

An optimized stability framework for three-dimensional Hartman flow via Chebyshev collocation simulations

Wafa F. Alfwzan^a, Zakir Hussain^b, Kamel Al-Khaled^c, Arshad Riaz^{d,*}, Talaat Abdelhamid^e, Sami Ullah Khan^f, Khurram Javid^g, El Sayed M. Tag El-Din^h, Wathek Chamman^{i,j}

^a Department of Mathematical Sciences, College of Science, Princess Nourah Bint Abdulrahman University, P.O. Box 84428, Riyadh 11671, Saudi Arabia

^b Department of Mathematics, COMSATS University Islamabad, Abbottabad Campus 22060, Pakistan

^c Department of Mathematics & Statistics, Jordan University of Science and Technology, P.O. Box 3030, Irbid 22110, Jordan

^d Department of Mathematics, Division of Science and Technology, University of Education, Lahore 54770, Pakistan

^e Physics and Mathematical Engineering Department, Faculty of Electronic Engineering, Menoufiya University, Egypt

^f Department of Mathematics, Namal University, Mianwali 42250, Pakistan

^g Department of Mathematics, Northern University, Wattar-Wallai Road, Nowshera 24110, KPK, Pakistan

^h Center of Research, Faculty of Engineering, Future University in Egypt, New Cairo 11835, Egypt

ⁱ Department of Mathematics, College of Science Al-Zulfi, Majmaah University, P.O. Box 66, Al-Majmaah 11952, Saudi Arabia

^j Research Laboratory Mathematics and Applications LR17ES11, Gabes University, Erriadh City 6072, Zrig, Gabes, Tunisia

ARTICLE INFO

Keywords:

Chebyshev collocation method
Three-dimensional Couette flow
Electrically conducting fluid
Linear instability

ABSTRACT

Hydrodynamics instability is studied for an electrical conducting fluid against small disturbance between the channels by using normal magnetized force. Chebyshev collocation technique is used to determine the stability of three-dimensional Hartmann flow problem. The distinctive case of the perturbations is calculated. It is also considered that perturbations intensity be determined by just on quantified by different parameters. We have considered one of the flow stabilities conditions to analyze our problem. QZ (Qualitat and Zuverlässigkeit) technique is applied to investigate the problem to draw stability curves. α_c, β_c are critical wave numbers in streamwise and span-wise respectively and for a big range of Hartmann value (Ha), we obtained critical Reynolds number (Re_c). It is found that Couette flow is destabilized for a specific value of Magnetize force while with greater or lesser magnitude than the particular one will stabilize the flow. Disturbances with particular oblique angle θ will grow while the others will decay for the three-dimensional disturbance. We observed from over results that for $Ha(>2.0)$ Re_c increases steadily and when Ha more than 3.886, Re_c decreases fast to the minimum. It is also established that for drop down from $Ha = 3.886$ and Re_c becomes very large. For distinct Hartmann values, there exist two Re_c numbers due to closed contour. The outcomes of current study are utilized in drug-delivery systems and photodynamic therapy.

Introduction

In the fluid mechanics, hydro-dynamic instability is the field of analyzing the stability and instability of fluids flow. The study of hydrodynamic instability objectives to discover whether a given fluid flow is unstable or stable, and therefore, how these instabilities will effect on the expansion of turbulence. Another important field is the magneto-hydrodynamics (MHD) is very effective and have various usages in the research fields. Hannes [1] is a famous Swedish physicist who first invented a new domain of research that is known as MHD fluid. He worked in the research field of plasma physics is unconscionable.

Analyzing of hydro-magnetic instabilities of the fluid flow is the most interesting comprehensive investigation. Darzin [2] demonstrated that fluid is unstable due to some disturbance of inertia, viscous stresses, and external forces of fluid. The developing interest of instability has many reasons such as applied magnetic field. The stability of different fluid systems is caused by electromagnetic forces are main fluid systems and interaction of the disturbance are distinct in according to systems of fluid. For plane Couette flow, it was found both in numerical simulation and in experiments (Anselmo et al. [3]; Kraheberger et al. [4]) that very low Reynolds number where the turbulence occurred between 300 and 400. They observed that the previous work of Weibel instability is

* Corresponding author.

E-mail address: arshad-riaz@ue.edu.pk (A. Riaz).

<https://doi.org/10.1016/j.rinp.2023.106497>

Received 18 December 2022; Received in revised form 12 April 2023; Accepted 25 April 2023

Available online 29 April 2023

2211-3797/© 2023 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).

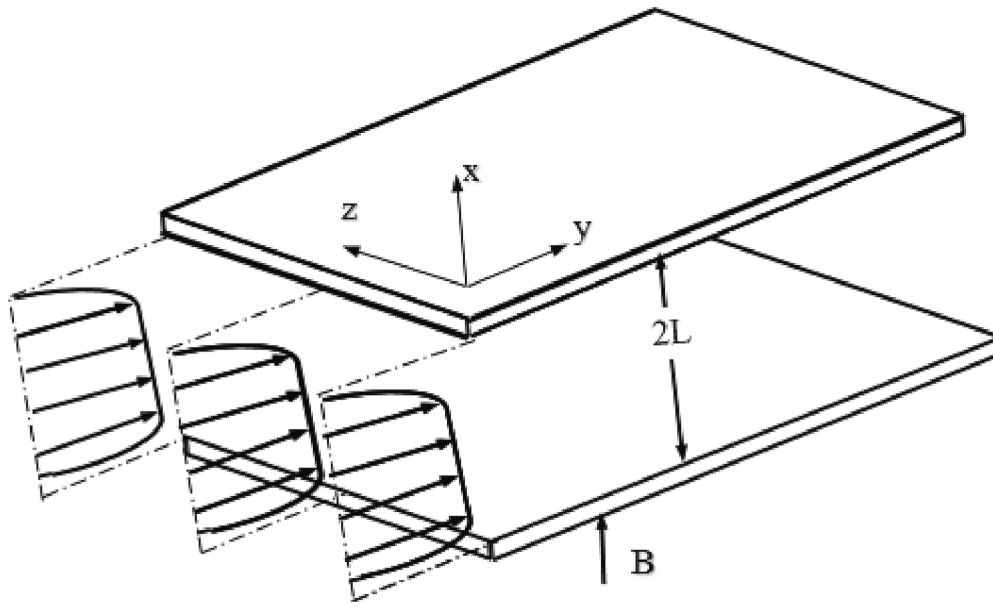


Fig. 1. Schematic diagram.

Table 1

Comparison of results with analysis of studies of Nabil [15] and Takashima [16].

Ha	Re_c Present	Takashima	Nabil	k_c Present	Takashima	Nabil	N_I Present	Takashima	Nabil
0	10E15			0.01			150		
3.887	10,826,409	10,826,766		0.020727	0.020717		150	150	
4	870000.8	869989.8	838,696	0.233069	0.233179	0.2338	120	120	120
5	338531.9	338528.2	334,005	0.64945	0.64950	0.65	110	110	100

because of overestimated number of the observed magnetic fields. Giribabu and Shankar [5] investigated the finite amplitude method by using plane Couette flow. The graphs are presented for least Reynolds numbers ($Re = 5125$) by numerical simulation is presented in steady states. With the help of the following technique was used by Philippou et al. [6] for plane Couette flow, wavy Taylor vortices and a new class of solution was found. The subcritical presence of three-dimensional flows was found for the onset of convection at low critical value Rayleigh number. Harisingh and Rama [7] investigated the shear flow by imposing the perturbation and concluded that results are approximately constant in a streamwise and circular direction. The finite amplitude of 3-D perturbations for Poiseuille flow is unstable in spanwise direction. Couette and Poiseuille flow are investigated by Orszag and Patera [8] and Hussain et al. [9,10]) found that as the wire was set in the flow in plane Couette flow, streamwise vortices could be generated for Re about 200. It is found by imposing the perturbation did not verify this approximation. Barkley [11] solved the plane Couette pattern instability by a numerical method when a ribbon is placed in the middle of the plan. He concluded that the results are about comparable when he used experimentally investigation. It has been obtained that by using the magnetize force is acting as a stabilizing factor, and some experimental data verifying this result.

This steady is an electrical conductive in infinite two planes by applying transverse magnetize force has been investigated that the results are showing by Lock [12] and Hussain et al. [13] observed that by applying Couette flow both stabilizing and destabilizing effect are appearing in the results. They determined the stability system is neglecting the velocity disturbance for different number of Ha . Zhang et al. [14] worked for the stability for the range of $0 < Ha < 3.91$ and destabilized by the magnetic field for the range of $3.91 < Ha < 5.4$ and stabilized again reversely for the range of $Ha > 5.4$. Nabil et al. [15]

presented the accurate results for the same flow and observed that flow is stabled when magnetic field is less than 3.9, destabilized for $3.88 \geq Ha \geq 5.542$ and then stabilized again for $Ha > 5.542$. Takashima [16] presented Couette flow with magnetic force impact. Some more studies in this topic can be seen in refs. [17–22].

The aim of current investigation is to observe a three-dimensional flow of parallel plate with applications of stability framework. The problem entertained the impact of magnetic force for Hartmann flow. The growth and disturbance of 3D problem is presented. Current model endorsed modifications in work of [14] and [15] for three-dimensional flow. Chebyshev collocation method with a different expansion is presented. The obtained results are more suitable as compared to [15]. Different expansion was used to solve this problem numerically with the help of QZ algorithm.

Mathematical formulation

Electrical conducting Hartmann flow is used to investigate the stability of this problem. Incompressible, viscous fluid properties are used with the help of perpendicular magnetize force. Electrically conductive fluid is moving from the parallel channel by setting $2L$ distance between the channels. The fluid is flowing inside the channel with basic velocity U_0 . We consider the flow in y -direction for the channel. The change in the velocity is appearing in x -axis of channel. Magnetize force is applying the normal direction of the channel that is in x -axis as presented in the schematic diagram of Fig. 1.

The flow is assumed to be induction less. The governing equations with non-dimensional parameters are stated as

$$\nabla \cdot \mathbf{v} = 0 \quad (1)$$

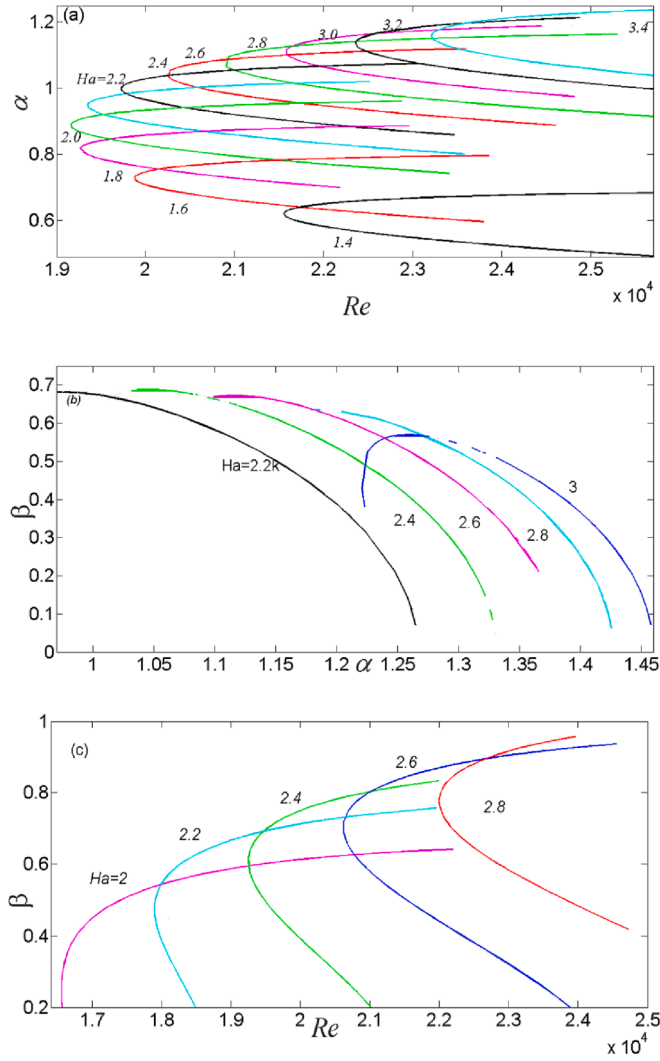


Fig. 2. Neutral curves of stability for (a) and, (b) and (c) and when.

$$\partial_t v + (v \cdot \nabla)v = -\nabla p + \frac{1}{Re} \Delta v + N(j \times \hat{x}) \quad (2)$$

In above equation, the Ohm law is represented by j which can be described as

$$j = -\nabla \phi + v \times \hat{x}, \quad (3)$$

and charge equation is represented by

$$\nabla \cdot j = 0. \quad (4)$$

The interaction number $N = \sigma LB^2 / \rho V$ (proportion of inertial & electromagnetic forces) and $Re = VL/\bar{\nu}$ (proportion of viscous force and inertial force are in ratio from) are two non-dimensional parameters. Interaction parameter N is the ratio electromagnetic forces and inertia and used in equation (2). Ha is represented by Hartmann number which is also depends on the strength of the magnetic field. The kinematic viscosity represents by the symbol $\bar{\nu}$. The boundary conditions considered for our system are shown in eq. (5).

$$\partial u / \partial x = \partial v / \partial y = 0, \quad \partial \phi / \partial x = u = 0, \text{ at } x = \pm L. \quad (5)$$

where v , ϕ , j , and p are represented as three-dimensional velocity vector, electric potential, current density and pressure V , σVB , VLB and ρV^2 , respectively. The fluid's material properties σ and ρ are density and electrical conductivity of the fluid and B is the magnetic field perpendicular to the channel.

Analysis of linear instability

The perturbation term for three dimensional Hartmann flow are represented as $u = u'$, $w = w'$, $v = \bar{v} + v'$, $\phi = \phi'$ and $p = p'$. By applying the new quantities of (u, v, p, ϕ) into equations (1) to (4) as:

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \quad (6)$$

$$\frac{\partial u'}{\partial t} + \bar{v} \frac{\partial u'}{\partial y} = -\frac{\partial p'}{\partial x} + \frac{1}{Re} \nabla^2 u' \quad (7)$$

$$\frac{\partial v'}{\partial t} + u' \frac{\partial \bar{v}}{\partial x} + \bar{v} \frac{\partial v'}{\partial y} = -\frac{\partial p'}{\partial y} + \frac{1}{Re} \nabla^2 v' - \frac{Ha^2}{Re} v' \quad (8)$$

$$\frac{\partial w'}{\partial t} + \bar{v} \frac{\partial w'}{\partial y} = -\frac{\partial p'}{\partial z} + \frac{1}{Re} \nabla^2 w' - \frac{Ha^2}{Re} w' \quad (9)$$

$$\frac{\partial^2 \phi'}{\partial x^2} + \frac{\partial^2 \phi'}{\partial y^2} + \frac{\partial^2 \phi'}{\partial z^2} + \frac{\partial w'}{\partial y} - \frac{\partial v'}{\partial z} = 0 \quad (10)$$

Calculating the basic velocity as:

$$\bar{v} = \frac{\sinh(Ha \cdot y)}{\cosh Ha} \quad (11)$$

Expression used in (6–10) can be further transformed as normal modes

$$(u', v', p', \phi') = [\hat{u}(x), \hat{v}(x), \hat{p}(x), \hat{\phi}(x)] e^{i(\alpha y + \beta z - \omega t)} \quad (12)$$

Here complex phase velocity $\omega = \omega_r + i\omega_i$, here ω_r is represented by oscillation frequency and ω_i is denoted by amplification frequency, α and β are y and z directional which are represented as wave numbers. (ω_i) and (ω_r) imaginary and real from represented as amplification and oscillation frequency. Applying the equation (12) in equation (6–10). The further simplification can be written as

$$D\hat{u} + i\alpha\hat{v} + i\beta\hat{w} = 0 \quad (13a)$$

$$-i\omega\hat{u} + i\alpha\hat{u} = -D\hat{p} + \frac{1}{Re} (D^2 - \alpha^2 - \beta^2)\hat{u} \quad (13b)$$

$$-i\omega\hat{v} + i\alpha\hat{v} + D\hat{v} = -i\alpha\hat{p} + \frac{1}{Re} (D^2 - \alpha^2 - \beta^2)\hat{v} + N(-i\beta\hat{\phi} - \hat{v}) \quad (13c)$$

$$-i\omega\hat{w} + i\alpha\hat{w} = -i\beta\hat{p} + \frac{1}{Re} (D^2 - \alpha^2 - \beta^2)\hat{w} + N(-i\alpha\hat{\phi} - \hat{w}) \quad (13d)$$

$$-(D^2 - \alpha^2 - \beta^2)\hat{\phi} + i\alpha\hat{w} - i\beta\hat{v} = 0 \quad (13e)$$

where D is represented as d/dx .

To write the systems (13-13e) in simplified form as

$$(\alpha\bar{v} - \omega)(D^2\hat{u} - k^2\hat{u}) - \alpha D^2\hat{v} - ND^2\hat{u} = -\frac{i}{Re} (D^4\hat{u} - 2k^2D^2\hat{u} + k^4\hat{u}) \quad (14)$$

The velocity in above equation in x axis and $k = (\alpha^2 + \beta^2)^{1/2}$ is represented as wave number. $\theta = \cos^{-1}(\alpha/k)$ represented as angle of flow direction of the wave speed, which is also known as Oblique angle.

By applying Squire's Theorem, we can rewrite Eq. (14) as

$$(\bar{v} - \omega)(D^2\hat{u} - k^2\hat{u}) - D^2\hat{v} - \frac{kN}{\alpha^2} D^2\hat{u} = -\frac{i}{kRe} (D^4\hat{u} - 2k^2D^2\hat{u} + k^4\hat{u}) \quad (15)$$

The equation (15) is simplified form of (14), Similarly boundary equation in (5) is simplified as

$$\hat{u} = 0 \quad D\hat{u} = 0 \text{ at } x = \pm L. \quad (16)$$

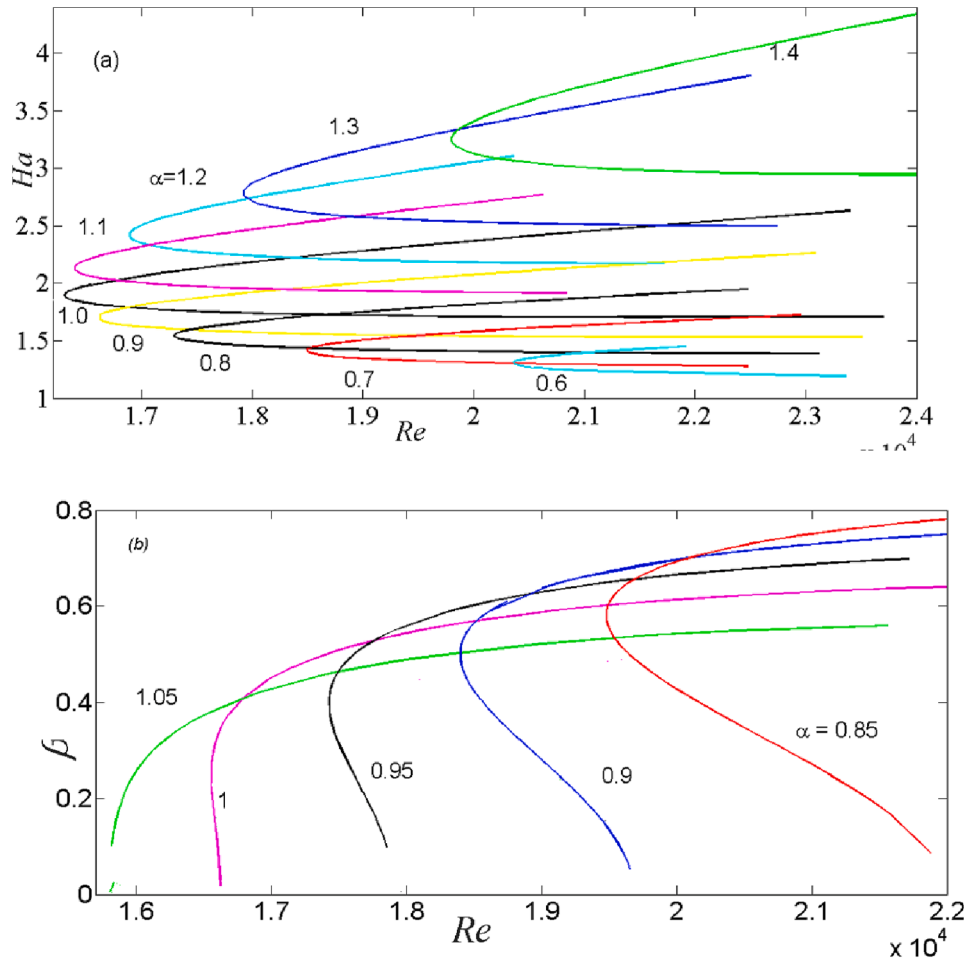


Fig. 3. Neutral curves of stability for various (a). and (b). and.

Numerical scheme

The numerical outcomes are presented via Chebyshev pseudospectral method. This scheme is based on performing the simulations for function presented via Eqs. (15) and (16) in $N_1 + 1$ terms. The velocity $\hat{u}$ and its derivatives used in (15–16) can be written in Chebyshev series expressed as follows:

$$\begin{aligned} u_m &= \sum_{n=0}^{N_1} a_n T_{nm} \\ u'_m &= \sum_{n=0}^{N_1} a_n T'_{nm} \\ u''_m &= \sum_{n=0}^{N_1} a_n T''_{nm} \\ u^{(4)}_m &= \sum_{n=0}^{N_1} a_n T^{(4)}_{nm} \end{aligned} \quad (m = 1, \dots, N_1) \quad (17)$$

Here, $T_n(x)$ is denoted as Chebyshev n th expansion of first kind, a_n is also the polynomial denoted as coefficient term, and $T'_{nm}, \dots, T^{(4)}_{nm}$ is expressed with Chebyshev polynomial with same collocation grid points as follow.

$$x_i = \cos\left(m \frac{\pi}{N_1}\right) m = 0, 1, 2, \dots, N_1 \quad (18)$$

Using $\hat{u}$ and its derivatives from equation (17), it can be represented as

$$\begin{aligned} \sum_{n=0}^{N_1} \left\{ T^{(4)} + \left[-k \frac{\overline{Ha}}{\alpha} - 2k - i\overline{v}k\overline{Re} \right] T'' + [k^2 + ik\overline{Re}\overline{v} + ik\overline{Re}D^2\overline{v}] T \right\} a_n \\ = \overline{\omega} \sum_{n=0}^{N_1} \left\{ -ik\overline{Re} [T'' - k^2 T] \right\} a_n \end{aligned} \quad (19)$$

The parameter used in our problem are redesigned as $\overline{Ha} = (\alpha/k)Ha$, $\overline{Re} = (\alpha/k)Re$, and $\overline{\omega} = \omega/\alpha$. The system satisfied by the expansion coefficients $X = (a_0, a_1, \dots, a_n)$ is expressed as

$$AX = \overline{\omega}BX \quad (20)$$

The $X^T = (a_0, a_1, a_3, \dots, a_n)$ is denoted as transpose of row vector matrix of X and A and B are matrices having the order $(N_1 + 1) \times (N_1 + 1)$.

To get the accurate results, the number of collocation points are used to get our results by setting $N_1 = 100$. System (19) in term of series can be simplified by using QZ technique.

Validation of results

The simulations are verified in Table 1 to ensure the accuracy of solution. The comparison of obtained data is verified with analysis of Nabil [15] and Takashima [16]. Clearly, a good agreement is noted with these studies.

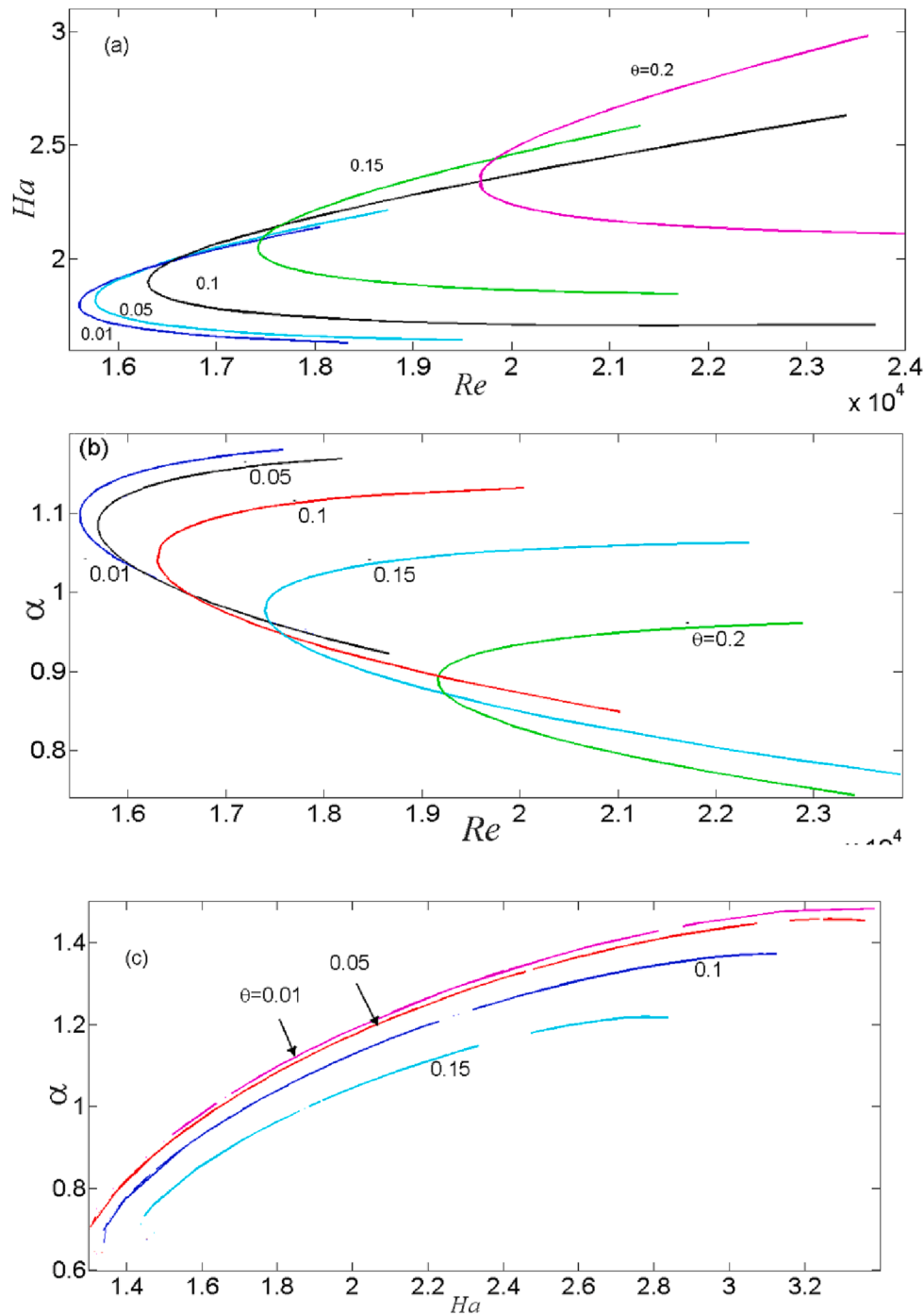


Fig. 4. Neutral curves of stability for various (a) and (b) and (c) and.

Results and discussion

The physical impact of flow parameters is presented in this section. The analysis is performed in view of fixed numerical values parameters like $\alpha = 0.83$, $Ha = 1.4$, $\theta = 0.2\pi$ and $Re = 19170$.

Effects of transverse magnetic field on 3-D instabilities

Fig. 2a presents the effect of Hartmann number Ha on 3D flow is investigated when $\theta = 0.2\pi$. It has been claimed that with variation of Ha from 1.4 to 2, Re is decreasing rapidly and flow system is destabilizing rapidly. However, variation of Ha from 2 to 3.4, the flow system destabilize steadily. It is further analyzed that for continues increasing

range of Ha from 1.4 to 3.4, two kind of stability outcomes are noted. First the flow is destabilizing than gradually starts stabilizing. The lowest values of Re_c appearing at $\alpha = 0.83$, $Ha = 2$ and $Re = 19170$. Upon enhancing Ha , the flow is destabilized gradually. For minor increase in starting range in Ha , the longitude and altitude area also increasing steadily. It has been known that for fluctuation of Ha from 2 to onward, the critical Reynolds number increases gradually. It can be concluded that the normal magnetic field destabilize and stabilize effect for various range of Ha . For given flow system, applications of magnetic field destabilizes for $1.4 \leq Ha \leq 2$. Small Hartmann number with given values of Reynolds number is observed to be 19170. Fig. 2b reports different neutral curves to analyses the stability for space parameter. The observations reveal that an increase in Ha with range 2.2 to 3, a

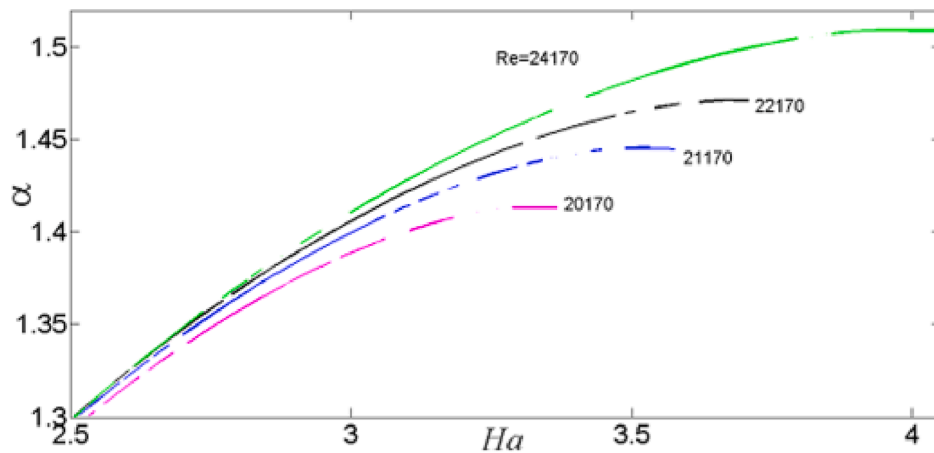


Fig. 5. Neutral curves for various values of.

destabilizing flow pattern is resulted. The upper side of different curve has a special shape and intersecting at one point from up side of the each curve. A gradual destabilizing pattern for enhancing Ha is noticed. Contribution of Ha for various values of Re and β is observed via Fig. 2c. For small change in Ha , the area in the unstable region become smaller. The shape of each curve is interesting and intersecting at one point in each curve. For leading values of Re , Ha also increases.

Effects of wave number on 3-D instabilities

The investigation regarding the instability of Hartman flow for different wave number is studied. Fig. 3 shows different neutral curves for $\theta = 0.2\pi$ with different wave number. Each curve showing the two effects for constant k . An unstable area is noted for k . Fig. 3b is sketched Re and β for distinct values of α . Different neutral curves are drawn to analyses the stability for space parameter. It is observed that an increase in α from 1.05 to 0.85 lead to stabilize the flow. The upper side of different curve has a special shape and intersecting at one point of each curve. Furthermore, for small increase in α , the flow is destabilizing gradually. The area between curves is increasing for enhancing wave number. It is further observed that a minor increasing fluctuation in Re is yield out for enriching wave number. The influence of α for Hartmann flow satisfies the stability of system.

Effects of θ on 3-D instabilities

The instability of Hartman flow for θ is important and predicted in various aspect. Fig. 4a implies the different neutral curves for θ along Re and Ha axes. Each curve showing two effects of k . The inside area become unstable whereas the outside regime of each curve become more stable. For small change in θ , a decreasing unstable region is noticed. The curves are diagonally shifting its position to the right direction. Fig. 4b is prepared to analyze the stability for space parameter for various values of θ . An increase in θ , the flow is stabilizing rapidly. The lower side of curves has been intersecting at one point of each curve. Moreover, for small increase in θ stabilize the flow rapidly. The area between the curves is increasing rapidly as the θ is increasing gradually. The influence of α for the Hartmann flow satisfies the stability of given system. The curves are diagonally shifting its position to the right direction. Fig. 4c has been drawn different neutral curves are drawn to analyses the stability for space parameter for various values of θ for Ha with α axis. We observed from this graph that the increase in the value of θ for the range 0.01–0.15 the flow is stabilizing rapidly as increase in θ . Two different observations are claimed for $k = 0.792$. The distance between curves is small for small Ha with small increment. Upon small increase in θ , the flow starts stabilizing rapidly. The influence of θ for Hartmann flow satisfies the stability when Ha is larger.

Effects of Re on neutral curves

Fig. 5 presents the different neutral curves between Ha and α for various values of Re . When Re is varies from 20,170 to 24,170, an unstable region steadily increases.

Conclusions

This study presents an analysis of the linear stability of 3D Hartmann flow in magnetohydrodynamics under the influence of normal magnetic force. The simulations were performed using the Chebyshev collocation technique. The study compared and discussed the instabilities of the fluid under different embedded physical parameters and growth rates. The following key observations were made:

- The transverse magnetic field has a stabilizing effect on the Hartmann flow.
- The normal magnetic field plays a crucial role in reducing the unstable region.
- The presence of a magnetized field enhances energy dissipation efficiency.
- The normal magnetized field induces a decreasing change in the velocity gradient along the span-wise direction and controls the unstable region.
- The magnetic field contributes to a more nominal energy dissipation, as the Lorentz force acts on the kinetic force of the electrical conducting fluid.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Acknowledgements

Princess Nourah bint Abdulrahman University Researchers Supporting Project number (PNURSP2023R 371), Princess Nourah bint Abdulrahman University, Riyadh, Saudi Arabia.

The authors would like to thank Deanship of Scientific Research at Majmaah University for supporting this work under Project No. R-2023-389.

References

- [1] Alfvén H. Existence of electromagnetic-hydrodynamic waves. *Nature* 1942;150 (3805):405–6.
- [2] P.G. Drazin W.H. Reid *Hydrodynamic Stability* 2 2004 Cambridge University Press.
- [3] Anselmo SP, Mompean G, Edson JS. Modeling and numerical simulation of polymer degradation in a drag reducing plane Couette flow. *J Fluid Mech* 2018; 256:1–7.
- [4] Kraheberger S, Hoyas S, Oberlack M. DNS of a turbulent Couette flow at constant wall transpiration up to $Re_\tau = 1000$. *J Fluid Mech* 2017;835:421–43.
- [5] Giribabu D, Shankar V. Stability of plane Couette flow of a power-law fluid past a neo-Hookean solid at arbitrary Reynolds number. *Phys Fluids* 2017;29: 074106–10.
- [6] Philippou M, Damianou Y, Miskouridou X, Georgiou GC. Cessation of Newtonian circular and plane Couette flows with wall slip and non-zero slip yield stress. *MECCA J* 2017;52(9):2081–99.
- [7] Harisingh SN, Rama KR. The effect of hall current on unsteady MHD free convective couette flow of a bingham fluid with thermal radiation. *Int J Endanger Adv Technol* 2014;3:1–16.
- [8] Orszag SA, Patera AT. Subcritical transition to turbulence in plane channel flows. *Phys Rev Lett* 1980;45(12):989–93.
- [9] Hussain Z, Khan N, Gul T, Ali M, Shahzad M, Sultan F. Instability of magneto hydro dynamics Couette flow for electrically conducting fluid through porous media. *Appl Nanosci* 2020;10(12):5125–34.
- [10] Hussain Z, Zeeshan R, Shahzad M. An optimised stability model for the magnetohydrodynamic fluid. *Pramana J Phys* 2021;95:27.
- [11] Barkley D, Tuckerman LS. Stability analysis of perturbed plane Couette flow. *Phys Fluid* 1999;11(5):1187–95.
- [12] Lock RC. The stability of the flow of an electrically conducting fluid between parallel planes under a transverse magnetic field. *Proc R Soc Lond A* 1955;221: 189–1125.
- [13] Hussain Z, Ali M, Shahzad M. Optimised wave perturbation for the linear instability of magnetohydrodynamics in plane Poiseuille flow. *Pramana J Phys* 2020;94:49.
- [14] Zhang Y, et al., Instability of magnetohydrodynamic flow of Hartmann layers between parallel plates. *AIP Adv* 2019;9:055003.
- [15] Eldabe NTM, El-Sabbagh MF, El-Sayed \(\text{Hajjaj}\) M A-S. Hydromagnetic stability of plane Poiseuille and Couette flow of viscoelastic fluid. *J Fluid Dyn Res* 2006;38 (10):699–715.
- [16] Takashima M. The stability of the modified plane Couette flow in the presence of a transverse magnetic field. *Fluid Dyn Res* 1998;22(2):105–21.
- [17] Hossain MM, Behera H. Shear-imposed falling thin Newtonian film over a porous slippery surface. *Phys Fluids* 2022;34(11):114124.
- [18] Samanta A. Effect of electric field on an oscillatory film flow. *Phys Fluids* 2019;31 (3):034109.
- [19] Paul D, Hossain MM, Behera H. Hydrodynamic instability of odd-viscosity-induced shear-imposed falling film. *Phys Fluids* 2023;35(3):034101.
- [20] Sengupta S, Ghosh S, Saha S, Chakraborty S. Rotational instabilities in microchannel flows. *Phys Fluids* 2019;31(5):054101.
- [21] Khan MB, Sani M, Ghosh S, Behera H. Poiseuille–Rayleigh–Benard instability of a channel flow with uniform cross-flow and thermal slip. *Phys Fluids* 2021. May 26; 33(5):053612.
- [22] Sani M, Selvan SA, Ghosh S, Behera H. Effect of imposed shear on the dynamics of a contaminated two-layer film flow down a slippery incline. *Phys Fluids* 2020;32 (10):102113.