{T}_\infty < \bar{T}_w$. Additionally, homogeneous and heterogeneous chemical reactions are considered for two species $\bar{A}_1$ and $\bar{A}_2$ respectively. The homogeneous chemical process for cubic autocatalysis is stated as



And heterogeneous chemical reaction on the catalyst sheet surface is



Here $\bar{a}$ and $\bar{b}$ represents the concentrations of two chemical species $\bar{A}_1$ and $\bar{A}_2$, whereas $\bar{k}_c$ and $\bar{k}_s$ are rate constants. By letting the aforementioned assumptions, the boundary layer equations for chemically reactive flow of couple stress fluid can be written as

$$\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{w}}{\partial \bar{y}} = 0, \quad (3)$$

$$\frac{\partial \bar{w}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{w}}{\partial \bar{x}} + \bar{w} \frac{\partial \bar{w}}{\partial \bar{y}} = \bar{\nu} \frac{\partial^2 \bar{w}}{\partial \bar{y}^2} - \bar{\eta}_0 \left(\frac{\partial^4 \bar{w}}{\partial \bar{y}^4} \right) - \frac{\bar{\sigma} B_0^2}{\bar{\rho}} \bar{w} - \frac{\bar{\nu} \phi}{\bar{k}} \bar{w}, \quad (4)$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{w} \frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} = \bar{\alpha} \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} - \frac{1}{\bar{\rho} \bar{c}_p} \frac{\partial \bar{q}_r}{\partial \bar{y}} + \frac{\bar{Q}}{\bar{\rho} \bar{c}_p} (\bar{T} - \bar{T}_\infty), \quad (5)$$

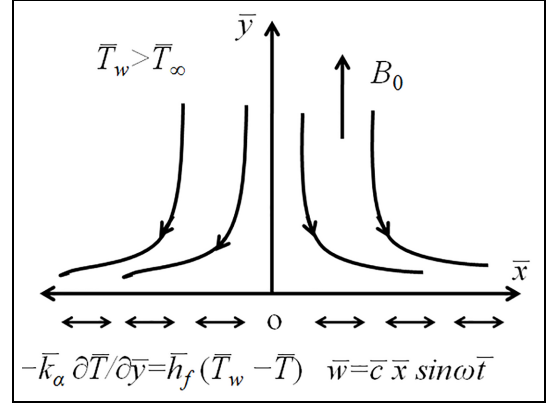


Figure 1. Schematic flow geometry.

$$\frac{\partial \bar{a}}{\partial \bar{t}} + \bar{w} \frac{\partial \bar{a}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{a}}{\partial \bar{y}} = \bar{D}_A \left(\frac{\partial^2 \bar{a}}{\partial \bar{y}^2} \right) - k_c \bar{a} \bar{b}^2, \quad (6)$$

$$\frac{\partial \bar{b}}{\partial \bar{t}} + \bar{w} \frac{\partial \bar{b}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{b}}{\partial \bar{y}} = \bar{D}_B \left(\frac{\partial^2 \bar{b}}{\partial \bar{y}^2} \right) + k_c \bar{a} \bar{b}^2, \quad (7)$$

The associated boundary conditions are

$$\left. \begin{aligned} \bar{v} = -\bar{v}_w, \quad \bar{w} = \bar{c}\bar{x}\sin\bar{\omega}\bar{t}, \quad \frac{\partial^2 \bar{w}}{\partial \bar{y}^2} \rightarrow 0, \quad \bar{D}_A \frac{\partial \bar{a}}{\partial \bar{y}} = \bar{k}_s \bar{a}, \\ \bar{D}_B \frac{\partial \bar{b}}{\partial \bar{y}} = -\bar{k}_s \bar{a}, \quad -\bar{k}_\alpha \frac{\partial \bar{T}}{\partial \bar{y}} = -\bar{h}_f (\bar{T} - \bar{T}_w) \text{ at } \bar{y} = 0, \\ \frac{\partial \bar{w}}{\partial \bar{r}} \rightarrow 0, \quad \bar{T} \rightarrow \bar{T}_\infty, \quad \bar{w} \rightarrow 0, \quad \bar{a} \rightarrow \bar{a}_0, \quad \bar{b} \rightarrow 0 \text{ as } \bar{y} \rightarrow \infty. \end{aligned} \right\} \quad (8)$$

here $(\bar{v}, \bar{w})$ are considered the fluid velocity parts along $(\bar{y}, \bar{x})$ directions, $(\bar{\rho}, \bar{\eta}_0, \bar{\nu}, \bar{T})$ are the density, couple stress viscosity, kinematic viscosity, and temperature, $\bar{v}_w > 0$ is the suction mass velocity and $\bar{v}_w < 0$ is the injection mass velocity, $\bar{c}_p$ is the specific heat capacitance, $(\bar{D}_A, \bar{D}_B)$ and are the diffusion coefficients and $\bar{q}_r$ indicates the radiative heat flux which is stated as

$$\bar{q}_r = -\frac{4\hat{\sigma}}{3\bar{k}_\gamma} \frac{\partial \bar{T}^4}{\partial \bar{y}}, \quad (9)$$

where $\bar{k}_\gamma$ expresses the absorption coefficient and $\hat{\sigma}$ represents the Stefan-Boltzmann constant.

By means of Taylor series, the term $\bar{T}^4$ which is used in equation (9) can be expanded as

$$\bar{T}^4 = 4\bar{T}_\infty^3 \bar{T} - 3\bar{T}_\infty^4. \quad (10)$$

With the help of equations (9) and (10), the energy equation (5) becomes

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{w} \frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} = \bar{\alpha} \left(\frac{16\bar{T}_\infty^3 \hat{\sigma}}{\bar{k}_\gamma \bar{k}_\alpha} + 1 \right) \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{\bar{Q}}{\bar{\rho} \bar{c}_p} (\bar{T} - \bar{T}_\infty), \quad (11)$$

We will initiate now the following dimensionless parameters as

$$\sqrt{\frac{\bar{c}}{\bar{\nu}}} = \bar{\eta}, \quad \bar{x}\bar{c}\bar{f}_{\bar{\eta}}(\bar{\eta}, \bar{\tau}) = \bar{w}, \quad \sqrt{\bar{\nu}\bar{c}\bar{f}}(\bar{\eta}, \bar{\tau}) = \bar{v}, \quad \bar{t}\bar{\omega} = \bar{\tau},$$

$$\bar{\theta}(\bar{\eta}, \bar{\tau}) = \bar{T} - \bar{T}_{\infty} / (\bar{T}_w - \bar{T}_{\infty}), \quad \bar{a} = \bar{a}_0\bar{\phi}(\bar{\xi}, \bar{\tau}), \quad \bar{b} = \bar{a}_0\bar{\varphi}(\bar{\xi}, \bar{\tau}). \quad (12)$$

By employing equation (12), equation (3) is proved identically, and rest equations (4), (6), (7), (8), and (11) yields

$$\bar{f}_{\bar{\eta}\bar{\eta}\bar{\eta}} + \bar{f}\bar{f}_{\bar{\eta}\bar{\eta}} - \bar{f}_{\bar{\eta}}^2 - S\bar{f}_{\bar{\eta}\bar{\tau}} - \delta\bar{f}_{\bar{\eta}\bar{\eta}\bar{\eta}\bar{\eta}} - \gamma\bar{f}_{\bar{\eta}} = 0. \quad (13)$$

$$(Rd + 1)\bar{\theta}_{\bar{\eta}\bar{\eta}} + Pr(\bar{f}\bar{\theta}_{\bar{\eta}} - S\bar{\theta}_{\bar{\tau}} + \lambda\bar{\theta}) = 0, \quad (14)$$

$$\frac{1}{Sc}\bar{\phi}_{\bar{\eta}\bar{\eta}} + \bar{f}\bar{\phi}_{\bar{\eta}} - S\bar{\phi}_{\bar{\tau}} - K_1\bar{\phi}\bar{\phi}^2 = 0, \quad (15)$$

$$\frac{\delta}{Sc}\bar{\varphi}_{\bar{\eta}\bar{\eta}} + \bar{f}\bar{\varphi}_{\bar{\eta}} - S\bar{\varphi}_{\bar{\tau}} - K_1\bar{\phi}\bar{\varphi}^2 = 0, \quad (16)$$

The associated boundary conditions has the following dimensionless form

$$\left. \begin{aligned} \bar{f}(0, \bar{\tau}) &= \alpha, \quad \bar{f}_{\bar{\eta}\bar{\eta}\bar{\eta}}(0, \bar{\tau}) = 0, \quad \bar{\theta}_{\bar{\eta}}(0, \bar{\tau}) = \beta(\bar{\theta}(0, \bar{\tau}) - 1), \quad \bar{f}_{\bar{\eta}}(0, \bar{\tau}) = \sin\bar{\tau}, \\ \bar{\phi}_{\bar{\eta}}(0, \bar{\tau}) &= K_2\bar{\phi}(0, \bar{\tau}), \quad \delta\bar{\varphi}_{\bar{\eta}}(0, \bar{\tau}) = -K_2\bar{\phi}(0, \bar{\tau}), \\ \bar{f}_{\bar{\eta}}(\infty, \bar{\tau}) &= 0, \quad \bar{f}_{\bar{\eta}\bar{\eta}}(\infty, \bar{\tau}) = 0, \quad \bar{\theta}(\infty, \bar{\tau}) = 0, \quad \bar{\phi}(\infty, \bar{\tau}) = 1, \quad \bar{\varphi}(\infty, \bar{\tau}) = 0. \end{aligned} \right\} \quad (17)$$

In above equations $\delta(= \bar{\eta}_0 \bar{c} / \bar{\rho} \bar{\nu}^2)$ is the dimensionless couple stress parameters, $S(= \bar{\omega} / \bar{c})$ is the relation parameter of surface oscillation frequency parameter to its stretched constant parameter, $\alpha(= -\bar{v}_w / \sqrt{\bar{\nu} \bar{c}})$ is the suction/injection parameter, $Pr(= \bar{\nu} / \bar{\alpha})$ is the Prandtl number, $\gamma(= \bar{\sigma} B_0^2 / \bar{\rho} \bar{c} + \bar{\nu} \varphi / \bar{k} \bar{c})$ is the magnetoporous parameter, $\lambda(= \bar{Q} / \bar{c} \bar{\rho} \bar{c}_p)$ is the heat production parameter, $K_1(= \bar{k}_c \bar{a}_0^2 / \bar{c})$ is the homogeneous reaction strength parameter, $Rd = 16\bar{T}_{\infty}^3 \bar{\sigma} / 3\bar{k}_{\gamma} \bar{k}_{\alpha}$ is the radiation parameter, and $K_2(= \bar{k}_s / \bar{D}_A \sqrt{\bar{c} / \bar{\nu}})$ is the heterogeneous reaction strength parameter respectively.

Here it is presumed that $\bar{D}_A = \bar{D}_B$, and by letting

$$\bar{\phi}(\bar{\eta}, \bar{\tau}) = 1 - \bar{\varphi}(\bar{\eta}, \bar{\tau}). \quad (18)$$

Then equations (15) and (16) yields

$$\bar{\phi}_{\bar{\eta}\bar{\eta}} + Sc(\bar{f}\bar{\phi}_{\bar{\eta}} - S\bar{\phi}_{\bar{\tau}} - K_1(1 - \bar{\phi})^2\bar{\phi}) = 0, \quad (19)$$

Subject to boundary conditions

$$\left. \begin{aligned} \bar{f}_{\bar{\eta}}(0, \bar{\tau}) &= \sin\bar{\tau}, \quad \bar{f}(0, \bar{\tau}) = \alpha, \quad \bar{\theta}_{\bar{\eta}}(0, \bar{\tau}) = \beta(\bar{\theta}(0, \bar{\tau}) - 1), \\ \bar{f}_{\bar{\eta}\bar{\eta}\bar{\eta}}(0, \bar{\tau}) &= 0, \quad \bar{\phi}_{\bar{\eta}}(0, \bar{\tau}) = K_2\bar{\phi}(0, \bar{\tau}), \\ \bar{\theta}(\infty, \bar{\tau}) &= 0, \quad \bar{f}_{\bar{\eta}}(\infty, \bar{\tau}) = 0, \quad \bar{f}_{\bar{\eta}\bar{\eta}}(\infty, \bar{\tau}) = 0, \quad \bar{\phi}(\infty, \bar{\tau}) = 1. \end{aligned} \right\} \quad (20)$$

The surface drag force, as well as the heat transmission rate along the sheet is characterized as

$$C_f = \frac{\tau_{\bar{x}\bar{y}}}{\bar{\rho} \bar{u}_w^2}, \quad \bar{N}u_x = \frac{\bar{s} \bar{q}_w}{(\bar{T}_w - \bar{T}_{\infty}) \bar{k}_{\alpha}}, \quad (21)$$

Here, $\tau_{\bar{x}\bar{y}}$ and $\bar{q}_w$ are depicted by

$$\tau_{\bar{x}\bar{y}} = \left(\bar{\mu} \left(\frac{\partial \bar{w}}{\partial \bar{y}} \right) - \bar{\eta}_0 \left(\frac{\partial^3 \bar{w}}{\partial \bar{y}^3} \right) \right) \Big|_{\bar{y}=0}, \quad \bar{q}_w = - \left(\bar{k}_{\alpha} \frac{\partial \bar{T}}{\partial \bar{y}} \right) \Big|_{\bar{y}=0}. \quad (22)$$

Making use of equation (12) in equations (21)–(22), we obtain

$$\sqrt{Re_s} C_f = \bar{f}_{\bar{\eta}\bar{\eta}}(0, \bar{\tau}) - \lambda \bar{f}_{\bar{\eta}\bar{\eta}\bar{\eta}\bar{\eta}}(0, \bar{\tau}), \quad (23)$$

$$\bar{N}u_x / \sqrt{Re_s} = -\bar{\theta}_{\bar{\eta}}(0, \bar{\tau}). \quad (24)$$

Here $\sqrt{Re_s} = \bar{c} \bar{x}^2 / \bar{\nu}$ represents the Reynolds number.

Solution method (HAM)

In this section, we have discussed the Homotopy analysis method, which is used to compute the analytical solution in series form of the governing equations (13), (14), and (19) with corresponding boundary conditions indicated in equation (20) by considering the following expressions²⁴

$$\bar{\phi}_0(\bar{\eta}, \bar{\tau}) = \frac{\beta}{1 + \beta} \exp(-\bar{\eta}), \quad \bar{\theta}_0(\bar{\eta}, \bar{\tau}) = \left(1 - \frac{1}{2} \exp(-K_2 \bar{\eta}) \right),$$

$$\bar{f}_0(\bar{\eta}, \bar{\tau}) = \alpha + \frac{\sin\bar{\tau}(3 - 3\exp(-\bar{\eta}) - \bar{\eta}\exp(-\bar{\eta}))}{2}, \quad (25)$$

$$\bar{M}_{\bar{\phi}}(\bar{\phi}) = \bar{\phi}_{\bar{\eta}\bar{\eta}} - \bar{\phi}, \quad \bar{M}_{\bar{\theta}}(\bar{\theta}) = \bar{\theta}_{\bar{\eta}\bar{\eta}} - \bar{\theta}, \quad \bar{M}_{\bar{f}}(\bar{f}) = \bar{f}_{\bar{\eta}\bar{\eta}\bar{\eta}} - \bar{f}_{\bar{\eta}}, \quad (26)$$

That satisfying

$$\begin{aligned} \bar{M}_{\bar{f}}[\bar{C}_3 \exp(\bar{\eta}) + \bar{C}_2 \exp(-\bar{\eta}) + \bar{C}_1] &= 0, \\ \bar{M}_{\bar{\theta}}[\bar{C}_4 \exp(\bar{\eta}) + \bar{C}_5 \exp(-\bar{\eta})] &= 0, \\ \bar{M}_{\bar{\phi}}[\bar{C}_6 \exp(\bar{\eta}) + \bar{C}_7 \exp(-\bar{\eta})] &= 0, \end{aligned} \quad (27)$$

Here $\bar{C}_i$ ($i = 1 - 7$) are denotes some arbitrary variables.

If $\bar{h}_\phi$, $\bar{h}_f$ and $\bar{h}_\theta$ are non- zero auxiliary variables then the zeroth-order deformation equations are established as follows

$$(1 - \bar{k}_1)\bar{M}_f[\bar{f}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1) - \bar{f}_0^*(\bar{\eta}, \bar{\tau})] = \bar{h}_f\bar{k}_1\bar{Q}_f[\bar{f}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)], \quad (28)$$

$$(1 - \bar{k}_1)\bar{M}_\theta[\bar{\theta}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1) - \bar{\theta}_0^*(\bar{\eta}, \bar{\tau})] = \bar{h}_\theta\bar{k}_1\bar{Q}_\theta[\bar{\theta}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)], \quad (29)$$

$$(1 - \bar{k}_1)\bar{M}_\phi[\bar{\phi}^*(\bar{\xi}, \bar{\tau}; \bar{k}_1) - \bar{\phi}_0^*(\bar{\xi}, \bar{\tau})] = \bar{h}_\phi\bar{k}_1\bar{Q}_\phi[\bar{\phi}^*(\bar{\xi}, \bar{\tau}; \bar{k}_1)], \quad (30)$$

Subject to boundary conditions

$$\left. \begin{aligned} \frac{\partial \bar{\theta}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\eta}} \Big|_{\bar{\eta}=0} &= \beta \left(\bar{\theta}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1) \Big|_{\bar{\eta}=0} - 1 \right), \quad \frac{\partial \bar{f}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\eta}} \Big|_{\bar{\eta}=0} = \sin \bar{\tau}, \quad \bar{f}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1) \Big|_{\bar{\eta}=0} = \alpha, \\ \frac{\partial^3 \bar{f}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\eta}^3} \Big|_{\bar{\eta}=0} &= 0, \quad \frac{\partial \bar{\phi}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\eta}} \Big|_{\bar{\eta}=0} = K_2 \bar{\phi}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1) \Big|_{\bar{\eta}=0} \\ \frac{\partial \bar{f}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\eta}} \Big|_{\bar{\eta}=\infty} &= \bar{\theta}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1) \Big|_{\bar{\eta}=\infty} = \frac{\partial^2 \bar{f}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\eta}^2} \Big|_{\bar{\eta}=\infty} = \bar{\phi}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1) \Big|_{\bar{\eta}=\infty} = 0. \end{aligned} \right\} \quad (31)$$

Where $k_1 \in [0, 1]$ implies an embedding parameter and the associated non- linear operators are of the form

$$\begin{aligned} \bar{Q}_f[\bar{f}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)] &= -\delta \frac{\partial^5 \bar{f}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\eta}^5} + \frac{\partial^3 \bar{f}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\eta}^3} - \frac{\partial^2 \bar{f}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\eta} \partial \bar{\tau}} S \\ &+ \frac{\partial^2 \bar{f}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\eta}^2} \bar{f}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1) - \left(\frac{\partial \bar{f}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\eta}} \right)^2 - \gamma \frac{\partial \bar{f}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\eta}}, \end{aligned} \quad (32)$$

$$\begin{aligned} \bar{Q}_\theta[\bar{\theta}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1), \bar{\phi}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1), \bar{f}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)] &= \frac{\partial^2 \bar{\theta}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\eta}^2} (1 + Rd) - SPr \frac{\partial \bar{\theta}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\tau}} \\ &+ Pr \left(\bar{f}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1) \frac{\partial \bar{\theta}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\eta}} + \lambda \bar{\theta} \right) \end{aligned} \quad (33)$$

$$\begin{aligned} \bar{Q}_\phi[\bar{\phi}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1), \bar{\theta}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1), \bar{f}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)] &= \frac{\partial^2 \bar{\phi}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\eta}^2} + Sc \bar{f}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1) \frac{\partial \bar{\phi}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\eta}} \\ &+ Sc \left(-K_1 (1 - \bar{\phi}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1))^2 \bar{\phi}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1) - S \frac{\partial \bar{\phi}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\tau}} \right), \end{aligned} \quad (34)$$

Put $\bar{k}_1 = 0$ and $\bar{k}_1 = 1$ in equations (28), (29), and (30), we obtain following solutions respectively

$$\bar{f}^*(\bar{\eta}, \bar{\tau}; 0) = \bar{f}_0^*(\bar{\eta}, \bar{\tau}), \quad \bar{f}^*(\bar{\eta}, \bar{\tau}; 1) = \bar{f}^*(\bar{\eta}, \bar{\tau}), \quad (35)$$

$$\bar{\theta}^*(\bar{\eta}, \bar{\tau}; 0) = \bar{\theta}_0^*(\bar{\eta}, \bar{\tau}), \quad \bar{\theta}^*(\bar{\eta}, \bar{\tau}; 1) = \bar{\theta}^*(\bar{\eta}, \bar{\tau}), \quad (36)$$

And

$$\bar{\phi}^*(\bar{\eta}, \bar{\tau}; 0) = \bar{\phi}_0^*(\bar{\eta}, \bar{\tau}), \quad \bar{\phi}^*(\bar{\eta}, \bar{\tau}; 1) = \bar{\phi}^*(\bar{\eta}, \bar{\tau}), \quad (37)$$

By employing Taylor series, we can develop $\bar{f}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)$, $\bar{\theta}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)$ and $\bar{\phi}^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)$ as

$$\bar{f}^*(\bar{\eta}, \bar{\tau}, \bar{k}_1) = \bar{f}_0^*(\bar{\eta}, \bar{\tau}) + \sum_{n=1}^{\infty} \bar{f}_n^*(\bar{\eta}, \bar{\tau}) \bar{k}_1^n, \quad \bar{f}_n^*(\bar{\eta}, \bar{\tau}) = \frac{1}{n!} \frac{\partial^n \bar{f}^*(\bar{\eta}, \bar{\tau}, \bar{k}_1)}{\partial \bar{k}_1^n} \Big|_{\bar{k}_1=0}, \quad (38)$$

$$\bar{\theta}^*(\bar{\eta}, \bar{\tau}, \bar{k}_1) = \bar{\theta}_0^*(\bar{\eta}, \bar{\tau}) + \sum_{n=1}^{\infty} \bar{\theta}_n^*(\bar{\eta}, \bar{\tau}) \bar{k}_1^n, \quad \bar{\theta}_n^*(\bar{\eta}, \bar{\tau}) = \frac{1}{n!} \frac{\partial^n \bar{\theta}^*(\bar{\eta}, \bar{\tau}, \bar{k}_1)}{\partial \bar{k}_1^n} \Big|_{\bar{k}_1=0}, \quad (39)$$

$$\bar{\phi}^*(\bar{\eta}, \bar{\tau}, \bar{k}_1) = \bar{\phi}_0^*(\bar{\eta}, \bar{\tau}) + \sum_{n=1}^{\infty} \bar{\phi}_n^*(\bar{\eta}, \bar{\tau}) \bar{k}_1^n, \quad \bar{\phi}_n^*(\bar{\eta}, \bar{\tau}) = \frac{1}{n!} \frac{\partial^n \bar{\phi}^*(\bar{\eta}, \bar{\tau}, \bar{k}_1)}{\partial \bar{k}_1^n} \bigg|_{\bar{k}_1=0}, \quad (40)$$

The convergence of the accomplished equations (38), (39), and (40) are highly influenced by the range of $\bar{h}_f$, $\bar{h}_\theta$ and $\bar{h}_\phi$. Consequently $\bar{h}_f$, $\bar{h}_\theta$ and $\bar{h}_\phi$ should be precisely designated so that the above equations (38), (39), and (40) are converge at $\bar{k}_1 = 1$.

Hence using equations (35), (36), and (37) in equations (38), (39), and (40), we have

$$\bar{f}^*(\bar{\eta}, \bar{\tau}; 1) = \bar{f}_0^*(\bar{\eta}, \bar{\tau}) + \sum_{n=1}^{\infty} \bar{f}_n^*(\bar{\eta}, \bar{\tau}), \quad (41)$$

$$\bar{\theta}^*(\bar{\eta}, \bar{\tau}; 1) = \bar{\theta}_0^*(\bar{\eta}, \bar{\tau}) + \sum_{n=1}^{\infty} \bar{\theta}_n^*(\bar{\eta}, \bar{\tau}), \quad (42)$$

$$\bar{\phi}^*(\bar{\eta}, \bar{\tau}; 1) = \bar{\phi}_0^*(\bar{\eta}, \bar{\tau}) + \sum_{n=1}^{\infty} \bar{\phi}_n^*(\bar{\eta}, \bar{\tau}), \quad (43)$$

Differentiate zeroth order deformation equations (28)–(30) and boundary condition equation (31) by n -times w.r.t embedding parameter $\bar{k}_1$, then set $\bar{k}_1 = 0$, and at the last divided by $n!$, on can attained

$$\bar{M}_f [\bar{f}_n^*(\bar{\eta}, \bar{\tau}) - \bar{\delta}_n \bar{f}_{n-1}^*(\bar{\eta}, \bar{\tau})] = R_n^{\bar{f}} \bar{h}_f^*(\bar{\eta}, \bar{\tau}), \quad (44)$$

$$\bar{M}_\theta [\bar{\theta}_n^*(\bar{\eta}, \bar{\tau}) - \bar{\delta}_n \bar{\theta}_{n-1}^*(\bar{\eta}, \bar{\tau})] = R_n^{\bar{\theta}} \bar{h}_\theta^*(\bar{\eta}, \bar{\tau}), \quad (45)$$

$$\bar{M}_\phi [\bar{\phi}_n^*(\bar{\eta}, \bar{\tau}) - \bar{\delta}_n \bar{\phi}_{n-1}^*(\bar{\eta}, \bar{\tau})] = R_n^{\bar{\phi}} \bar{h}_\phi^*(\bar{\eta}, \bar{\tau}), \quad (46)$$

$$\left. \begin{aligned} \frac{\partial \bar{\theta}_n^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\eta}} \bigg|_{\bar{\eta}=0} - \beta \bar{\theta}_n^*(\bar{\eta}, \bar{\tau}; \bar{k}_1) \bigg|_{\bar{\eta}=0} &= \frac{\partial \bar{f}_n^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\eta}} \bigg|_{\bar{\eta}=0} = \bar{f}_n^*(\bar{\eta}, \bar{\tau}; \bar{k}_1) \bigg|_{\bar{\eta}=0} = 0, \\ \frac{\partial^3 \bar{f}_n^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\eta}^3} \bigg|_{\bar{\eta}=0} &= 0, \quad \frac{\partial \bar{\phi}_n^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\eta}} \bigg|_{\bar{\eta}=0} - K_2 \bar{\phi}_n^*(\bar{\eta}, \bar{\tau}; \bar{k}_1) \bigg|_{\bar{\eta}=0} = 0, \\ \frac{\partial \bar{f}_n^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\eta}} \bigg|_{\bar{\eta}=\infty} &= \bar{\theta}_n^*(\bar{\eta}, \bar{\tau}; \bar{k}_1) \bigg|_{\bar{\eta}=\infty} = \frac{\partial^2 \bar{f}_n^*(\bar{\eta}, \bar{\tau}; \bar{k}_1)}{\partial \bar{\eta}^2} \bigg|_{\bar{\eta}=\infty} = \bar{\phi}_n^*(\bar{\eta}, \bar{\tau}; \bar{k}_1) \bigg|_{\bar{\eta}=\infty} = 0. \end{aligned} \right\} \quad (47)$$

Where

$$R_n^{\bar{f}} = -\delta \frac{\partial^5 \bar{f}_{n-1}^*}{\partial \bar{\eta}^5} + \frac{\partial^3 \bar{f}_{n-1}^*}{\partial \bar{\eta}^3} - \frac{\gamma \partial \bar{f}_{n-1}^*}{\partial \bar{\eta}} - \frac{S \partial^2 \bar{f}_{n-1}^*}{\partial \bar{\eta} \partial \bar{\tau}} + \sum_{i=0}^{n-1} \left(-\frac{\partial \bar{f}_{n-i-1}^*}{\partial \bar{\eta}} \frac{\partial \bar{f}_i^*}{\partial \bar{\eta}} + \bar{f}_{n-i-1}^* \frac{\partial^2 \bar{f}_i^*}{\partial \bar{\eta}^2} \right), \quad (48)$$

$$R_n^{\bar{\theta}} = (1 + Rd) \frac{\partial^2 \bar{\theta}_{n-1}^*}{\partial \bar{\xi}^2} - Spr \frac{\partial \bar{\theta}_{n-1}^*}{\partial \bar{\tau}} + Pr \left(\sum_{i=0}^{n-1} \bar{f}_{n-i-1}^* \frac{\partial \bar{\theta}_i^*}{\partial \bar{\xi}} - \bar{\theta}_{n-1}^* \lambda \right), \quad (49)$$

$$R_n^{\bar{\phi}} = \frac{\partial^2 \bar{\phi}_{n-1}^*}{\partial \bar{\xi}^2} - S \frac{\partial \bar{\phi}_{n-1}^*}{\partial \bar{\tau}} Sc - Sc K_1 \bar{\phi}_{n-1}^* - Sc K_1 \sum_{i=0}^{n-1} \left(2 \bar{\phi}_{n-i-1}^* \bar{\phi}_i^* + \bar{\phi}_{n-i-1}^* \sum_{j=0}^{n-1} \bar{\phi}_{i-j}^* \bar{\phi}_i^* \right), \quad (50)$$

And

$$\bar{\delta}_n = \begin{cases} 0 & n \leq 1, \\ 1 & n > 1. \end{cases} \quad (51)$$

Results and discussion

The principal objective of this part is to demonstrate the graphical and tabular outcomes of dissimilar involved flow parameters, likely the couple stress parameter (δ), ratio parameter of sheet oscillatory frequency parameter to its constant rate parameter (S), radiation parameter (Rd) and magneto-porous parameter (γ), heat production

Table 1. Comparison of the numerical outcomes of $\bar{f}''(0, \bar{\tau})$ with existing outcomes for dissimilar values of $\bar{\tau}$ when $\delta = 0$.

$\bar{\tau}$	Ali et al. ¹²	Sheikh and Abbas ¹⁵	Present results
$\bar{\tau} = 3\pi/2$	11.678656	11.678565	11.678657
$\bar{\tau} = 11\pi/2$	11.678707	11.678706	11.678707
$\bar{\tau} = 19\pi/2$	11.678656	11.678656	11.678656

Table 2. Numerical assessment of $(\text{Re}_x^{-1/2} \bar{N}u_x)$ at $\bar{\tau} = 0.5\pi$ with $S = \alpha = 0.1$, $\gamma = 1.5$ and $\delta = 1.0$ fixed.

λ	Pr	Rd	β	$\text{Re}_x^{-1/2} \bar{N}u_x$
0.5	1.0	0.5	0.2	0.249538
2.0				0.248909
3.5				0.248274
0.5	1.5			0.249619
	3.0			0.249863
	4.5			0.250108
	1.0	0.3		0.216338
		0.6		0.266130
		0.9		0.315872
		0.5	0.3	0.345269
			0.4	0.427215
			0.5	0.498153

parameter (λ) and homogeneous reaction strength variable (K_1), Biot number (β), Schmidt number (Sc), Prandtl number (Pr), suction/injection parameter (α) and heterogeneous reaction parameter (K_2) on the fluid motion velocity field, the concentration and temperature distribution, the drag surface force coefficient and on Nusselt number respectively in detail.

Table 1 is constructed to perform the comparison between the outcomes of the present research and the results of the existing literature by taking couple stress parameter (δ) = 0 at different time instants, and they are found to be in good agreement.

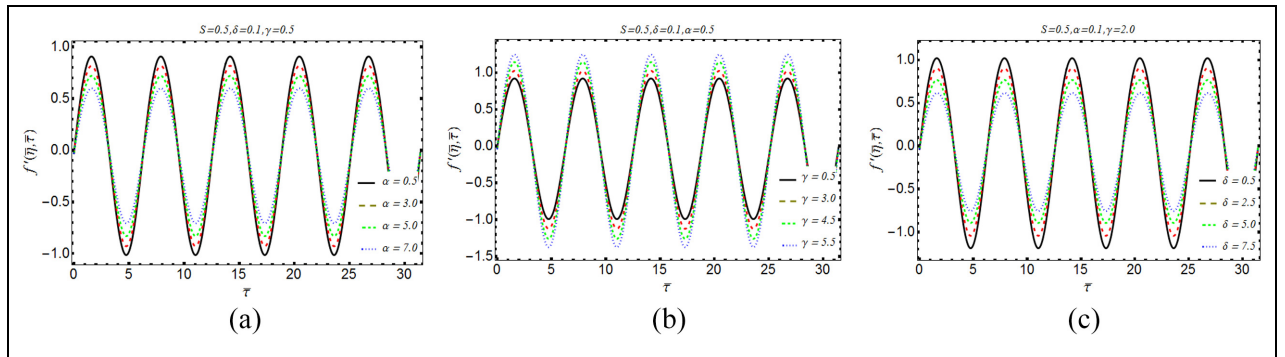
Table 2 detects that the magnitude of local Nusselt number depicts elevating behavior with uplifting values of (β), (Pr), (Rd). However, its magnitude decreases with heat production parameter (λ).

Figure 2(a) to (c) expound the consequences of (α), (γ), and (δ) on the profile of velocity $\bar{f}'(\bar{\eta}, \bar{\tau})$ at fixed position ($\bar{\eta} = 0.1$) from the wall of the sheet for intervals of time $0 \leq \bar{\tau} \leq 10\pi$. This figure clearly represents that the liquid velocity amplitude diminishes for rising values of suction/injection parameter (α) and parameter of couple stress (δ). Whereas, it grows for upward values of parameter (γ).

The influences of (α) on $\bar{f}'(\bar{\eta}, \bar{\tau})$ at some fixed time instant $\bar{\tau} = 8.5\pi$ and $\bar{\tau} = 9.5\pi$ are illustrates in Figure 3(a) and (b). The profile $\bar{f}'(\bar{\eta}, \bar{\tau})$ shows diminishing behavior with parametric values of (α). Figure 4(a) and (b) is drawn to explain the outcomes of (γ) on $\bar{f}'(\bar{\eta}, \bar{\tau})$ at $\bar{\tau} = 8.5\pi$ and $\bar{\tau} = 9.5\pi$ respectively. It is clearly detected that profile $\bar{f}'(\bar{\eta}, \bar{\tau})$ is decreased by an increase in (γ). Figure 5(a) and (b) presents the effects of couple stress variable (δ) on $\bar{f}'(\bar{\eta}, \bar{\tau})$ at $\bar{\tau} = 8.5\pi$ and $\bar{\tau} = 9.5\pi$. This figure confirmed that velocity profile is reduced for varying (δ). Figure 6(a) and (b) demonstrates the impacts of (S) on $\bar{f}'(\bar{\eta}, \bar{\tau})$ at $\bar{\tau} = 8.5\pi$ and $\bar{\tau} = 9.5\pi$ fixed. It is witnessed that $\bar{f}'(\bar{\eta}, \bar{\tau})$ profile is declining function of (S).

The variation in temperature distribution profile $\bar{\theta}(\bar{\eta}, \bar{\tau})$ under the influences of different parameters namely (β), (λ), (Pr) and (Rd) at $\bar{\tau} = 0.5\pi$ is discussed through Figure 7(a) to (d). From Figure 7(a), (b) and (d) it can be noticed that the temperature profile shows increasing response for improving values of (β), (λ), and (Rd). However, from Figure 7(c) it is clear that $\bar{\theta}(\bar{\eta}, \bar{\tau})$ is decreased by mounting values of (Pr).

The impacts of (K_1), (K_2), (S), and (Sc) on concentration distribution profile $\bar{\phi}(\bar{\xi}, \bar{\tau})$ at $\bar{\tau} = 0.5\pi$ is explained in Figure 8(a) to (d). Figure 8(a) and (c) show increasing response of concentration profile with varying values of (K_1) and (S), while Figure 8(b) and (d)

**Figure 2.** Variation in amplitude of $\bar{f}'(\bar{\eta}, \bar{\tau})$: (a) influences of α , (b) influences of γ , and (c) influences of δ by keeping other parameters.

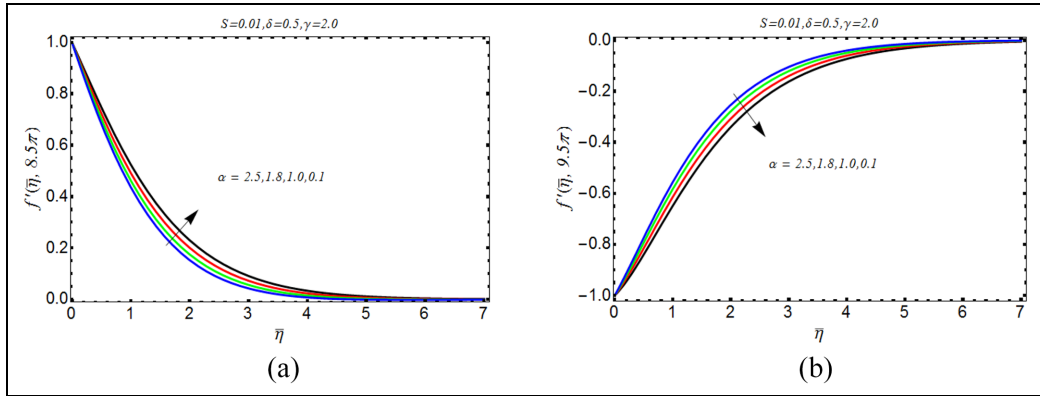


Figure 3. Consequences of α on $\bar{f}'(\bar{\eta}, \bar{\tau})$ for unlike intervals of time: (a) $\bar{\tau} = (8.5)\pi$ and (b) $\bar{\tau} = (9.5)\pi$.

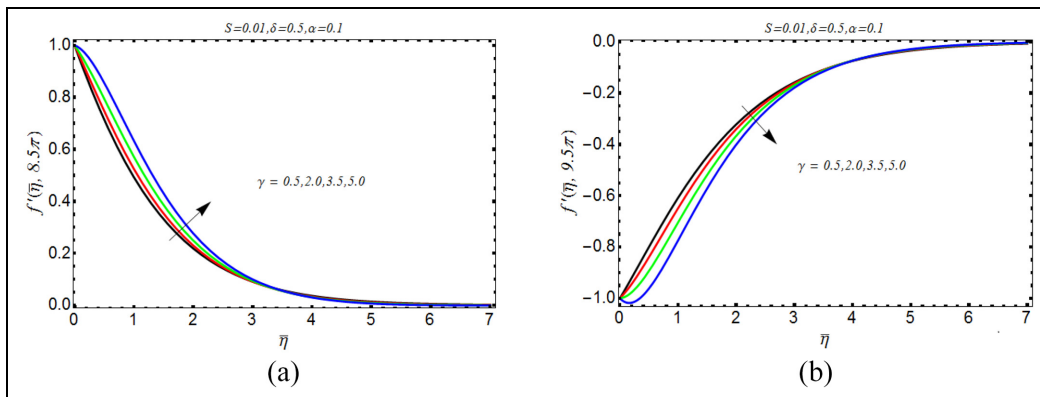


Figure 4. Impacts of γ on $\bar{f}'(\bar{\eta}, \bar{\tau})$: (a) $\bar{\tau} = (8.5)\pi$ and (b) $\bar{\tau} = (9.5)\pi$.

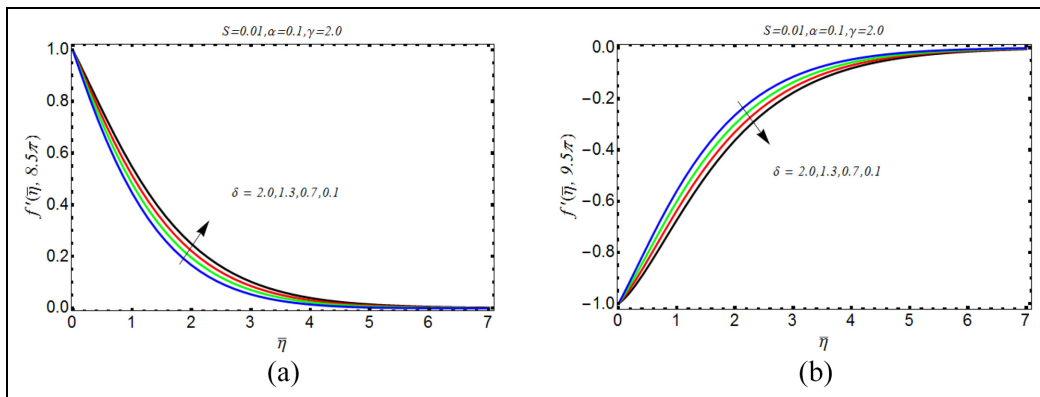


Figure 5. Consequences of δ on profile $\bar{f}'(\bar{\eta}, \bar{\tau})$ for unlike intervals of time: (a) $\bar{\tau} = (8.5)\pi$ and (b) $\bar{\tau} = (9.5)\pi$.

depicts reducing behavior of $\bar{\phi}(\bar{\xi}, \bar{\tau})$ for enhanced values (K_2) and (Sc).

Figure 9(a) to (d) interprets the influences of dissimilar parameters such as (α), (γ), (δ), and (S) on ($Re_s^{1/2}C_f$) for $\bar{\tau} \in (0, 10\pi)$. It is clearly seen that ($\sqrt{Re_s}C_f$) amplitude is diminished with (δ) and is enlarged with uplifting values of (γ), (α), and (S).

Figure 10(a) and (b) elucidates the consequences of (γ) versus (α) and (S) versus (δ) ($Re_s^{1/2}C_f$) at $\bar{\tau} = 0.5\pi$. The magnitude of ($\sqrt{Re_s}C_f$) is enhanced for with (δ) and (α), however it is declined with enhancing values of (γ) and (S).

Figure 11(a) to (d) illustrates the outcomes of various parameters (K_1), (Sc), (S), and (K_2) on surface

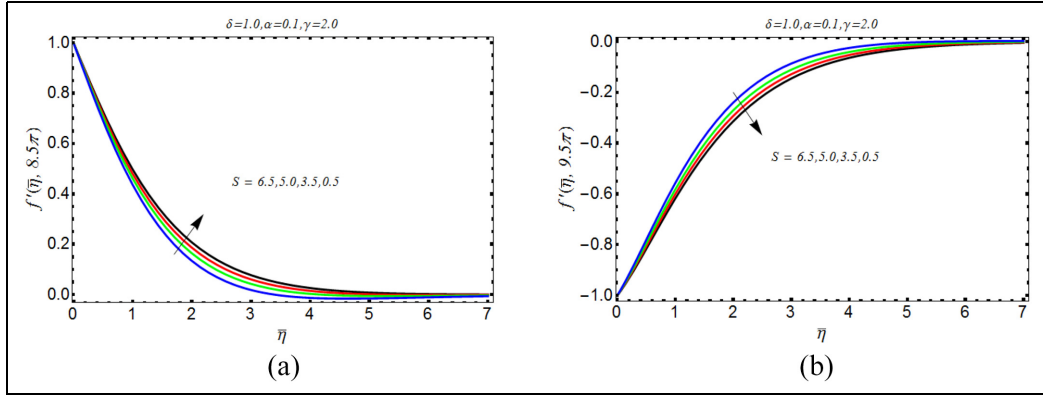


Figure 6. Consequences of S on $\bar{f}'(\eta, \bar{\tau})$: (a) $\bar{\tau} = (8.5)\pi$ (b) $\bar{\tau} = (9.5)\pi$.

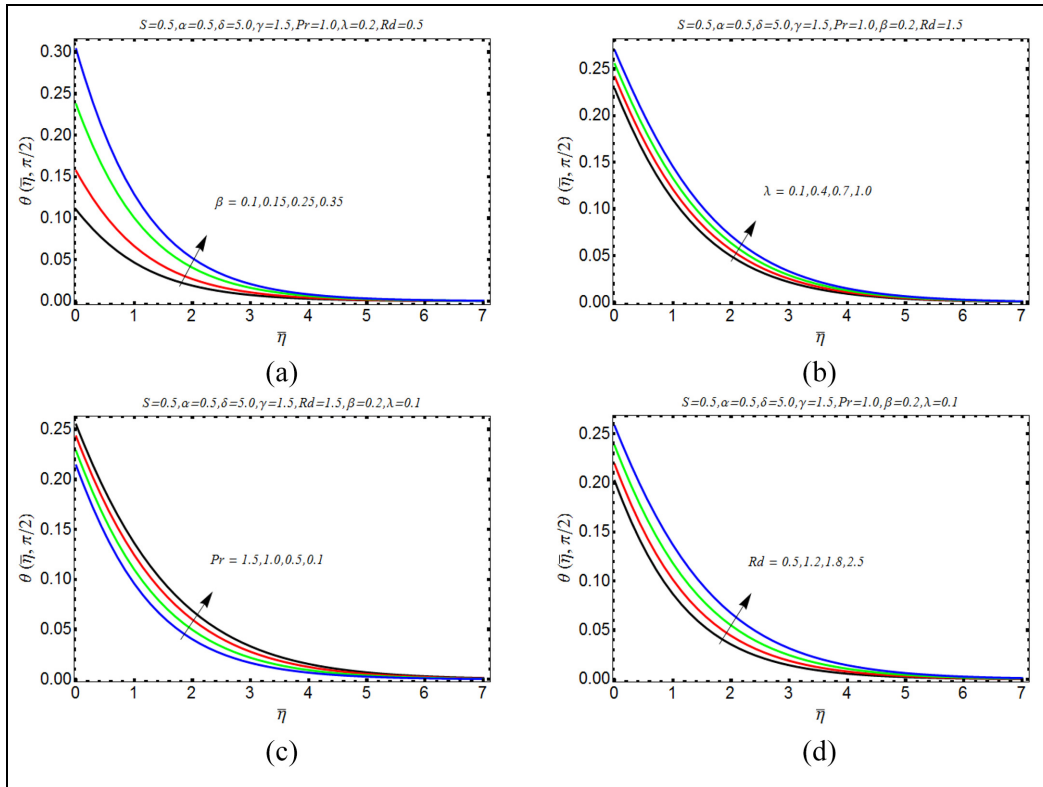


Figure 7. Change in temperature profile $\theta(\eta, \bar{\tau})$: (a) influences of β , (b) influences of λ , (c) influences of Pr , and (d) influences of Rd .

concentration profile $\bar{\phi}(0, \bar{\tau})$ at the instants $0 \leq \bar{\tau} \leq 10\pi$. It is noticed that the amplitude of $\bar{\phi}(0, \bar{\tau})$ is reduced with (K_1) and (K_2) . Whereas, it is increased with growing (S) and (Sc) .

Concluding remarks

In this research inquiry, the consequences of homogeneous and heterogeneous chemical reactions on thermally radiative unsteady flow of couple stress liquid over an oscillatory stretching porous surface have been

observed. Also the consequences of porous medium and constant magnetic field are considered in momentum equation. Here, the major key findings of the current assessment are presented as follows

- The $\bar{f}'(\eta, \bar{\tau})$ amplitude is declined for increasing values of (α) and (δ) . Whilst, it improved for (γ) .
- At time $\bar{\tau} = 8.5\pi$ and $\bar{\tau} = 9.5\pi$, the velocity of the profile show diminishing behavior with varying values of (α) , (δ) , and (S) . However, it shows increasing response with (γ) .

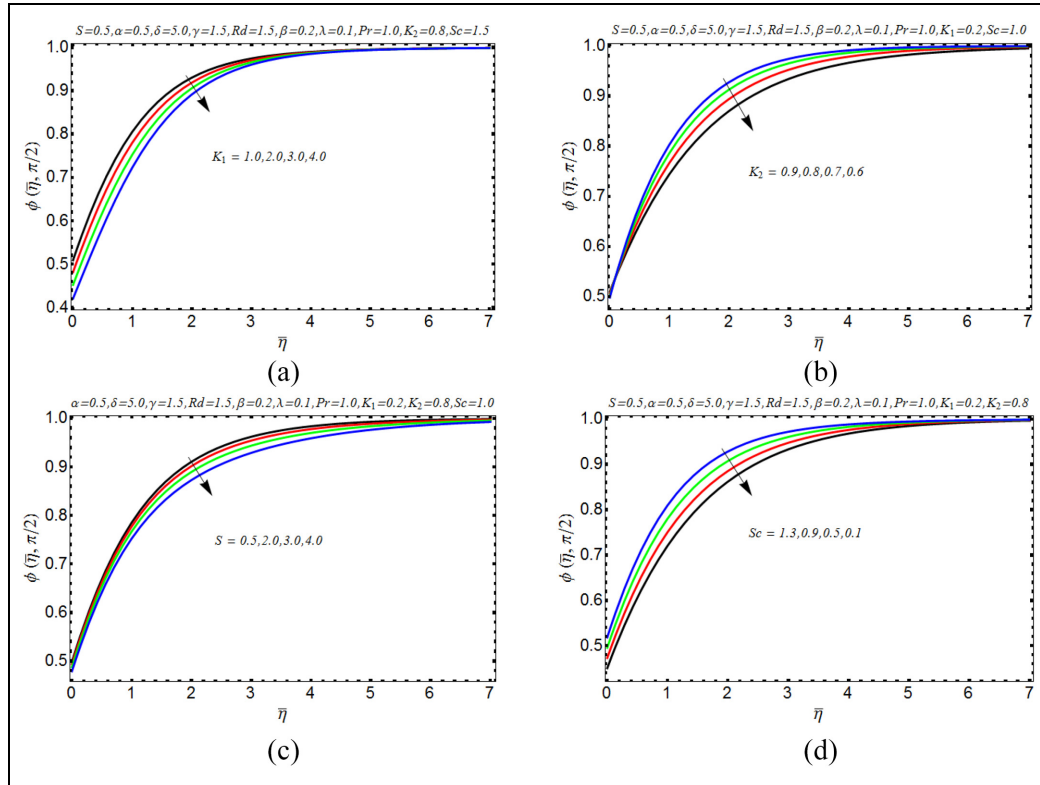


Figure 8. Alteration in concentration profile $\phi(\bar{\eta}, \bar{\tau})$ for $\bar{\tau} = \pi/2$: (a) influences of C_1 , (b) influences of S , (c) influences of Sc , and (d) influences of Sr

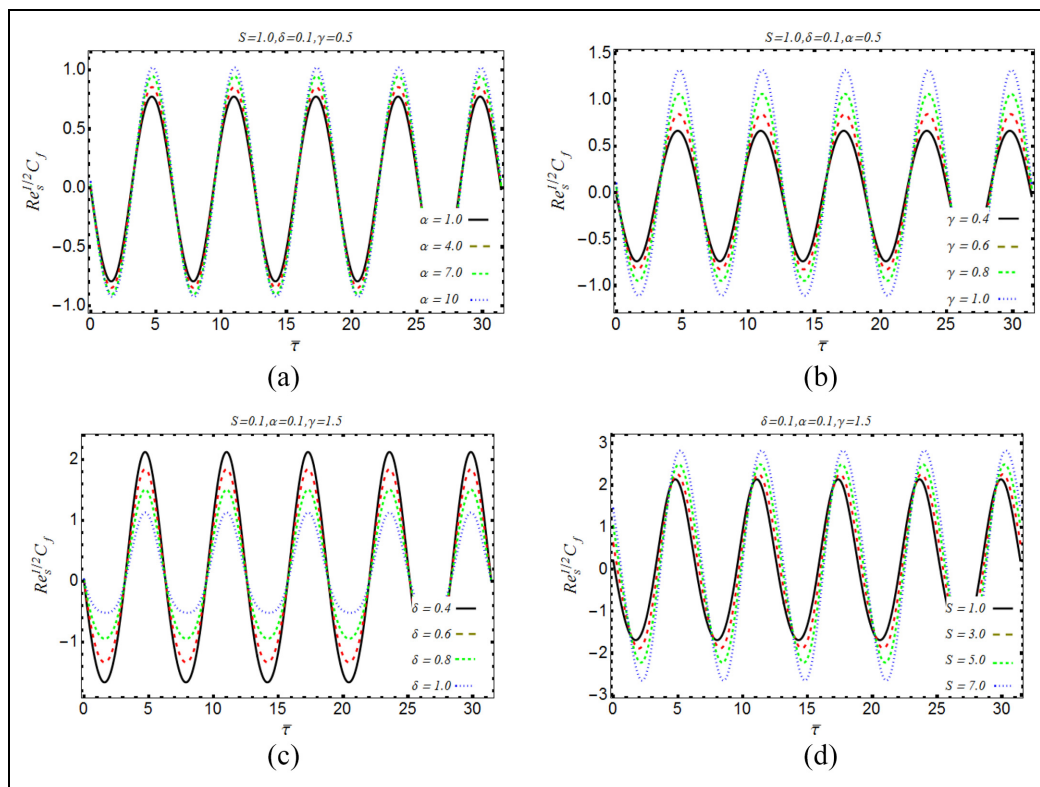


Figure 9. Variation in skin friction coefficient ($Re_s^{1/2} C_f$): (a) influences of α , (b) influences of γ , (c) influences of δ , and (d) influences of S .

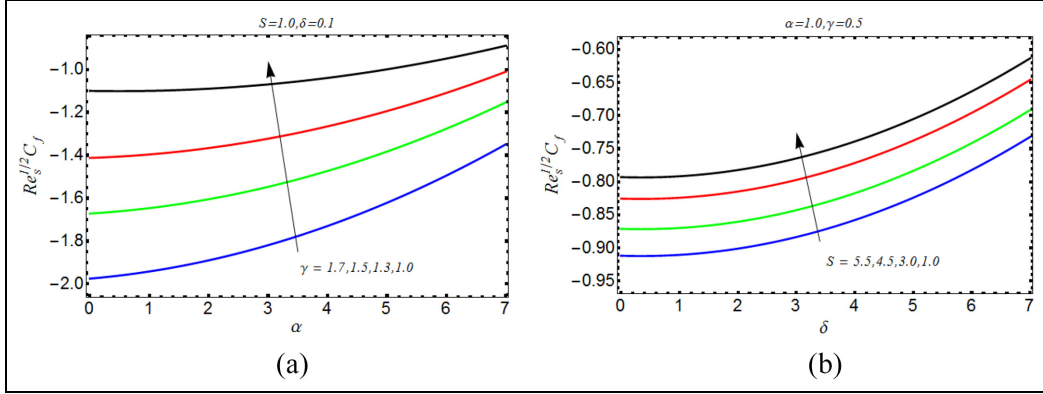


Figure 10. (a) Variations in $(Re_s^{1/2} C_f)$ with γ versus α and (b) variations in $(Re_s^{1/2} C_f)$ with S versus δ when $\bar{\tau} = (0.5)\pi$.

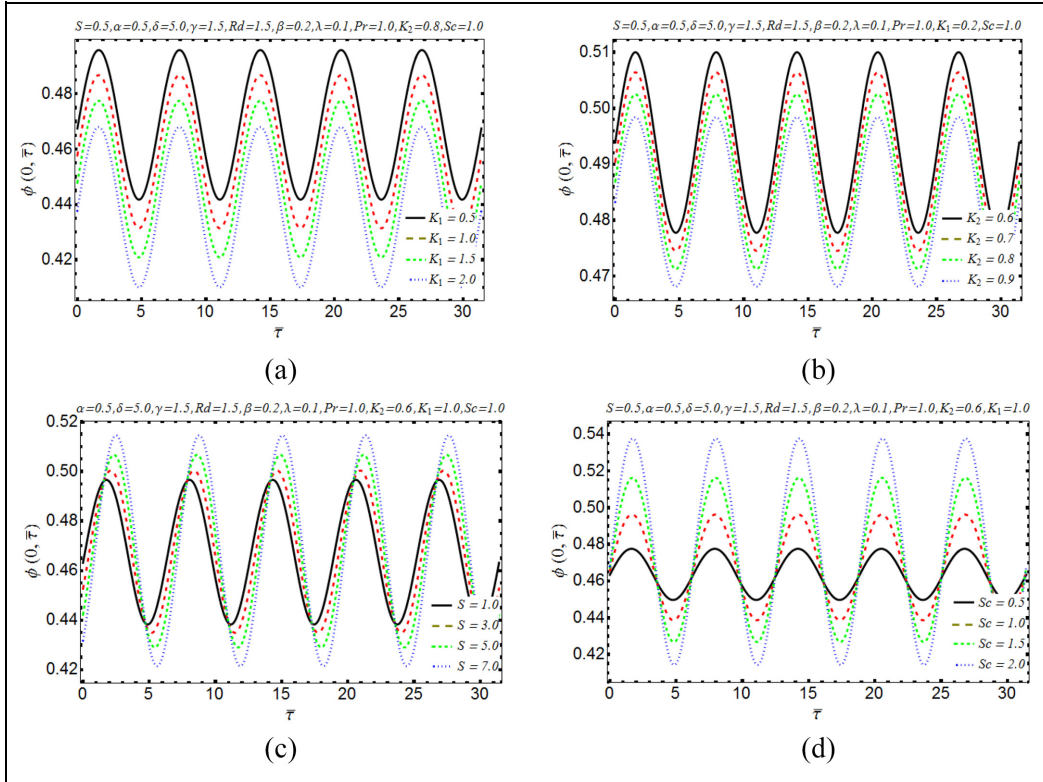


Figure 11. Variation in profile of surface concentration $\phi(0, \bar{\tau})$ at: (a) influences of K_1 , (b) influences of K_2 , (c) influences of S , and (d) influences of Sc by keeping other parameters fixed.

- The temperature distribution is reducing function of Prandtl number (Pr). While, it increases for (β), (λ), and (Rd).
- The improvement in profile of concentration distribution is noted with an enhancement in (K_1) and (S). However, it shows opposite trend for (K_2) and (Sc).
- For mounting values of (α), (S), and (γ) the skin friction coefficient amplitude depicts growing response and it declines with (δ).
- At time $\bar{\tau} = 0.5\pi$, the magnitude of skin friction coefficient enhances with upward values of (α) and (δ), and for higher (S) and (γ), it shows the opposite trend.
- The amplitude of surface concentration $\bar{\phi}(0, \bar{\tau})$ reduces by increasing (K_1) and (K_2) and it rises up for uplifting values of (S) and (Sc).
- The Nusselt number magnitude is growing function of (β), (Pr), (Rd). Whilst, its magnitude depicts opposite manner with (λ).

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ORCID iDs

Muhammad Naveed  <https://orcid.org/0000-0002-9616-3087>
Muhammad Imran  <https://orcid.org/0000-0002-2760-3595>

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Appendix

Notation

$\bar{A}_1, \bar{A}_2$	chemical species
$\bar{a}, \bar{b}$	concentrations of the chemical species $\bar{A}_1$ and $\bar{A}_2$
B_0	strength of magnetic field
$\bar{c}_p$	specific heat ($J/kg K$)
$\bar{c}$	stretching rate (s^{-1})
$\bar{C}_f$	dimensionless Coefficient of skin friction
$\bar{C}$	concentration of the fluid (mol/V^3)
$\bar{C}_w$	concentration at surface (mol/V^3)
$\bar{C}_\infty$	Ambient concentration (mol/V^3)
$\bar{D}_A, \bar{D}_B$	concentration diffusion coefficient (m^2/s)
$\bar{f}_\eta$	non-dimensional velocity
$\bar{h}_f$	heat transfer coefficient
$\bar{k}$	permeability of the porous medium
$\bar{k}_c, \bar{k}_s$	rate constants
$\bar{k}_\gamma$	mean absorption coefficient
K_1	measure of the strength of the homogeneous reaction
K_2	measure of the strength of the heterogeneous (surface) reaction
$\bar{Nu}_x$	non-dimensional local Nusselt number
Pr	Prandtl number
$\bar{Q}$	Volumetric rate of heat production
$\bar{q}_r$	radiative heat flux
Rd	radiation parameter
S	Relation of oscillatory frequency of the surface to stretching rate constant
Sc	Schmidt number
$\bar{t}$	time (s)

$\bar{T}$	temperature of the fluid (K)
$\bar{T}_w$	temperature of the oscillatory surface (K)
$\bar{T}_\infty$	ambient fluid temperature (K)
Sc	Schmidt number
$\bar{S} \bar{h}_s$	dimensionless Sherwood number
$\bar{v}$	component of velocity along $\bar{y}$ - direction (m/s)
$\bar{v}_w$	suction/injection mass velocity
$\bar{w}$	component of velocity along $\bar{x}$ - direction (m/s)
$\bar{x}$	along stretching surface (m)
$\bar{y}$	normal direction to the stretching surface (m)

Greek letter

α	suction/injection parameter
$\bar{\alpha}$	thermal diffusivity
β	Biot number
γ	magneto-porous parameter
δ	Couple stress parameter
$\bar{\rho}$	density of the fluid
$\bar{\eta}$	dimensionless variable
$\bar{\eta}_0$	Couple stress viscosity
$\bar{\nu}$	Kinematic viscosity
$\bar{\sigma}$	electrical conductivity (Sm^{-1})
$\hat{\sigma}$	Stefan-Boltzmann constant ($W/m^2 K^4$)
$\bar{\omega}$	angular frequency (s^{-1})
$\bar{\theta}$	dimensionless fluid temperature
$\bar{\phi}, \bar{\varphi}$	dimensionless reduced concentrations of species $\bar{A}_1, \bar{A}_2$