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Novel wave behaviors of the generalized Kadomtsev–Petviashvili modified equal width-burgers equation via modified mathematical methods

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In this paper, we have employed two different forms of mathematical methods, namely the extended simple equation method, and modified F-expansion method to establish several types of solutions of the generalized Kadomtsev–Petviashvili modified equal width-Burgers (G-KP-MEW-B) equation that is used to designate the propagation of long-wave with dissipation and dispersion in nonlinear media. A suitable transform is applied to convert into an ordinary differential equation. As a result, after implementation of the proposed schemes, distinct types of solutions are obtained in the form of exponential hyperbolic, trigonometric and rational functions. To analyze the physical phenomena of the model, some constructed solutions are plotted in 2-dimensional and 3-dimensional by inserting the specific values to attached parameters. Hence, the recommended schemes are highly admirably mathematical tools to evaluate the wave solutions of various models in nonlinear science.

Keywords: The generalized KP-MEW-B model; modified methods; exact solutions.

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1. Introduction

Nonlinear evolution equations (NLEEs) have been supportive tools for demonstrating the natural phenomena of sciences and engineering.[1,2] NLEEs are applied in

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numerous areas of science, like physics, mathematics and engineering.[3–12] Lately, research on the propagation of nonlinear equations among researchers has received a great deal of attention, such as Alberto *et al.* explored the relation between the Langmuir wave field and the transverse electromagnetic field.[10] The behavior of ion solitary waves in plasma was examined by Pakzad.[11] Salahuddin *et al.* scrutinized the ion-acoustic cover solitons in a collisionless unmagnetized electron-positron-ion plasma.[12] Several numerical and computational methods have been framed to grip the solutions of these types of nonlinear models, such as the exp-function method,[13] the Homotopy perturbation technique,[14] the Adomian decomposition approach,[15] the sin-Gordon method,[16] the (G/G')-expansion method,[17] the Shooting approach[18] and Hirota's simple scheme[19–24] and many more.[25–35]

The Kadomtsev–Petviashvili (KP) model was introduced by two Soviet physicists, Kadomtsev and Petviashvili in 1970,[36] this model is signified by the Korteweg–de Vries (KdV) equation. The KP equation was promptly espoused as a natural extension of the classical KdV equation to two locative dimensions and was subsequently duplicated as a model for surface and inner water waves,[37] and in nonlinear optics in Ref. 38. The investigation of the KP equation transpired closely meanwhile with the progress of the inverse scattering transform.[39,40]

The generalized KP-MEW equation is[41]

$$(R_t + \alpha(R^n)x + \gamma R_{xxt})_x + \delta R_{yy} = 0. \tag{1}$$

Currently, various authors have explored different solutions of generalized (KP-MEW) equations with the assistance of altered schemes. Wazwaz[41] constructed wave solutions generalized (KP-MEW) equation via tanh scheme, and the sine-cosine method. Saha[42] explored wave solutions of generalized (KPMEW) equation via bifurcation method. Wei *et al.*[43] investigated wave solutions of generalized (KP-MEW) (2,2) equation by employing the differential equations qualitative theory. Li and Song[44] built the kink-type wave and compacton-type wave solutions of generalized KP-MEW (2,2) equation and similarly some other more techniques to explore wave solutions of KP-MEW equation.[45–47]

The generalized KP-MEW-Burgers equation[48] is

$$(R_t + \alpha(R^n)x + \beta R_{xx} - \gamma R_{xxt})_x + \delta R_{yy} = 0. \tag{2}$$

Distinct authors studied KP-MEW-Burgers equation with different schemes. Saha[48] explored the bifurcation behavior of Eq. (2) with stable oscillations. Seadawy *et al.*[49] applied the modified extended auxiliary equation mapping method to construct variety of wave solutions. Now currently Karmina *et al.*[50] employed two different schemes $(1/G')$-expansion and the Bernoulli sub-equation techniques to establish solutions of Eq. (2).

In this study, we have investigated different results of KP-MEW-Burgers model via improved simple equation method and modified F-expansion methods, see details in Ref. 51.

The remaining portion of the work is arranged as: In Sec. 2, the proposed methods are presented. In Sec. 3, the mathematical calculation of the concern model as well as the application of both novel techniques are addressed. Conclusion is discussed in Sec. 4.

2. Formation of the Schemes

Let the general NLFDE have the following form:

$$Q_1(R, R_x, R_{xx}, R_{tt}, R_{xt}, R_{xy}, R_{xyt}, \ldots) = 0. \tag{3}$$

Let the wave transformation,

$$R = U(\xi), \quad \xi = k_1 x + k_2 y - \omega t. \tag{4}$$

Putting Eq. (3) into Eq. (4), we get

$$Q_2(U, U', U'', U''', \ldots) = 0. \tag{5}$$

2.1. *Improved simple equation method*

Let (5) have a solution,

$$U(\xi) = \sum_{i=-N}^{N} A_i \Psi^i(\xi). \tag{6}$$

Let Ψ satisfy,

$$\Psi' = c_0 + c_1\Psi + c_2\Psi^2 + c_3\Psi^3. \tag{7}$$

Put (6) with (7) in (5) to achieve the solution of (3).

2.2. *Modified F-expansion method*

Let the solution of (5) be

$$U = a_0 + \sum_{i=1}^{N} a_i F^i(\xi) + \sum_{i=1}^{N} b_i F^{-i}(\xi). \tag{8}$$

Suppose F satisfies Eq. (8),

$$F' = A + BF + CF^2. \tag{9}$$

Put (8) with (9) in (5) and solve the solution of Eq. (3).

3. Applications

3.1. *Application of improved simple equation method*

Let $n = 2$, then Eq. (2) has the following form:

$$(R_t + \alpha(R^2)x + \beta R_{xx} - \gamma R_{xxt})_x + \delta R_{yy} = 0. \tag{10}$$

Subsitute (4) into (10):

$$\alpha k_1^2 U^2 + \gamma k_1^3 \omega U'' + \beta k_1^3 U' + U(\delta k_2^2 - k_1\omega) = 0. \tag{11}$$

Let the solution of (11) be

$$U = A_2\Psi^2 + A_1\Psi + \frac{A_{-2}}{\Psi^2} + \frac{A_{-1}}{\Psi} + A_0. \tag{12}$$

Putting (12) with (7) in (11), we get

Case 1: $c_3 = 0$.

FAMILY-I

$$\begin{aligned}
A_0 &= -\frac{3\left(\sqrt{\beta^2\gamma^4(c_1^2-4c_0c_2)^3\delta^2 k_1^6} + \beta\gamma\delta k_1^3\left(2c_0c_2\sqrt{\gamma^2(c_1^2-4c_0c_2)} + \gamma c_1^3 - 4\gamma c_0c_2c_1\right)\right)}{5\alpha\gamma^2(c_1^2-4c_0c_2)\delta k_1^2},\\
A_{-2} &= A_{-1} = 0, \quad A_2 = -\frac{6\beta\gamma c_2^2 k_1}{5\alpha\sqrt{\gamma^2(c_1^2-4c_0c_2)}},\\
A_1 &= \frac{6\left(-\frac{\beta\gamma c_1c_2k_1}{\sqrt{\gamma^2(c_1^2-4c_0c_2)}} - \beta c_2k_1\right)}{5\alpha},\\
k_2 &= \frac{\sqrt{\frac{6\sqrt{\beta^2\gamma^4(c_1^2-4c_0c_2)^3\delta^2k_1^6} + \beta\delta k_1\sqrt{\gamma^2(c_1^2-4c_0c_2)}}{\gamma^2(c_1^2-4c_0c_2)\delta^2}}}{\sqrt{5}},\\
\omega &= \frac{\beta}{5\sqrt{\gamma^2c_1^2 - 4\gamma^2c_0c_2}}.
\end{aligned} \tag{13}$$

Put (13) in (12),

$$\begin{aligned}
U_1 = &\left(-\frac{3\left(\sqrt{\beta^2\gamma^4(c_1^2-4c_0c_2)^3\delta^2 k_1^6} + \beta\gamma\delta k_1^3\left(2c_0c_2\sqrt{\gamma^2(c_1^2-4c_0c_2)} + \gamma c_1^3 - 4\gamma c_0c_2c_1\right)\right)}{5\alpha\gamma^2(c_1^2-4c_0c_2)\delta k_1^2}\right)\\
&-\left(\frac{\left(c_1 - \sqrt{4c_2c_0-c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0-c_1^2}(h+\xi)\right)\right)\left(6\left(-\frac{\beta\gamma c_1c_2k_1}{\sqrt{\gamma^2(c_1^2-4c_0c_2)}} - \beta c_2k_1\right)\right)}{2(5\alpha)}\right)\\
&\times\left(\frac{\left(\frac{1}{2}\left(c_1 - \sqrt{4c_2c_0-c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0-c_1^2}(h+\xi)\right)\right)\right)^2(6\beta\gamma c_2^2k_1)}{5\alpha\sqrt{\gamma^2(c_1^2-4c_0c_2)}}\right),\\
&4c_0c_2 > c_1^2.
\end{aligned} \tag{14}$$

FAMILY-II

$$
\begin{aligned}
A_0 &= \frac{\begin{array}{c}3\beta\gamma\delta k_1^3\left(-2c_0c_2\sqrt{\gamma^2(c_1^2-4c_0c_2)}+\gamma c_1^3-4\gamma c_0c_2c_1\right)\\ -3\sqrt{\beta^2\gamma^4(c_1^2-4c_0c_2)^3\delta^2k_1^6}\end{array}}{5\alpha\gamma^2(c_1^2-4c_0c_2)\delta k_1^2},\\
A_{-2} &= -\frac{6\beta\gamma c_0^2k_1}{5\alpha\sqrt{\gamma^2(c_1^2-4c_0c_2)}},\\
k_2 &= \frac{\begin{array}{c}3\beta\gamma\delta k_1^3\left(-2c_0c_2\sqrt{\gamma^2(c_1^2-4c_0c_2)}+\gamma c_1^3-4\gamma c_0c_2c_1\right)\\ -3\sqrt{\beta^2\gamma^4(c_1^2-4c_0c_2)^3\delta^2k_1^6}\end{array}}{5\alpha\gamma^2(c_1^2-4c_0c_2)\delta k_1^2},\\
A_{-1} &= \frac{6\left(\beta c_0k_1-\frac{\beta\gamma c_0c_1k_1}{\sqrt{\gamma^2(c_1^2-4c_0c_2)}}\right)}{5\alpha},\\
A_2 &= A_1 = 0,\quad \omega = \frac{\beta}{5\sqrt{\gamma^2c_1^2-4\gamma^2c_0c_2}}.
\end{aligned}
\tag{15}
$$

Substitute (15) in (12),

$$
\begin{aligned}
U_2 = &\left(\frac{\begin{array}{c}3\beta\gamma\delta k_1^3\left(-2c_0c_2\sqrt{\gamma^2(c_1^2-4c_0c_2)}+\gamma c_1^3-4\gamma c_0c_2c_1\right)\\ -3\sqrt{\beta^2\gamma^4(c_1^2-4c_0c_2)^3\delta^2k_1^6}\end{array}}{5\alpha\gamma^2(c_1^2-4c_0c_2)\delta k_1^2}\right)\\
&-\left(\frac{6\beta\gamma c_0^2k_1}{5\alpha\sqrt{\gamma^2(c_1^2-4c_0c_2)}}\right)
\end{aligned}
$$

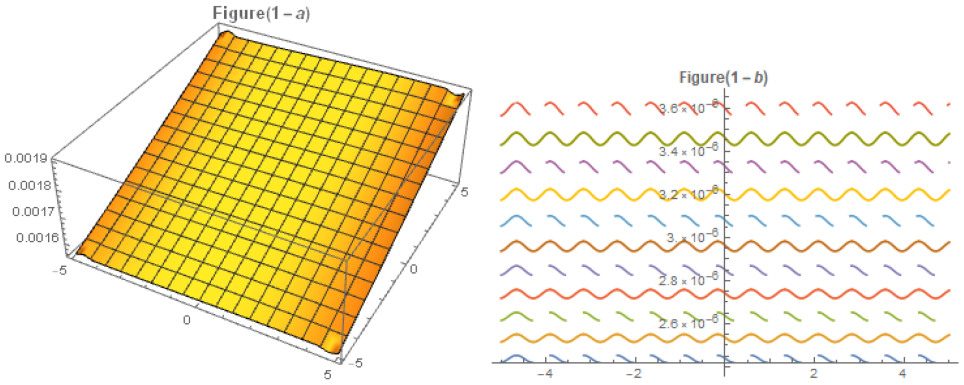

Fig. 1. (Color online) Solution U_1 with $\alpha=-3$, $\beta=-0.1$, $c_0=1$, $c_1=0.05$, $c_2=1$, $\gamma=1.03$, $\delta=-0.01$, $h=-1.03$, $k_1=4.2$, $y=1$.

$$\times\left(\frac{1}{2}\left(c_1-\sqrt{4c_2c_0-c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0-c_1^2}(h+\xi)\right)\right)\right)^{-2}$$

$$+\left(\frac{6\left(\beta c_0k_1-\frac{\beta\gamma c_0c_1k_1}{\sqrt{\gamma^2(c_1^2-4c_0c_2)}}\right)}{5\alpha}\right)$$

$$\times\left(\frac{1}{2}\left(c_1-\sqrt{4c_2c_0-c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0-c_1^2}(h+\xi)\right)\right)\right)^{-2},$$

$$4c_0c_2>c_1^2. \tag{16}$$

Case 2: $c_0=c_3=0$,

$$A_0=A_{-2}=A_{-1}=0,\quad A_2=-\frac{6\beta c_2^2k_1}{5\alpha c_1},\quad A_1=-\frac{12\beta c_2k_1}{5\alpha},$$
$$k_2=-\frac{\sqrt{\beta k_1-6\beta\gamma c_1^2k_1^3}}{\sqrt{5}\sqrt{\gamma}\sqrt{c_1}\sqrt{\delta}},\quad \omega=\frac{\beta}{5\gamma c_1}. \tag{17}$$

Put (17) in (12),

$$U_3=-\left(\frac{(6\beta c_2^2k_1)\left(\frac{c_1\exp(c_1(h+\xi))}{1-c_2\exp(c_1(h+\xi))}\right)^2}{5\alpha c_1}\right)-\left(\frac{(12\beta c_2k_1)(c_1\exp(c_1(h+\xi)))}{(5\alpha)(1-c_2\exp(c_1(h+\xi)))}\right),$$

$$c_1>0, \tag{18}$$

$$U_4=-\left(\frac{(6\beta c_2^2k_1)\left(-\frac{c_1\exp(c_1(h+\xi))}{c_2\exp(c_1(h+\xi))+1}\right)^2}{5\alpha c_1}\right)-\left(\frac{(12\beta c_2k_1)(-c_1\exp(c_1(h+\xi)))}{(5\alpha)(c_2\exp(c_1(h+\xi))+1)}\right),$$

$$c_1<0. \tag{19}$$

Case 3: $c_1=0, c_3=0$.

FAMILY-I

$$A_0=\frac{3i\beta\sqrt{c_0}\sqrt{c_2}k_1}{5\alpha},\quad A_{-2}=0,\quad A_{-1}=0,$$
$$A_2=-\frac{3i\beta c_2^{3/2}k_1}{5\alpha\sqrt{c_0}},\quad A_1=-\frac{6\beta c_2k_1}{5\alpha},$$
$$k_2=-\frac{\sqrt[4]{-1}\sqrt{\frac{\beta k_1}{\gamma\sqrt{c_0}\sqrt{c_2}}-24\beta\sqrt{c_0}\sqrt{c_2}k_1^3}}{\sqrt{10}\sqrt{\delta}},\quad \omega=\frac{i\beta}{10\gamma\sqrt{c_0}\sqrt{c_2}}. \tag{20}$$

Put (20) in (12),

$$U_5=\left(-\frac{\left(3i\beta c_2^{3/2}k_1\right)\left(\frac{\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(h+\xi))}{c_2}\right)^2}{5\alpha\sqrt{c_0}}\right)$$

$$
\begin{aligned}
&- \left(\frac{(6\beta c_2 k_1)(\sqrt{c_0 c_2}\tan(\sqrt{c_0 c_2}(h+\xi)))}{(5\alpha)c_2} \right) \\
&+ \left(\frac{3i\beta\sqrt{c_0}\sqrt{c_2}k_1}{5\alpha} \right), \quad c_0 c_2 > 0,
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
U_6 = &\left(- \frac{(3i\beta c_2^{3/2} k_1)\left(-\frac{\sqrt{-c_0 c_2}\tanh(\sqrt{-c_0 c_2}(h+\xi))}{c_2}\right)^2}{5\alpha\sqrt{c_0}} \right) \\
&- \left(\frac{(6\beta c_2 k_1)(-\sqrt{-c_0 c_2}\tanh(\sqrt{-c_0 c_2}(h+\xi)))}{(5\alpha)c_2} \right) \\
&+ \left(\frac{3i\beta\sqrt{c_0}\sqrt{c_2}k_1}{5\alpha} \right), \quad c_0 c_2 < 0.
\end{aligned} \tag{22}
$$

FAMILY-II

$$
\begin{aligned}
&A_0 = -\frac{9i\beta\sqrt{c_0}\sqrt{c_2}k_1}{5\alpha}, \quad A_{-2} = -\frac{3i\beta c_0^{3/2} k_1}{5\alpha\sqrt{c_2}}, \quad A_{-1} = \frac{6\beta c_0 k_1}{5\alpha}, \quad A_2 = 0, \\
&A_1 = 0, \quad k_2 = \frac{\sqrt[4]{-1}\sqrt{\frac{\beta k_1}{\gamma\sqrt{c_0}\sqrt{c_2}} + 24\beta\sqrt{c_0}\sqrt{c_2}k_1^3}}{\sqrt{10}\sqrt{\delta}}, \quad \omega = \frac{i\beta}{10\gamma\sqrt{c_0}\sqrt{c_2}}.
\end{aligned} \tag{23}
$$

Put (23) in (12),

$$
\begin{aligned}
U_7 = &\left(\frac{6\beta c_0 k_1}{\frac{(5\alpha)(\sqrt{c_0 c_2}\tan(\sqrt{c_0 c_2}(h+\xi)))}{c_2}} - \frac{3i\beta c_0^{3/2} k_1}{(5\alpha\sqrt{c_2})\left(\frac{\sqrt{c_0 c_2}\tan(\sqrt{c_0 c_2}(h+\xi))}{c_2}\right)^2} \right. \\
&\left. - \frac{9i\beta\sqrt{c_0}\sqrt{c_2}k_1}{5\alpha} \right), \quad c_0 c_2 > 0,
\end{aligned} \tag{24}
$$

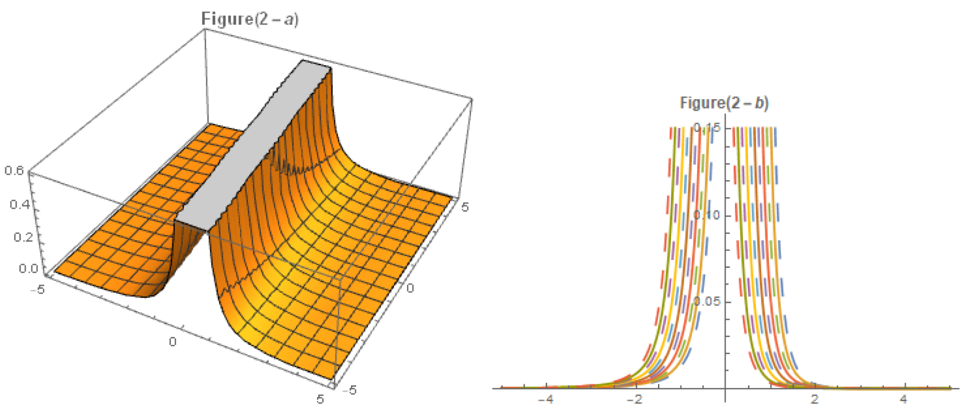

Fig. 2. (Color online) Solution U_3 with $\alpha = -3$, $\beta = -0.1$, $c_1 = 0.05$, $c_2 = 1$, $\gamma = 1.03$, $\delta = -0.01$, $h = -1.03$, $k_1 = 4.2$, $y = 1$.

$$U_8 = \left(\frac{6\beta c_0 k_1}{\frac{(5\alpha)(-\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(h+\xi)))}{c_2}} - \frac{3i\beta c_0^{3/2}k_1}{(5\alpha\sqrt{c_2})\left(-\frac{\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(h+\xi))}{c_2}\right)^2} \right.$$
$$\left. - \frac{9i\beta\sqrt{c_0}\sqrt{c_2}k_1}{5\alpha} \right), \quad c_0c_2 < 0. \tag{25}$$

FAMILY-III

$$A_0 = -\frac{3i\beta\sqrt{c_0}\sqrt{c_2}k_1}{\alpha}, \quad A_{-2} = -\frac{3i\beta c_0^{3/2}k_1}{10\alpha\sqrt{c_2}}, \quad A_{-1} = \frac{6\beta c_0 k_1}{5\alpha},$$
$$A_2 = -\frac{3i\beta c_2^{3/2}k_1}{10\alpha\sqrt{c_0}}, \quad A_1 = -\frac{6\beta c_2 k_1}{5\alpha}, \tag{26}$$
$$k_2 = \frac{\sqrt[4]{-1}\sqrt{\frac{\beta k_1}{\gamma\sqrt{c_0}\sqrt{c_2}} + 96\beta\sqrt{c_0}\sqrt{c_2}k_1^3}}{2\sqrt{5}\sqrt{\delta}}, \quad \omega = \frac{i\beta}{20\gamma\sqrt{c_0}\sqrt{c_2}}.$$

Put (26) in (12),

$$U_9 = \left(-\frac{3i\beta\sqrt{c_0}\sqrt{c_2}k_1}{\alpha} - \frac{(6\beta c_2k_1)(\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(h+\xi)))}{(5\alpha)c_2} \right.$$
$$\left. - \frac{(3i\beta c_2^{3/2}k_1)\left(\frac{\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(h+\xi))}{c_2}\right)^2}{10\alpha\sqrt{c_0}} \right)$$
$$+ \left(\frac{6\beta c_0k_1}{\frac{(5\alpha)(\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(h+\xi)))}{c_2}} - \frac{3i\beta c_0^{3/2}k_1}{(10\alpha\sqrt{c_2})\left(\frac{\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(h+\xi))}{c_2}\right)^2} \right),$$
$$c_0c_2 > 0, \tag{27}$$

$$U_{10} = \left(-\frac{3i\beta\sqrt{c_0}\sqrt{c_2}k_1}{\alpha} - \frac{(6\beta c_2k_1)(-\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(h+\xi)))}{(5\alpha)c_2} \right.$$
$$\left. - \frac{(3i\beta c_2^{3/2}k_1)\left(-\frac{\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(h+\xi))}{c_2}\right)^2}{10\alpha\sqrt{c_0}} \right)$$
$$+ \left(\frac{6\beta c_0k_1}{\frac{(5\alpha)(-\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(h+\xi)))}{c_2}} \right.$$
$$\left. - \frac{3i\beta c_0^{3/2}k_1}{(10\alpha\sqrt{c_2})\left(-\frac{\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(h+\xi))}{c_2}\right)^2} \right), \quad c_0c_2 < 0. \tag{28}$$

3.2. *Application of modified F-expansion method*

Let the solution of (11) be

$$U = a_2F^2 + a_1F + a_0 + \frac{b_2}{F^2} + \frac{b_1}{F}. \tag{29}$$

Putting (29) in (11) with (9), we get

A = 0, **B = 1**, **C = −1**,

$$a_0 = 0, \quad a_2 = -\frac{6\beta k_1}{5\alpha}, \quad a_1 = \frac{12\beta k_1}{5\alpha}, \quad b_1 = 0,$$
$$b_2 = 0, \quad \omega = \frac{\beta}{5\gamma}, \quad k_2 = \frac{\sqrt{\beta k_1 - 6\beta\gamma k_1^3}}{\sqrt{5}\sqrt{\gamma}\sqrt{\delta}}. \tag{30}$$

Putting (30) in (29), we get

$$U_{11} = \left(\frac{12\beta k_1}{5\alpha} + \left(\frac{1}{2}\tanh\left(\frac{\xi}{2}\right) + \frac{1}{2}\right) - \left(\frac{6\beta k_1}{5\alpha}\right)\left(\frac{1}{2}\tanh\left(\frac{\xi}{2}\right) + \frac{1}{2}\right)^2\right). \tag{31}$$

A = 0, $B = $ **−1**, **C = 1**,

$$a_0 = \frac{6\beta k_1}{5\alpha}, \quad a_2 = -\frac{6\beta k_1}{5\alpha}, \quad a_1 = 0, \quad b_1 = 0,$$
$$b_2 = 0, \quad \omega = \frac{\beta}{5\gamma}, \quad k_2 = \frac{\sqrt{\beta k_1 - 6\beta\gamma k_1^3}}{\sqrt{5}\sqrt{\gamma}\sqrt{\delta}}. \tag{32}$$

Substitute (32) into (29):

$$U_{12} = \left(\frac{6\beta k_1}{5\alpha} - \frac{6\beta k_1}{5\alpha}\left(\frac{1}{2} - \frac{1}{2}\coth\left(\frac{\xi}{2}\right)\right)^2\right). \tag{33}$$

A = 1/2, **B = 0**, **C = −1/2**.

FAMILY-I

$$a_0 = \frac{9\beta k_1}{10\alpha}, \quad a_2 = -\frac{3\beta k_1}{10\alpha}, \quad a_1 = \frac{3\beta k_1}{5\alpha}, \quad b_1 = 0,$$
$$b_2 = 0, \quad \omega = \frac{\beta}{5\gamma}, \quad k_2 = \frac{\sqrt{\beta k_1 - 6\beta\gamma k_1^3}}{\sqrt{5}\sqrt{\gamma}\sqrt{\delta}}. \tag{34}$$

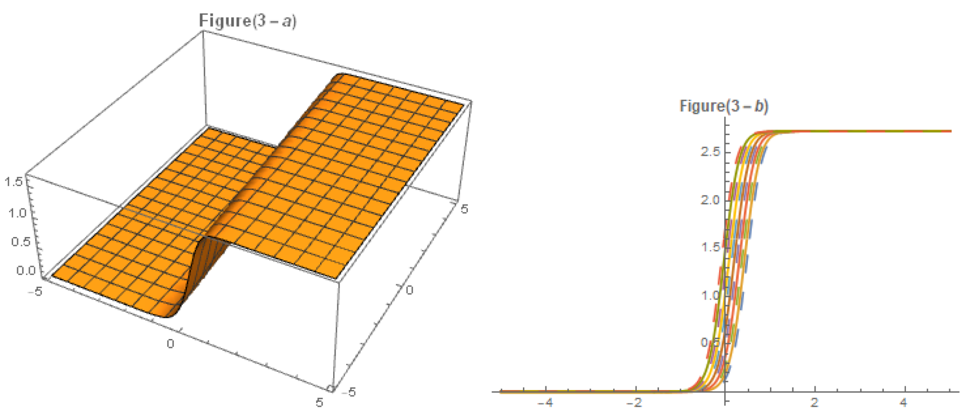

Fig. 3. (Color online) Solution U_{10} with $\alpha = -3$, $\beta = -1.1$, $c_0 = -0.05$, $c_2 = 1$, $\gamma = 1.03$, $\delta = -0.01$, $h = -1.03$, $k_1 = 4$.

Putting (34) in (29), we get

$$U_{13,1} = \left(\frac{9\beta k_1}{10\alpha} + \frac{3\beta k_1}{5\alpha}\coth(\xi) + \operatorname{csch}(\xi) - \frac{3\beta k_1}{10\alpha}(\coth(\xi) + \operatorname{csch}(\xi))^2\right). \tag{35}$$

FAMILY-II

$$\begin{gathered} a_0 = \frac{9\beta k_1}{10\alpha}, \quad a_2 = 0, \quad a_1 = 0, \quad b_1 = \frac{3\beta k_1}{5\alpha}, \\ b_2 = -\frac{3\beta k_1}{10\alpha}, \quad \omega = \frac{\beta}{5\gamma}, \quad k_2 = \frac{\sqrt{\beta k_1 - 6\beta\gamma k_1^3}}{\sqrt{5}\sqrt{\gamma}\sqrt{\delta}}. \end{gathered} \tag{36}$$

Putting (36) in (29), we get

$$U_{13,2} = \left(\frac{9\beta k_1}{10\alpha} + \frac{3\beta k_1}{5\alpha}\frac{1}{\coth(\xi) + \operatorname{csch}(\xi)} - \frac{3\beta k_1}{10\alpha}\frac{1}{(\coth(\xi) + \operatorname{csch}(\xi))^2}\right). \tag{37}$$

FAMILY-III

$$\begin{gathered} a_0 = \frac{3\beta k_1}{2\alpha}, \quad a_2 = -\frac{3\beta k_1}{20\alpha}, \quad a_1 = \frac{3\beta k_1}{5\alpha}, \quad b_1 = \frac{3\beta k_1}{5\alpha}, \\ b_2 = -\frac{3\beta k_1}{20\alpha}, \quad \omega = \frac{\beta}{10\gamma}, \quad k_2 = \frac{\sqrt{\beta k_1 - 24\beta\gamma k_1^3}}{\sqrt{10}\sqrt{\gamma}\sqrt{\delta}}. \end{gathered} \tag{38}$$

Putting (38) in (29), we get

$$\begin{aligned} U_{13,3} = {} & \left(\frac{3\beta k_1}{2\alpha} - \frac{3\beta k_1}{20\alpha}(\coth(\xi) + \operatorname{csch}(\xi))^2 + \frac{3\beta k_1}{5\alpha}(\coth(\xi) + \operatorname{csch}(\xi))^1 + \frac{3\beta k_1}{5\alpha}\right) \\ & + \left(\frac{1}{\coth(\xi) + \operatorname{csch}(\xi)} - \frac{3\beta k_1}{(20\alpha)(\coth(\xi) + \operatorname{csch}(\xi))^2}\right). \end{aligned} \tag{39}$$

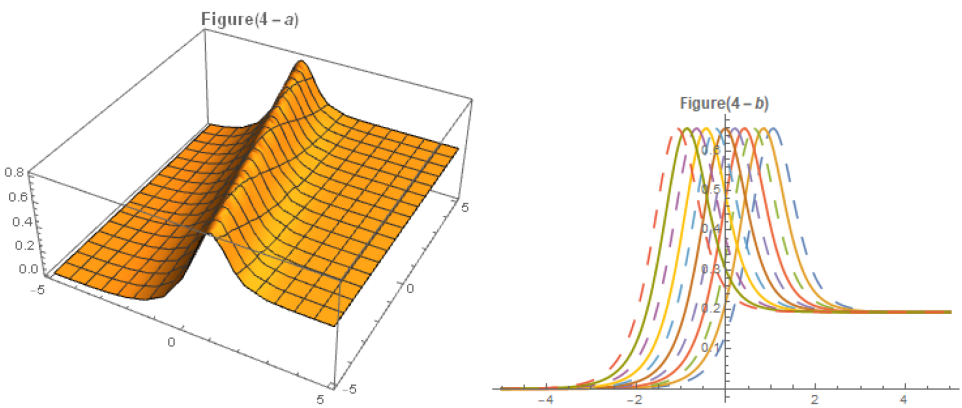

Fig. 4. (Color online) Solution $U_{13,3}$ with $\alpha = -3$, $\beta = -1.1$, $\gamma = 1.03$, $\delta = -0.01$, $k_1 = 0.5$.

$\mathbf{A = 1}$, $\mathbf{B = 0}$, $\mathbf{C = -1}$.

FAMILY-I

$$\begin{aligned} &a_0 = \frac{9\beta k_1}{5\alpha}, \quad a_2 = -\frac{3\beta k_1}{5\alpha}, \quad a_1 = \frac{6\beta k_1}{5\alpha}, \quad b_1 = 0, \\ &b_2 = 0, \quad \omega = \frac{\beta}{10\gamma}, \quad k_2 = \frac{\sqrt{\beta k_1 - 24\beta\gamma k_1^3}}{\sqrt{10}\sqrt{\gamma}\sqrt{\delta}}. \end{aligned} \tag{40}$$

Putting (40) in (29), we get

$$U_{14,1} = \left(\frac{9\beta k_1}{5\alpha} + \frac{6\beta k_1}{5\alpha}\tanh(\xi) - \frac{3\beta k_1}{5\alpha}\tanh^2(\xi)\right). \tag{41}$$

FAMILY-II

$$\begin{aligned} &a_0 = \frac{3\beta k_1}{5\alpha}, \quad a_2 = 0, \quad a_1 = 0, \quad b_1 = \frac{6\beta k_1}{5\alpha}, \\ &b_2 = \frac{3\beta k_1}{5\alpha}, \quad \omega = -\frac{\beta}{10\gamma}, \quad k_2 = \frac{\sqrt{-24\beta\gamma k_1^3 - \beta k_1}}{\sqrt{10}\sqrt{\gamma}\sqrt{\delta}}. \end{aligned} \tag{42}$$

Putting (42) in (29), we get

$$U_{14,2} = \left(\frac{3\beta k_1}{5\alpha} + \frac{6\beta k_1}{5\alpha}\frac{1}{\tanh(\xi)} + \frac{3\beta k_1}{5\alpha}\frac{1}{\tanh^2(\xi)}\right). \tag{43}$$

FAMILY-III

$$\begin{aligned} &a_0 = \frac{3\beta k_1}{\alpha}, \quad a_2 = -\frac{3\beta k_1}{10\alpha}, \quad a_1 = \frac{6\beta k_1}{5\alpha}, \quad b_1 = \frac{6\beta k_1}{5\alpha}, \\ &b_2 = -\frac{3\beta k_1}{10\alpha}, \quad \omega = \frac{\beta}{20\gamma}, \quad k_2 = \frac{\sqrt{\beta k_1 - 96\beta\gamma k_1^3}}{2\sqrt{5}\sqrt{\gamma}\sqrt{\delta}}. \end{aligned} \tag{44}$$

Putting (44) in (29), we get

$$\begin{aligned} U_{14,3} = \Bigg(&\frac{3\beta k_1}{\alpha} + \frac{6\beta k_1}{5\alpha}\tanh(\xi) - \frac{3\beta k_1}{10\alpha}\tanh^2(\xi) \\ &+ \frac{6\beta k_1}{5\alpha}\frac{1}{\tanh(\xi)} - \frac{3\beta k_1}{10\alpha}\frac{1}{\tanh^2(\xi)}\Bigg). \end{aligned} \tag{45}$$

$\mathbf{A = B = 1/2}$, $\mathbf{C = 0}$.

FAMILY-I

$$\begin{aligned} &a_0 = -\frac{3i\beta k_1}{10\alpha}, \quad a_2 = \frac{3i\beta k_1}{10\alpha}, \quad a_1 = -\frac{3\beta k_1}{5\alpha}, \quad b_1 = 0, \\ &b_2 = 0, \quad \omega = -\frac{i\beta}{5\gamma}, \quad k_2 = -\frac{\sqrt[4]{-1}\sqrt{\beta}\sqrt{k_1}\sqrt{6\gamma k_1^2 - 1}}{\sqrt{5}\sqrt{\gamma}\sqrt{\delta}}. \end{aligned} \tag{46}$$

Putting (46) in (29), we get

$$U_{15,1} = \left(-\frac{3\beta k_1}{5\alpha} - \frac{3i\beta k_1}{10\alpha}(\tan(\xi)+\sec(\xi))^2 - \frac{3\beta k_1}{5\alpha}(\tan(\xi)+\sec(\xi))\right). \quad (47)$$

FAMILY-II

$$\begin{aligned} a_0 &= \frac{3i\beta k_1}{10\alpha}, \quad a_2 = 0, \quad a_1 = 0, \quad b_1 = \frac{3\beta k_1}{5\alpha}, \\ b_2 &= -\frac{3i\beta k_1}{10\alpha}, \quad \omega = \frac{i\beta}{5\gamma}, \quad k_2 = \frac{(-1)^{3/4}\sqrt{\beta}\sqrt{k_1}\sqrt{6\gamma k_1^2 - 1}}{\sqrt{5}\sqrt{\gamma}\sqrt{\delta}}. \end{aligned} \quad (48)$$

Putting (48) in (29), we get

$$U_{15,2} = \left(\frac{3i\beta k_1}{10\alpha} + \frac{3\beta k_1}{5\alpha}\frac{1}{\tan(\xi)+\sec(\xi)} - \frac{3i\beta k_1}{10\alpha}\frac{1}{(\tan(\xi)+\sec(\xi))^2}\right). \quad (49)$$

FAMILY-III

$$\begin{aligned} a_0 &= -\frac{3i\beta k_1}{2\alpha}, \quad a_2 = -\frac{3i\beta k_1}{20\alpha}, \quad a_1 = -\frac{3\beta k_1}{5\alpha}, \quad b_1 = \frac{3\beta k_1}{5\alpha}, \\ b_2 &= -\frac{3i\beta k_1}{20\alpha}, \quad \omega = \frac{i\beta}{10\gamma}, \quad k_2 = \frac{\sqrt[4]{-1}\sqrt{\beta}\sqrt{k_1}\sqrt{24\gamma k_1^2 + 1}}{\sqrt{10}\sqrt{\gamma}\sqrt{\delta}}. \end{aligned} \quad (50)$$

Putting (50) in (29), we get

$$\begin{aligned} U_{15,3} = &\left(-\frac{3i\beta k_1}{2\alpha} - \frac{3i\beta k_1}{20\alpha}(\tan(\xi)+\sec(\xi))^2 - \frac{3\beta k_1}{5\alpha}(\tan(\xi)+\sec(\xi))\right) \\ &+ \left(\frac{3\beta k_1}{5\alpha}\frac{1}{\tan(\xi)+\sec(\xi)} - \frac{3i\beta k_1}{20\alpha}\frac{1}{(\tan(\xi)+\sec(\xi))^2}\right). \end{aligned} \quad (51)$$

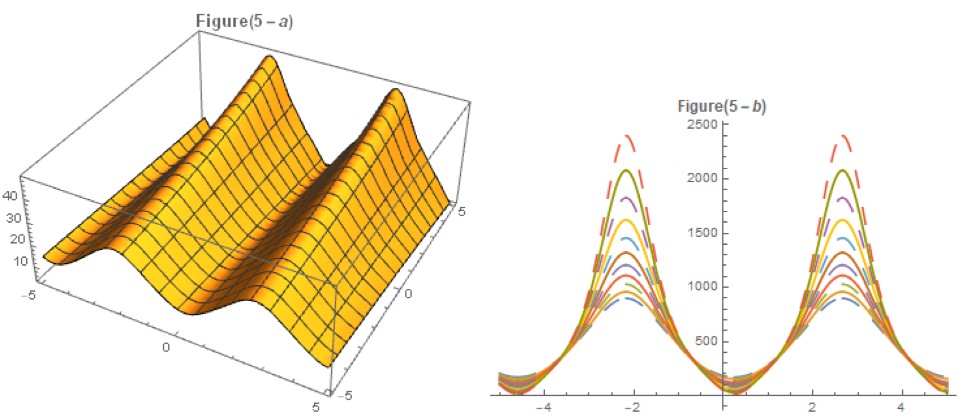

Fig. 5. (Color online) Solution $U_{15,3}$ with $\alpha = -0.1$, $\beta = -1.1$, $\gamma = 1.3$, $\delta = -1.01$, $k_1 = -1.3$, $y = 1$.

$\mathbf{A = B = -1/2}$, $\mathbf{C = 0}$.

FAMILY-I

$$\begin{aligned} &a_0 = -\frac{9i\beta k_1}{10\alpha}, \quad a_2 = -\frac{3i\beta k_1}{10\alpha}, \quad a_1 = \frac{3\beta k_1}{5\alpha}, \quad b_1 = 0, \\ &b_2 = 0, \quad \omega = \frac{i\beta}{5\gamma}, \quad k_2 = \frac{\sqrt[4]{-1}\sqrt{\beta}\sqrt{k_1}\sqrt{6\gamma k_1^2+1}}{\sqrt{5}\sqrt{\gamma}\sqrt{\delta}}. \end{aligned} \tag{52}$$

Putting (52) in (29), we get

$$U_{16,1} = \left(-\frac{9i\beta k_1}{10\alpha} - \frac{3i\beta k_1}{10\alpha}(\sec(\xi) - \tan(\xi))^2 + \frac{3\beta k_1}{5\alpha}(\sec(\xi) - \tan(\xi))^2\right). \tag{53}$$

FAMILY-II

$$\begin{aligned} &a_0 = -\frac{9i\beta k_1}{10\alpha}, \quad a_2 = 0, \quad a_1 = 0, \quad b_1 = -\frac{3\beta k_1}{5\alpha}, \\ &b_2 = -\frac{3i\beta k_1}{10\alpha}, \quad \omega = \frac{i\beta}{5\gamma}, \quad k_2 = \frac{\sqrt[4]{-1}\sqrt{\beta}\sqrt{k_1}\sqrt{6\gamma k_1^2+1}}{\sqrt{5}\sqrt{\gamma}\sqrt{\delta}}. \end{aligned} \tag{54}$$

Putting (54) in (29), we get

$$U_{16,2} = \left(-\frac{9i\beta k_1}{10\alpha} - \frac{3\beta k_1}{5\alpha}\frac{1}{\sec(\xi) - \tan(\xi)} - \frac{3i\beta k_1}{10\alpha}\frac{1}{(\sec(\xi) - \tan(\xi))^2}\right). \tag{55}$$

FAMILY-III

$$\begin{aligned} &a_0 = -\frac{3i\beta k_1}{2\alpha}, \quad a_2 = -\frac{3i\beta k_1}{20\alpha}, \quad a_1 = \frac{3\beta k_1}{5\alpha}, \quad b_1 = -\frac{3\beta k_1}{5\alpha}, \\ &b_2 = -\frac{3i\beta k_1}{20\alpha}, \quad \omega = \frac{i\beta}{10\gamma}, \quad k_2 = \frac{\sqrt[4]{-1}\sqrt{\beta}\sqrt{k_1}\sqrt{24\gamma k_1^2+1}}{\sqrt{10}\sqrt{\gamma}\sqrt{\delta}}. \end{aligned} \tag{56}$$

Putting (56) in (29), we get

$$\begin{aligned} U_{16,3} = &\left(-\frac{3i\beta k_1}{2\alpha} - \frac{3i\beta k_1}{20\alpha}\frac{1}{(\sec(\xi) - \tan(\xi))^2} - \frac{3\beta k_1}{5\alpha}\frac{1}{\sec(\xi) - \tan(\xi)}\right) \\ &+ \left(\frac{3\beta k_1}{5\alpha}\sec(\xi) - \tan(\xi) + -\frac{3i\beta k_1}{20\alpha}(\sec(\xi) - \tan(\xi))^2\right). \end{aligned} \tag{57}$$

$\mathbf{A = B = -1}$, $\mathbf{C = 0}$.

FAMILY-I

$$\begin{aligned} &a_0 = -\frac{9i\beta k_1}{5\alpha}, \quad a_2 = -\frac{3i\beta k_1}{5\alpha}, \quad a_1 = \frac{6\beta k_1}{5\alpha}, \quad b_1 = b_2 = 0, \\ &\omega = \frac{i\beta}{10\gamma}, \quad k_2 = \frac{\sqrt[4]{-1}\sqrt{\beta}\sqrt{k_1}\sqrt{24\gamma k_1^2+1}}{\sqrt{10}\sqrt{\gamma}\sqrt{\delta}}. \end{aligned} \tag{58}$$

Putting (58) in (29), we get

$$U_{17,1} = \left(-\frac{9i\beta k_1}{5\alpha} - \frac{3i\beta k_1}{5\alpha}\tan^2(\xi) + \frac{6\beta k_1}{5\alpha}\tan(\xi)\right). \tag{59}$$

FAMILY-II

$$\begin{aligned} a_0 &= -\frac{9i\beta k_1}{5\alpha}, \quad a_2 = 0, \quad a_1 = 0, \quad b_1 = -\frac{6\beta k_1}{5\alpha}, \\ b_2 &= -\frac{3i\beta k_1}{5\alpha}, \quad \omega = \frac{i\beta}{10\gamma}, \quad k_2 = \frac{\sqrt[4]{-1}\sqrt{\beta}\sqrt{k_1}\sqrt{24\gamma k_1^2+1}}{\sqrt{10}\sqrt{\gamma}\sqrt{\delta}}. \end{aligned} \tag{60}$$

Putting (60) in (29), we get

$$U_{17,2} = \left(-\frac{9i\beta k_1}{5\alpha} - \frac{6\beta k_1}{5\alpha}\frac{1}{\tan(\xi)} - \frac{3i\beta k_1}{5\alpha}\frac{1}{\tan^2(\xi)}\right). \tag{61}$$

FAMILY-III

$$\begin{aligned} a_0 &= -\frac{3i\beta k_1}{\alpha}, \quad a_2 = -\frac{3i\beta k_1}{10\alpha}, \quad a_1 = \frac{6\beta k_1}{5\alpha}, \quad b_1 = -\frac{6\beta k_1}{5\alpha}, \\ b_2 &= -\frac{3i\beta k_1}{10\alpha}, \quad \omega = \frac{i\beta}{20\gamma}, \quad k_2 = \frac{\sqrt[4]{-1}\sqrt{\beta}\sqrt{k_1}\sqrt{96\gamma k_1^2+1}}{2\sqrt{5}\sqrt{\gamma}\sqrt{\delta}}. \end{aligned} \tag{62}$$

Putting (62) in (29), we get

$$\begin{aligned} U_{17,3} = \Bigg(&-\frac{3i\beta k_1}{\alpha} - \frac{6\beta k_1}{5\alpha}\frac{1}{\tan(\xi)} - \frac{3i\beta k_1}{10\alpha}\frac{1}{\tan^2(\xi)} \\ &+ \frac{6\beta k_1}{5\alpha}\tan(\xi) - \frac{3i\beta k_1}{10\alpha}\tan^2(\xi)\Bigg). \end{aligned} \tag{63}$$

$\mathbf{C = 0}$,

$$\begin{aligned} a_0 &= \frac{6\beta B k_1}{5\alpha}, \quad a_2 = 0, \quad a_1 = 0, \quad b_1 = 0, \\ b_2 &= -\frac{6A^2\beta k_1}{5\alpha B}, \quad \omega = \frac{\beta}{5B\gamma}, \quad k_2 = \frac{\sqrt{\beta k_1 - 6\beta B^2\gamma k_1^3}}{\sqrt{5}\sqrt{B}\sqrt{\gamma}\sqrt{\delta}}. \end{aligned} \tag{64}$$

Putting (64) in (29), we get

$$U_{18} = \left(\frac{6\beta B k_1}{5\alpha} - \frac{6A^2\beta k_1}{5\alpha B}\left(\frac{1}{\frac{\exp(B\xi)-A}{B}}\right)^2\right). \tag{65}$$

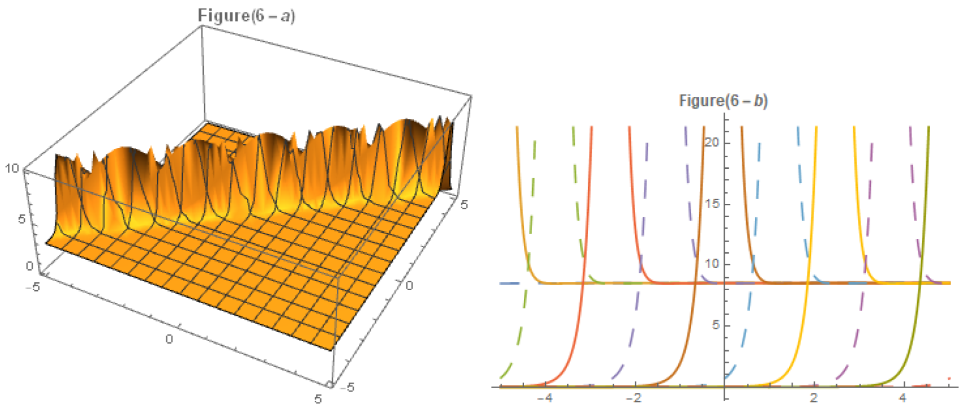

Fig. 6. (Color online) Solution U_{18} with $\alpha = 2.1$, $A = 1.01$, $\beta = 3.1$, $B = 0.5$, $\gamma = 0.3$, $\delta = 4.01$, $k_1 = 3.3$, $y = 1$.

4. Conclusion

We have reconnoitered enlightened and well-organized solutions that are constructed of generalized Kadomtsev–Petviashvili modified equal width-Burgers equation by successfully implementing two mathematical methods. The investigated solutions are in various functional forms like trigonometric, hyperbolic, rational and exponential. To demonstrate the physical phenomena of the KP-MEW-Burgers equation, some solutions are plotted in two and three dimensions by employing certain values to the parameters (Figs. 1–6). Hence, the proposed schemes are meritorious for advanced studies for other NFPDEs.

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