

Energy density inhomogenization in relativistic spheres with Maxwell- $f(\mathcal{G}, T)$ theory

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Abstract: In this paper, the accountable factors are brought out for irregularity in energy density of spherically symmetric radiating fluid in $f(\mathcal{G}, T)$ gravity under electromagnetic field, where $\mathcal{G}$ is a Gauss–Bonnet term and T is the trace of the energy–momentum tensor. The expressions by varying mass function are composed by taking its radial and temporal derivatives. The relation between Weyl scalar, mass function, and modified field equations is developed, which would then be used to obtain Ellis equations. The role of parameters of fluid, Weyl curvature, correction terms, and electromagnetic field in the growth of inhomogeneity is explored by taking different types of realistic fluids. It is inferred that the electromagnetic field is trying to produce hindrances in the emergence of inhomogeneous energy density from the homogeneous one with $f(\mathcal{G}, T)$ terms.

Keywords: Gravitation; Charged fluid; Weyl scalar; Ellis equations

1. Introduction

The well-known general relativity (GR) is often regarded as the most effective realistic theory for studying the phenomena of gravitational interactions in self-gravitating systems and related circumstances. Einstein proposed this theory by considering the cosmos to be static. The investigation of its alternatives cannot be overlooked, which comes from the current conflict that GR is facing [1–4]. The discovery of different phenomena, like dark energy (DE), dark matter (DM), and universe expansion, urged scientists to modify the theory of GR. To address such issues many ideas have been proposed in the literature. The modified theories are proposed for further experiments by generalizing Einstein–Hilbert action with more general functions in order to deal with the dark sector of the cosmos.

The simplest framework in which one can easily ask the general question of which modified version of gravitational theories is permitted is $f(R)$ theory of gravity [5–9]. This modified version is obtained by replacing curvature scalar with more general scalar invariant function. Santos *et al.*

[10] discussed about the bounds urged by energy conditions on $f(R)$ theory and calculated strong and null energy conditions through Raychaudhuri equations. Sharif and Yousaf [11] studied the dynamical instability of collapsing fluid in cylindrical symmetric geometry under $f(R)$ theory of gravity by taking locally isotropic fluid with electromagnetic field and deducing that different features of fluids like corrections terms, energy density, and pressure are responsible for producing instability in the collapsing system. Dynamical instability of the spherical self-gravitating body was observed by Sharif and Yousaf [12] in the framework of $f(R)$ theory, where they assumed fluid to be anisotropic with usable CDTT model. After evaluating the useful expressions with a perturbation scheme, they concluded that dynamical properties are affected by dark sources coming from the CDTT model.

Then they [13] extend their work to evaluate dynamical equations by adopting the same procedure and deduced that charge is an additional term that affects the dynamical behavior of spherical isotropic fluid in $f(R)$ theory. The structure of a family of spherically symmetric static spacetimes formed by an anisotropic fluid and regulated by $f(R)$ gravity was investigated by Olmo and Garcia [14]. They discovered that such solutions represent black holes with a limited-size wormhole in place of the center

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singularity for a range of physical parameters of interest. Yousaf [15] explained the role of corrections terms and matter quantities in the formation of structure scalar and their growth through complexity factor. He concluded that the significant impact of the scalar is the presence of structural spherical effects resulting from the energy density inhomogeneity.

For more interesting results, a new theory was established by considering the curvature quantities like Riemann tensor ($R_{\xi\gamma\delta\eta}$), Ricci tensor ($R_{\xi\eta}$) and curvature scalar (R) in the GR action named as, $f(\mathcal{G})$ theory of gravity [16], where $\mathcal{G}$ is the topological invariant and stands for Gauss–Bonnet term. The primary benefit of $\mathcal{G}$ theory over others is that applicable $f(\mathcal{G})$ [17] models stay consistent under the constraints of the solar system. Myrzakula [18] discovered various cosmological solutions by taking Gauss–Bonnet gravity and observed that $f(\mathcal{G})$ theory can define Λ CDM model. It can be used as a substitute for DE and can reconstruct different kinds of solutions. Houndjo *et al.* [19] proposed the formation of anisotropic compact cylindrical systems in $f(\mathcal{G})$ gravity and explained the remarkable properties of the obtained compact star. Bamba *et al.* [20] developed the energy bounds for $f(\mathcal{G})$ gravity by considering FLRW metric for perfect fluid. Silva and Rodrigues [21] investigated the feasibility of generalizing GR solutions of regular black holes through an electric charge for the $f(\mathcal{G})$ theory. Bhatti *et al.* [22] contributed in $f(\mathcal{G})$ gravity by claiming that pressure anisotropy, energy density, and $f(\mathcal{G})$ terms affect the dynamical properties of self-gravitating fluid with the use of dynamical equations for curvature tensor. Mustafa *et al.* [23] inferred that gravitational force in modified $f(\mathcal{G})$ gravity is significantly stronger than in GR.

Kanti *et al.* [24] devoted their research to the early evolution of the cosmos, revealing that the Ricci scalar R is sub-dominant to Gauss–Bonnet (GB) term and thus can be ignored. They observed that during inflation, the scalar field reduces exponentially, and the GB terms provide the required potential for the field. Oikonomou [25] looked at how the $f(\mathcal{G})$ gravity behaved when a Type IV singularity appear in the bouncing domain. They found that the $f(\mathcal{G})$ theory effectively captures singular bouncing cosmology for the appropriate Hubble parameter. The $f(\mathcal{G}, T^2)$ theory is a novel theory that allows Lagrangian to rely on some analytical function of T^2 along with the GB term. It is plausible to assume that the T^2 will only be important in regions of high energy, such as the early-time universe or black holes. Yousaf *et al.* [26, 27] studied how this new gravitational model, the $f(\mathcal{G}, T^{\delta\gamma} T_{\delta\gamma})$ model, affect the complexity of time-dependent non-radiating and radiating spherical celestial structures. They revealed that the presence of GB and higher-order matter terms increases the

complexity, which leads to significant changes in the gravitational source.

In the realm of string-inspired dilatonic Einstein- $f(\mathcal{G})$ gravity, Kanti *et al.* [28] examined the existence of wormhole solutions without using exotic matter or phantom scalar fields, revealing that the $f(\mathcal{G})$ terms are completely responsible for the stable models of wormholes. In the realm of Einstein-scalar- $f(\mathcal{G})$ theory, Antoniou *et al.* [29] produced solutions for black holes and demonstrated that the creation of these solutions is a generic feature of this theory, and no-hair theorems may be conveniently avoided. Avik De *et al.* [30] investigated the role of geometrical constraints of $(GR)_4$ spacetime in $f(R, \mathcal{G})$ gravity. They explored the weak, null, strong, and dominant energy conditions in a generalized Ricci recurrent conformally flat space-time and proposed that their assumed models are in good agreement with current observational values. Antoniou *et al.* [31] supported the claims in [29] by analyzing the presence of scalar hair black hole solutions by enforcing a specific constraint and in a certain background. They also calculated the corresponding cosmic solutions in $f(\mathcal{G})$ theory.

The $f(\mathcal{G}, T)$ theory of gravity is a modified form of $f(\mathcal{G})$ theory, where T is the trace of the stress–energy tensor, presented by Sharif and Ikram [32]. The work of Harko *et al.* [33] in the building of $f(R, T)$ theory from $f(R)$ gravity inspired them the inclusion of T in the analysis of [16]. By using power law and de sitter solutions Sharif and Ikram [32] formulated the corresponding energy conditions in the framework of $f(\mathcal{G}, T)$ and obtained plausible constraints on the free variables. The $f(\mathcal{G}, T)$ theory of gravity models was utilized to reconstruct Λ CDM cosmology without the usage of a cosmological constant [34] to obtain solutions by taking Noether symmetry strategy. Yousaf [35, 36] observed structure scalars in the light of $f(\mathcal{G}, T)$ theory of gravity and found the behavior of scalars in this context. The effect of $f(\mathcal{G}, T)$ on expanding star was studied by Bhatti *et al.* [37]. By using energy conditions with physical variables, they deduced that the compactness of a star model's core grows as the energy conditions hold. Ilyas *et al.* [38] discussed the physical behavior of a charged sphere using the $f(\mathcal{G}, T)$ formalism and studied the structures of realistic charged sphere. Maurya *et al.* [39] presented a spherical solution of a charged fluid by embedding it in the $f(\mathcal{G}, T)$ formalism and the results suggested that the solution obtained depicts a physically viable and stable compact star. In $f(\mathcal{G}, T)$ theory, Yousaf *et al.* [40] looked at the effect of charge on the complexity of a considered static fluid and showed that complexity is affected by charge and $f(\mathcal{G}, T)$ terms.

Gravitational collapse is a significant phenomenon in the relativistic self-gravitating system when it experiences

irregularity in the energy density. This phenomenon has enforced numerous researchers to investigate the variables that contribute to the appearance of inhomogeneity in the initially regular radiating objects [41, 42]. Herrera [43] discovered that any deviation from equilibrium (or quasi-equilibrium) of a matter distribution is strongly governed by changes in the Weyl curvature. Herrera and his collaborators [44] studied the expansion-free shearing distribution of matter with constant energy density and by using dust case they deduced that expansion-free condition requires irregularity. Gravitational behavior of radiating star in spherically symmetric geometry by taking geodesic motion was explained by Thirukkanesh and Maharaj [45], where they found different equations satisfying special and elementary functions. Herrera *et al.* [46] looked at the irregularity in LTB spacetimes for tilted congruence in which they identified and defined the factor that contributed to the existence of inhomogeneity and compared their findings to those obtained by a non tilted observer. Finally, they showed a strange circumstance in which the non-tilted congruence may be unstable.

Herrera [47] observed the causes of collapsing fluid inhomogeneity and homogeneity by taking different features of considered fluid and concluded that the vanishing of Weyl curvature with correction terms implies homogeneity. The role of irregularity in energy density and anisotropy of pressure for collapsing phenomena was studied by Sharma and Tikekar [48]. Govender *et al.* [49] investigated how the collapse phenomenon reacts in the existence of shear. They showed that shear causes an increase in core temperature, underscoring the importance of relaxations effects when a star exits hydrostatic equilibrium. Herrera *et al.* [50] investigated the physical infeasibility of the radiating geodesic dust and showed that until the dissipative flow diminishes, a geodesic, rotational radiating dust violates regularity criteria at the center of the distribution. Yousaf and his colleagues [51–54] recently investigated how (non)-tilted congruence influences the dynamical features of a self-gravitating fluid in which he looked at the stability of irregular energy density in the setting of modified gravity.

It is quite interesting to start an investigation of the charged self-gravitating systems. In this direction, Roselland [55] began his research on the phenomenon of collapse in charged radiating fluids. A lot of work has been done to investigate the contribution of the electromagnetic field in the evolution of self-gravitating systems [56–58]. In the existence of cosmological constant, structure scalar is investigated for charged radiating spherical sphere and dust to understand the change in inhomogeneity by Herrera *et al.* [59]. Pinheiro and Chan [60] studied the role of

charge in inhomogeneity by taking radiating fluids and concluded that conformal flatness implies homogeneity. Di Prisco *et al.* [61] studied the collapse of charged relativistic spherical systems. They also calculated the impact of electric charge in the formation of inhomogeneous energy density by exploring the corresponding relation. The role of electric charge has been observed by Nyonyi *et al.* [62] by simulating a spherical sphere with a radiating matter distribution that dissipates in the heat and radiation density mode. They found that their calculated charged models are in fact the generalizations of conformally flat spacetime. The significance of Palatini $f(R)$ theory and electromagnetic field in the energy density irregularity of plane-symmetric anisotropic fluid was proposed by Bhatti and Yousaf [63] with the results that charge and correction terms try to keep the system homogeneous. Yousaf [64] calculated the exact solution of radiating and non-radiating fluid with the condition of conformal flatness and concluded that dark source terms complicate the system but they do not affect the temperature of the system.

We extend the work of Herrera [47] to understand the energy-density irregularity of relativistic non-adiabatic sphere using the $f(\mathcal{G}, T)$ theory of gravity under the effect of an electromagnetic field. The following is how it is organized: The distribution of a radiating compact charged sphere, which is spherically symmetric and confined by Σ , is described in Sect. 2. The kinematical variables, modified and conservation equations, and $f(\mathcal{G}, T)$ terms will be explored in the form of subsections of this section. The key expressions of Sect. 3 are the mass function, dynamical equations, modified differential equations for Weyl curvature, and the associated heat flow transfer equation. Using the updated $f(\mathcal{G}, T)$ theory and various fluid features, Sect. 4 investigates various reasons for the causes of energy-density irregularity of the spherical system. Finally, we talk about how our efforts have turned out.

2. Fluid distribution in $f(\mathcal{G}, T)$ theory of gravity

The generalization of Einstein–Hilbert action in $f(\mathcal{G}, T)$ theory takes the form

$$\mathcal{S}_{f(\mathcal{G}, T)} = \frac{1}{2\kappa^2} \int [R + f(\mathcal{G}, T)] \sqrt{-g} d^4x + \int \mathcal{L}_m \sqrt{-g} d^4x, \quad (1)$$

where T , R , $\mathcal{G}$ and $\mathcal{L}_m$ represent trace of energy–momentum tensor, curvature scalar, Gauss–Bonnet terms, and Lagrangian matter density, respectively. The energy–momentum tensor for the matter content can be described as

$$T_{\delta\beta} = -\frac{2}{(-g)^{\frac{1}{2}}} \frac{\delta((-g)^{\frac{1}{2}} \mathcal{L}_m)}{\partial g^{\delta\beta}}, \quad (2)$$

which yields

$$T_{\delta\beta} = -\frac{2\partial \mathcal{L}_m}{\partial g^{\delta\beta}} + g_{\delta\beta} \mathcal{L}_m. \quad (3)$$

Using a variational strategy, Eq. (3) becomes

$$\frac{\delta T_{\delta\beta}}{\delta g_{\nu\mu}} = -\frac{2\partial^2 \mathcal{L}_m}{\partial g_{\nu\mu} \partial g^{\delta\beta}} + g_{\delta\beta} \frac{\partial \mathcal{L}_m}{\partial g_{\nu\mu}} + \mathcal{L}_m \frac{\delta g_{\delta\beta}}{\delta g_{\nu\mu}}. \quad (4)$$

We assume a radiating collapsing charged fluid in spacetime, which is spherically symmetric with comoving coordinates in this paper. The general line element for a spherical surface Σ is defined as

$$ds^2 = Y^2 dr^2 - X^2 dt^2 + C^2 d\theta^2 + C^2 \sin^2 \theta d\phi^2, \quad (5)$$

where the metric components are positive and coordinates are demonstrated as $x^1 = t$, $x^2 = r$, $x^3 = \theta$, $x^4 = \phi$. We suppose that the geometry above is coupled with a locally anisotropic charged fluid that is radiating in the form of null radiations and heat under an electromagnetic field.

2.1. Matter stress–energy tensor

The following equation denotes the matter stress–energy tensor

$$T_{\delta\beta}^{(m)} = (P_{\perp} + \mu)V_{\delta}V_{\beta} + \epsilon l_{\delta}l_{\beta} + (-P_{\perp} + P_r)\chi_{\delta}\chi_{\beta} + q_{\delta}V_{\beta} + P_{\perp}g_{\delta\beta} + V_{\delta}q_{\beta}, \quad (6)$$

where χ_{γ} , l_{γ} , V_{γ} , ϵ , q_{γ} , P_r , μ and $P_{\perp}$ stand for unit four vector, null radial vector, four velocity, null-fluid density, heat flux, radial pressure, energy density, and tangential pressure and are functions of r and t . The above factors satisfy

$$\chi^{\delta}\chi_{\delta} = 1, \quad V^{\delta}V_{\delta} = -1, \quad q_{\delta}V^{\delta} = 0, \quad (7)$$

$$l^{\delta}l_{\delta} = 0, \quad V_{\delta}l^{\delta} = -1, \quad V_{\delta}\chi^{\delta} = 0. \quad (8)$$

where $q^{\delta} = q\chi^{\delta}$. The expansion scale (Θ) and acceleration (a_{δ}) of fluid are written as

$$\Theta = V^{\delta}{}_{;\delta}, \quad a_{\delta} = V^{\beta}V_{\delta;\beta}. \quad (9)$$

The form of shear tensor is

$$\sigma_{\delta\beta} = a_{(\delta}V_{\beta)} - \frac{1}{3}\Theta h_{\delta\beta} + V_{(\delta;\beta)}, \quad (10)$$

where $h_{\delta\beta} = V_{\beta}V_{\delta} + g_{\delta\beta}$. We assume the considered metric (5) is resting in a comoving reference frame. Therefore

$$\chi^{\nu} = \frac{1}{Y}\delta_2^{\nu}, \quad q^{\nu} = \frac{q}{Y}\delta_2^{\nu}, \quad l^{\nu} = \frac{1}{X}\delta_1^{\nu} + \frac{1}{Y}\delta_2^{\nu}, \quad V^{\nu} = \frac{1}{X}\delta_1^{\nu}. \quad (11)$$

Equation (9) with Eq. (11) yields

$$a^2 = a_{\delta}a^{\delta} = \left(\frac{\partial X}{\partial r}\right)^2 \left(\frac{1}{YX}\right)^2, \quad a_2 = \left(\frac{\partial X}{\partial r}\right)\frac{1}{X}, \quad (12)$$

and for expansion Θ , we get

$$\Theta = \frac{1}{XYC}(C\dot{Y} + 2Y\dot{C}), \quad (13)$$

using Eq. (11) in Eq. (10), we obtain scalar and nonzero expressions of shear tensor as follows

$$\sigma_{\delta\beta}\sigma^{\delta\beta} = \frac{2}{3}\sigma^2, \quad \sigma = \frac{1}{XYC}(\dot{Y}C - \dot{C}Y), \quad (14)$$

$$\sigma_{22} = \frac{2\sigma}{3}Y^2, \quad \sigma_{33} = \frac{\sigma_{44}}{\sin^2 \theta} = -\frac{\sigma}{3}C^2. \quad (15)$$

2.2. Modified field equations in $f(\mathcal{G}, T)$ gravity

The field equations for the charged radiating fluid in $f(\mathcal{G}, T)$ theory are written as

$$G_{\delta\beta} = T_{\delta\beta}^{(tot)} = T_{\delta\beta}^{(m)} + E_{\delta\beta} + T_{\delta\beta}^{(\mathcal{G}, T)}, \quad (16)$$

in which the first term in right hand side is for matter part, the second term denotes the presence of charge and the third term is for modified gravity part.

2.3. Electromagnetic stress–energy tensor

The electromagnetic stress–energy tensor is given by

$$E_{\delta\beta} = \frac{1}{4\pi} \left(-\frac{1}{4}F^{\eta\zeta}F_{\eta\zeta}g_{\delta\beta} + F_{\delta}^{\eta}F_{\beta\eta} \right), \quad (17)$$

where $F_{\eta\zeta}$ is Maxwell field tensor, expressed as

$$F_{\delta\beta} = \phi_{\beta;\delta} - \phi_{\delta;\beta} \quad \text{and} \quad \phi_{\delta} = \phi\delta_{\delta}^1, \quad \phi_1 = \phi(t, r),$$

here ϕ_{δ} and ϕ denote four and scalar potential, respectively. $E = \frac{q}{C^2 4\pi}$ is electric intensity of charge. The Maxwell field equation, in which J^{δ} is four current, can be written as

$$F^{\delta\beta}{}_{;\beta} = 4\pi J^{\delta}. \quad (18)$$

Equation (18) gives

$$\frac{\partial^2 \phi}{\partial r^2} - 4\pi XY^2 \mu_0 = \left(\frac{Y'}{Y} + \frac{2C'}{C} + \frac{X'}{X} \right) \frac{\partial \phi}{\partial r}, \quad (19)$$

$$\frac{\partial^2 \phi}{\partial r \partial t} = \left(\frac{\dot{Y}}{Y} - \frac{2\dot{C}}{C} + \frac{\dot{X}}{X} \right) \frac{\partial \phi}{\partial r}. \quad (20)$$

Equation (19) becomes by taking $A = \frac{\partial \phi}{\partial r}$

$$\frac{\partial A}{\partial r} + \left(\frac{2C'}{C} - \frac{Y'}{Y} - \frac{X'}{X} \right) A = 4\pi X \mu_0 Y^2. \quad (21)$$

Its integration gives

$$A = \frac{XY}{C^2} \mathbf{q}, \quad (22)$$

where $\mathbf{q} = 4\pi \int_0^r \mu_0 C^2 Y dr$ is an electric charge. The independent components of (17) are

$$\begin{aligned} E_{11} &= 2\pi E^2 X^2, & E_{22} &= -2\pi E^2 Y^2, \\ E_{33} &= 2\pi E^2 C^2, & E_{44} &= E_{33} \sin^2 \theta. \end{aligned} \quad (23)$$

2.4. Modified stress–energy tensor

The modified version of stress–energy tensor in $f(\mathcal{G}, T)$ gravity is

$$\begin{aligned} T_{\delta\beta}^{(\mathcal{G}, T)} &= \frac{1}{2} g_{\delta\beta} f(\mathcal{G}, T) - (\Theta_{\delta\beta} + T_{\delta\beta}^{(m)}) f_T(\mathcal{G}, T) \\ &\quad + 2[(2R_{\delta\beta\alpha\nu} R^{\alpha\nu} + 2R_{\delta\alpha} R_{\beta}^{\alpha} - RR_{\delta\beta} \\ &\quad - R_{\delta\beta\eta\nu} R_{\alpha}^{\eta\nu}) f_{\mathcal{G}}(\mathcal{G}, T) + (2R_{\delta\beta} - Rg_{\delta\beta}) \nabla^2 f_{\mathcal{G}}(\mathcal{G}, T) \\ &\quad - 2R_{\delta}^{\alpha} \nabla_{\alpha} \nabla_{\beta} f_{\mathcal{G}}(\mathcal{G}, T) + R \nabla_{\delta} \nabla_{\beta} f_{\mathcal{G}}(\mathcal{G}, T) \\ &\quad + 2g_{\delta\beta} R^{\alpha\nu} \nabla_{\alpha} \nabla_{\nu} f_{\mathcal{G}}(\mathcal{G}, T) - 2R_{\beta}^{\alpha} \nabla_{\delta} \nabla_{\alpha} f_{\mathcal{G}}(\mathcal{G}, T) \\ &\quad - 2R_{\delta\alpha\beta\eta} \nabla^{\alpha} \nabla^{\eta} f_{\mathcal{G}}(\mathcal{G}, T)], \end{aligned} \quad (24)$$

where d'Alembert operator is given by $\nabla^2 = \nabla_{\delta} \nabla^{\delta}$, $f_T(\mathcal{G}, T) \equiv \frac{\partial f(\mathcal{G}, T)}{\partial \mathcal{G}}$ and $f_{\mathcal{G}}(\mathcal{G}, T) \equiv \frac{\partial f(\mathcal{G}, T)}{\partial \mathcal{G}}$.

In cosmology, DHOST theories can be incorporated into a unified efficient description of modified gravity and dark energy. The Lagrangian of DHOST theories contain scalar and tensor terms [65], which causes instability in the analysis of any system. It also raises extra degrees of freedom which is not easy to tackle in normal circumstances. One may need to impose some extra conditions to reduce the order of field equations because it is commonly believed in recent studies that those Euler–Lagrangian which results in second-order field equations are exempt from extra degrees of freedom.

Further, $f(R)$ and $f(\mathcal{G})$ gravity could be taken as interesting gravitation models for the description of DE and relativistic fluid in a systematic way. Such theories can be made ghost free and thus could be useful in describing various aspects such as inflation, deceleration-to-acceleration phase, and current cosmic dynamics. It specifies the role of matter and curvature terms exclusively, to understand the mutual effects of matter and curvature at a large gravitational scale, it could be worthwhile to take $f(\mathcal{G}, T)$ into consideration. Due to this coupling, test particles

experience an extra force, which is discovered to be orthogonal to the four velocities, causing the motion to become non-geodesic. This non-geodesic motion under $f(\mathcal{G}, T)$ gravity indicates the acceleratory nature of the cosmos. Moreover, it effectively deals with various astrophysical issues such as late-time expansion, inflation, and current cosmological behavior.

To invoke the curvature terms of the $f(\mathcal{G}, T)$ gravity, separate formulations for the functions $\mathcal{G}$ and T might be utilized. As a result, we choose a model of the type $f(\mathcal{G}, T)$ shown below [36]

$$f(\mathcal{G}, T) = f_2(T) + f_1(\mathcal{G}). \quad (25)$$

These types of models are thought to be possible corrections in $f(\mathcal{G})$ theory. The models above might be used as toy models to learn more about the universe's dark area. We use linear $f_2(T)$ to investigate the role of curvature terms deriving from $f(\mathcal{G})$ gravity on the dynamics of a spherical star in this work. As a result, we have

$$f(\mathcal{G}, T) = \lambda T + f_1(\mathcal{G}), \quad (26)$$

where λ is regarded as constant. For Gauss–Bonnet terms now we take model of $f(\mathcal{G})$ as

$$f_1(\mathcal{G}) = \alpha \mathcal{G}^2 + \mathcal{G}, \quad (27)$$

where α is any fixed real number. As a result, we may write

$$f(\mathcal{G}, T) = \mathcal{G} + \lambda T + \alpha \mathcal{G}^2.$$

By using this form of the model, we explored interesting results for various forms of fluids. The curvature defining term $\mathcal{G}$ reacts differently on the dynamical stability and existence of the considered system for different kinds of spacetime. The spacetime symmetry determines the effect of $\mathcal{G}$ on the stability; for example, it is seen that for the stability analysis of wormhole [66], the role of $\mathcal{G}$ is significant for different spacetime such as it has no effect on the stability of planar symmetric wormholes. However, it destabilized the wormhole in spherical symmetry but stabilized the wormhole for hyperbolical symmetry. Similarly in our case, the role of $\mathcal{G}$ is significant. We explored the role of this curvature term for spherical symmetric non-static spacetime, in which it rises as a function of r and t . It is clear from the observation that coupling of this term with T affects the system at a large scale and curvature terms along with matter make the system irregular, i.e., for spherical symmetry $\mathcal{G}$ produces irregularity and leads the system to collapse region for making new objects. Consequently, the results that are provided in the influence of $\mathcal{G}$

terms reveal a better way to understand the evolution of the spherical structures at a high curvature epoch.

In the assumed model, the α and λ are regarded as constants, which are used here in a general form. From further formulations, one can be seen that these constants define the existence of inhomogeneity. It is obvious from our findings that, all the results reduce to GR if we assume $\alpha = \lambda = 0$, which indicate the significance of these parameters.

In Eq. (24) $\Theta_{\delta\beta}$ is given by

$$\Theta_{\delta\beta} = g_{\alpha\eta} \frac{\partial T_{\alpha\eta}}{\partial g_{\delta\beta}}, \quad (28)$$

Equations (28) and (4) provide

$$\Theta_{\delta\beta} = g_{\delta\beta} \mathcal{L}_m - 2T_{\delta\beta} - 2g^{\alpha\eta} \frac{\partial^2 \mathcal{L}_m}{\partial g^{\delta\beta} \partial g^{\alpha\eta}}, \quad (29)$$

We can take the Lagrangian density of considered matter as $\mathcal{L}_m = -P$ [33]. Thus Eq. (29) becomes

$$\Theta_{\alpha\gamma} = -2T_{\alpha\gamma} - Pg_{\alpha\gamma}.$$

Equations (5), (6), (11), (23), and (24) yield us to independent components of modified form of Einstein's field equations

$$\begin{aligned} (T_{11}^{(m)} + E_{11} + T_{11}^{(G,T)}) &= \tilde{\mu}(1 + \lambda) - \lambda P \\ &\quad - \frac{(\mathcal{G} + \lambda T + \alpha \mathcal{G}^2)}{2} \\ &\quad + 2 \left(\frac{\chi_1}{2X^2} + \frac{\chi_1 \alpha \mathcal{G}}{X^2} + \frac{\alpha \chi_1^*}{X^2} + \pi E^2 \right) \\ &= \left(\frac{2Y'C'}{Y^3 C} - \frac{C'^2}{C^2 Y^2} - \frac{2C''}{CY^2} + \frac{1}{C^2} \right) \\ &\quad - \left(\frac{2\dot{Y}\dot{C}}{X^2 Y C} + \frac{\dot{C}^2}{X^2 C^2} \right), \end{aligned} \quad (30)$$

$$\begin{aligned} (T_{22}^{(m)} + E_{22} + T_{22}^{(G,T)}) &= \tilde{P}_r(1 + \lambda) + \lambda P \\ &\quad + \frac{(\mathcal{G} + \lambda T + \alpha \mathcal{G}^2)}{2} \\ &\quad + 2 \left(\frac{\chi_2}{2Y^2} + \frac{\chi_2 \alpha \mathcal{G}}{Y^2} + \frac{\alpha \chi_2^*}{Y^2} - \pi E^2 \right) \\ &= \left(\frac{2\dot{X}\dot{C}}{X^3 C} - \frac{2\dot{C}^2}{C^2 X^2} - \frac{2\ddot{C}}{X^2 C} \right) \\ &\quad + \left(\frac{C'^2}{C^2 Y^2} - \frac{1}{C^2} + \frac{2C'X'}{CXY^2} \right), \end{aligned} \quad (31)$$

$$\begin{aligned} (T_{12}^{(m)} + T_{12}^{(G,T)}) &= -\tilde{q}(1 + \lambda) + \frac{2\alpha Z_1}{YX} = \left(\frac{2\dot{Y}C'}{Y^2 X C} \right. \\ &\quad \left. + \frac{2\dot{C}X'}{X^2 C Y} - \frac{2\dot{C}'}{XYC} \right), \end{aligned} \quad (32)$$

$$\begin{aligned} (T_{33}^{(m)} + E_{33} + T_{33}^{(G,T)}) &= P_\perp(1 + \lambda) + \lambda P + \frac{(\mathcal{G} + \lambda T + \alpha \mathcal{G}^2)}{2} \\ &\quad + 2 \left(\frac{Z_2}{2C^2} + \frac{Z_2 \alpha \mathcal{G}}{C^2} + \frac{\alpha Z_2^*}{C^2} + \pi E^2 \right) \\ &= - \left(\frac{\ddot{C}}{CX^2} - \frac{C''}{CY^2} + \frac{\dot{Y}\dot{C}}{CYX^2} - \frac{\dot{X}\dot{C}}{CX^3} + \frac{\ddot{Y}}{YX^2} \right) \\ &\quad + \left(\frac{X''}{XY^2} + \frac{\dot{X}\dot{Y}}{YX^3} + \frac{X'C'}{CXY^2} - \frac{X'Y'}{XY^3} - \frac{Y'C'}{CY^3} \right), \end{aligned} \quad (33)$$

$$\begin{aligned} (T_{44}^{(m)} + E_{44} + T_{44}^{(G,T)}) &= P_\perp(1 + \lambda) + \lambda P + \frac{(\mathcal{G} + \lambda T + \alpha \mathcal{G}^2)}{2} \\ &\quad + 2 \left(\frac{Z_2}{2C^2 \sin^2 \theta} + \frac{Z_2 \alpha \mathcal{G}}{C^2 \sin^2 \theta} + \frac{\alpha Z_2^*}{C^2 \sin^2 \theta} + \pi E^2 \right) \\ &= - \frac{1}{\sin^2 \theta} \left(\frac{\ddot{C}}{CX^2} - \frac{C''}{CY^2} + \frac{\dot{Y}\dot{C}}{CYX^2} - \frac{\dot{X}\dot{C}}{CX^3} + \frac{\ddot{Y}}{YX^2} \right) \\ &\quad + \frac{1}{\sin^2 \theta} \left(\frac{X''}{XY^2} + \frac{\dot{X}\dot{Y}}{YX^3} + \frac{X'C'}{CXY^2} - \frac{X'Y'}{XY^3} - \frac{Y'C'}{CY^3} \right). \end{aligned} \quad (34)$$

where $\tilde{P}_r = P_r + \epsilon$, $\tilde{q} = q + \epsilon$, $\tilde{\mu} = \mu + \epsilon$ and the quantities χ_2 , χ_2^* , χ_1 , χ_1^* , Z_2 , Z_1 and Z_2^* are for modified parts and their values are given in "Appendix."

3. Dynamical equations and Ellis equations

In the influence of charge, mass [67] function is described as

$$\begin{aligned} m &= \frac{C}{2} R_{232}^3 + \frac{q^2}{2C} = 8\pi^2 C^3 E^2 + \left[\frac{XYC}{2} \right. \\ &\quad \left. + \frac{YCC'^2}{2} - \frac{XCC'^2}{2} \right] \frac{1}{XY}. \end{aligned} \quad (35)$$

The radial and proper time derivatives are

$$D_C = \frac{1}{C'} \frac{\partial}{\partial r}, \quad D_T = \frac{1}{X} \frac{\partial}{\partial t}. \quad (36)$$

The velocity of charged radiating fluid is

$$U = X^{-1} \dot{C}. \quad (37)$$

Equation (35) now becomes

$$\mathbb{E} \equiv \frac{C'}{Y} = \sqrt{-\frac{2m}{C} + U^2 + 1 + 8\pi^2 C^3 E^2}. \quad (38)$$

The variation in Eq. (35) tells us about the role of different variables on mass function and change is given by considering Eqs. (30)–(32) as

$$D_T m = -C^2 \left[\left(\tilde{P}_r(1+\lambda) + \lambda P + \frac{(\mathcal{G} + \lambda T + \alpha \mathcal{G}^2)}{2} \right. \right. \\ \left. \left. + 2 \left(\frac{\chi_2}{2Y^2} + \frac{\chi_2 \alpha \mathcal{G}}{Y^2} \right. \right. \right. \\ \left. \left. + \frac{\alpha \chi_2^*}{Y^2} - \pi E^2 \right) \right) U + \left(\tilde{q}(1+\lambda) - \frac{2\alpha Z_1}{XY} \right) \mathbb{E} \\ \left. - \frac{8\pi^2}{X} (3\dot{C}E^2 + 2E\dot{E}C) \right], \quad (39)$$

$$D_C m = C^2 \left[\tilde{\mu}(1+\lambda) - \lambda, P - \frac{(\mathcal{G} + \lambda T + \alpha \mathcal{G}^2)}{2} \right. \\ \left. + 2 \left(\frac{\chi_1}{2X^2} + \frac{\chi_1 \alpha \mathcal{G}}{X^2} \right. \right. \\ \left. \left. + \frac{\alpha \chi_1^*}{X^2} + \pi E^2 \right) + \left(\tilde{q}(1+\lambda) - \frac{2\alpha Z_1}{XY} \right) \frac{U}{\mathbb{E}} + \frac{8\pi^2}{C'} (3C'E^2 \right. \\ \left. + 2EE'C) \right]. \quad (40)$$

Integration of latter equation yields

$$m = \int_0^r C' C^2 \left[\tilde{\mu}(1+\lambda) - \lambda P - \frac{(\mathcal{G} + \lambda T + \alpha \mathcal{G}^2)}{2} \right. \\ \left. + 2 \left(\frac{\chi_1}{2X^2} + \frac{\chi_1 \alpha \mathcal{G}}{X^2} \right. \right. \\ \left. \left. + \frac{\alpha \chi_1^*}{X^2} + \pi E^2 \right) + \left(\tilde{q}(1+\lambda) - \frac{2\alpha Z_1}{XY} \right) \frac{U}{\mathbb{E}} \right. \\ \left. + \frac{8\pi^2}{C'} (3C'E^2 + 2EE'C) \right] dr, \quad (41)$$

where $m(0) = 0$ for regular charged fluid distribution is taken. The integrating of Eq. (41) gives

$$\frac{3m}{C^3} = \tilde{\mu}(1+\lambda) - \frac{1}{C^3} \int_0^r C^3 \left[D_C \tilde{\mu}(1+\lambda) \right. \\ \left. + \frac{3\lambda P}{C} + \frac{3(\mathcal{G} + \lambda T + \alpha \mathcal{G}^2)}{2C} \right. \\ \left. - \frac{6}{C} \left(\frac{\chi_1}{2X^2} + \frac{\chi_1 \alpha \mathcal{G}}{X^2} + \frac{\alpha \chi_1^*}{X^2} + \pi E^2 \right) - \frac{3U}{C\mathbb{E}} \left(\tilde{q}(1+\lambda) \right. \right. \\ \left. \left. - \frac{2\alpha Z_1}{XY} \right) - \frac{24\pi^2}{C'C} \right. \\ \left. (3C'E^2 + 2EE'C) \right] C' dr. \quad (42)$$

In $f(\mathcal{G}, T)$ theory, the nonzero divergence of Eq. (24) is expressed as

$$\nabla^\delta T_{\delta\beta} = \frac{f_T(\mathcal{G}, T)}{1 - f_T(\mathcal{G}, T)} [(T_{\delta\beta}^m + \Theta_{\delta\beta}) \nabla^\delta (\ln f_T(\mathcal{G}, T)) \\ + \nabla^\delta \Theta_{\delta\beta} - \frac{1}{2} g_{\delta\beta} \nabla^\delta T]. \quad (43)$$

whose nonzero components are

$$(1+3\lambda) \left[\tilde{\mu} + \frac{\tilde{q}X}{Y} + \frac{2q(XC)'}{YC} + (\tilde{\mu} + \tilde{P}_r) \frac{\dot{Y}}{Y} + (2\tilde{\mu} + 2P_\perp) \frac{\dot{C}}{C} \right] = \\ \left(\frac{\dot{T}\lambda}{2} + 2\dot{P}\lambda - l_0 - l_0^* \right), \quad (44)$$

$$(1+3\lambda) \left[\tilde{q} + \tilde{q} \left(\frac{2\dot{C}}{C} + \frac{2\dot{Y}}{Y} \right) + (\tilde{\mu} + \tilde{P}_r) \frac{X'}{Y} + 2\Pi \frac{C'X}{CY} + (\tilde{P}_r)' \frac{X}{Y} \right] = \\ - \left(\frac{T'X\lambda}{2Y} + \frac{2P'X\lambda}{Y} + l_1 + l_1^* \right), \quad (45)$$

where the expressions for l_0 , l_0^* , l_1 and l_1^* are present in “[Appendix](#).” The above equations that are evaluated are referred as dynamical equations. These will be utilized to comprehend irregularity in energy density of the charged spherical system.

The Weyl tensor is made up of magnetic and electric components. In a system, which is spherically symmetric the first component is zero. The electric component [68] is listed below.

$$E_{\delta\beta} = C_{\delta\alpha\beta\eta} V^\alpha V^\eta, \quad (46)$$

that can be shown as

$$E_{\delta\beta} = \mathcal{E} \left(\chi_\delta \chi_\beta - \frac{1}{3} h_{\delta\beta} \right). \quad (47)$$

The independent components are

$$E_{22} = \frac{2\mathcal{E}Y^2}{3}, \quad E_{33} = -\frac{\mathcal{E}C^2}{3}, \quad E_{44} = E_{33} \sin^2 \theta, \quad (48)$$

here $\mathcal{E}$ is Weyl scalar, given by

$$\mathcal{E} = \frac{1}{2} \left[\frac{C'^2}{C^2} + \frac{Y'C'}{YC} + \frac{Y\dot{Y}\dot{C}}{C} + \frac{Y^2\ddot{C}}{C} - \frac{C''}{C} - Y\ddot{Y} - \frac{Y^2\dot{C}^2}{C^2} \right. \\ \left. - \frac{Y^2}{2C^2} \right] \frac{1}{Y^2}. \quad (49)$$

The link between the Weyl scalar, field equations, mass function, and charge can be described by taking Eqs. (30), (31), and (33) with Eqs. (35) and (49) as

$$\begin{aligned}
2\mathcal{E} + \frac{6m}{C^3} - 48\pi^2 E^2 &= (\tilde{\mu} - \Pi) + (\tilde{\mu} - \Pi - P)\lambda \\
&- \frac{(\mathcal{G} + \lambda T + \alpha \mathcal{G}^2)}{2} \\
&+ 2 \left(\frac{\chi_1}{2X^2} + \frac{\chi_1 \alpha \mathcal{G}}{X^2} + \frac{\alpha \chi_1^*}{X^2} - \frac{\chi_2}{2Y^2} - \frac{\chi_2 \alpha \mathcal{G}}{Y^2} - \frac{\alpha \chi_2^*}{Y^2} + \frac{Z_2}{2C^2} \right. \\
&+ \frac{Z_2 \alpha \mathcal{G}}{C^2} + \frac{\alpha Z_2^*}{C^2} \\
&\left. + 3\pi E^2 \right). \tag{50}
\end{aligned}$$

In the context of $f(\mathcal{G}, T)$ gravity, the set of equations defining the development of self-gravitating matter distribution with anisotropic stresses is employed and utilized to conduct a study on the behavior of such systems. These two differential equations for the Weyl curvature are called Ellis equations, which are significant in our study. They were initially discovered by Ellis [69]. Now, we reconstructed them by using the approach described in [70], in which they emphasized their attention to the relation between the pressure anisotropy, Weyl tensor, shear tensor, and inhomogeneity. They also exhibited some solutions to explain the required discussion in the arena of GR. Here, we have used the same approach to develop the equations which specify link between Weyl curvature and matter variables by adding extra factors. With the help of this relation, we studied the inhomogeneity of the system and concluded that the evolutionary behavior of the system under investigation will be homogeneous if the Weyl scalar vanishes with curvature terms. The form of the obtained equations are

$$\begin{aligned}
&\left[2\mathcal{E} - \left((\tilde{\mu} - \Pi) + (\tilde{\mu} - \Pi - P)\lambda - \frac{(\mathcal{G} + \lambda T + \alpha \mathcal{G}^2)}{2} \right. \right. \\
&\quad + 2 \left(\frac{\chi_1}{2X^2} + \frac{\chi_1 \alpha \mathcal{G}}{X^2} + \frac{\alpha \chi_1^*}{X^2} - \frac{\chi_2}{2Y^2} - \frac{\chi_2 \alpha \mathcal{G}}{Y^2} - \frac{\alpha \chi_2^*}{Y^2} + \frac{Z_2}{2C^2} + \frac{Z_2 \alpha \mathcal{G}}{C^2} + \frac{\alpha Z_2^*}{C^2} \right. \\
&\quad \left. \left. + 3\pi E^2 \right) \right] \\
&= \frac{3\dot{C}}{C} \left[(\tilde{\mu} + P_\perp)(1 + \lambda) + 2 \left(\frac{\chi_1}{2X^2} + \frac{\chi_1 \alpha \mathcal{G}}{X^2} + \frac{\alpha \chi_1^*}{X^2} + \frac{Z_2}{2C^2} + \frac{Z_2 \alpha \mathcal{G}}{C^2} + \frac{\alpha Z_2^*}{C^2} \right. \right. \\
&\quad \left. \left. + 2\pi E^2 \right) - 2\mathcal{E} \right] + 3 \left(\tilde{q}(1 + \lambda) - \frac{2\alpha Z_1}{XY} \right) \frac{XC'}{YC} \\
&\quad - 24\pi^2 E^2 \left(\frac{3\dot{C}}{C} - 4 \right), \tag{51}
\end{aligned}$$

and

$$\begin{aligned}
&\left[2\mathcal{E} - \left((\tilde{\mu} - \Pi) + (\tilde{\mu} - \Pi - P)\lambda - \frac{(\mathcal{G} + \lambda T + \alpha \mathcal{G}^2)}{2} \right. \right. \\
&\quad + 2 \left(\frac{\chi_1}{2X^2} + \frac{\chi_1 \alpha \mathcal{G}}{X^2} + \frac{\alpha \chi_1^*}{X^2} - \frac{\chi_2}{2Y^2} - \frac{\chi_2 \alpha \mathcal{G}}{Y^2} - \frac{\alpha \chi_2^*}{Y^2} + \frac{Z_2}{2C^2} + \frac{Z_2 \alpha \mathcal{G}}{C^2} + \frac{\alpha Z_2^*}{C^2} \right. \\
&\quad \left. \left. + 3\pi E^2 \right) \right] \\
&= -\frac{3C'}{C} \left[\Pi(1 + \lambda) + 2 \left(\frac{\chi_2}{2Y^2} + \frac{\chi_2 \alpha \mathcal{G}}{Y^2} + \frac{\alpha \chi_2^*}{Y^2} - \frac{Z_2}{2C^2} - \frac{Z_2 \alpha \mathcal{G}}{C^2} - \frac{\alpha Z_2^*}{C^2} \right. \right. \\
&\quad \left. \left. - 2\pi E^2 \right) + 2\mathcal{E} \right] - 3 \left(\tilde{q}(1 + \lambda) - \frac{2\alpha Z_1}{XY} \right) \frac{YC'}{XC} \\
&\quad - 24\pi^2 E^2 \left(\frac{3C'}{C} + 4 \right). \tag{52}
\end{aligned}$$

The corresponding heat flow transport equation [71, 72] is stated as,

$$\tau h^{\delta\beta} V^\eta q_{\beta;\eta} + q^\delta = -\kappa h^{\delta\beta} \left(T_{;\delta} + T a_\delta \right) - \frac{1}{2} \kappa T^2 \left(\frac{\tau V^\delta}{\kappa T^2} \right)_{;\delta} q^\beta, \tag{53}$$

where T , τ and κ indicate temperature, relaxation time, and thermal conductivity. Its independent component is written as

$$\dot{q}\tau = - \left[\frac{\kappa}{Y} (TX)' + Xq + \frac{qX}{2} \tau \Theta + \frac{1}{2} \kappa q T^2 \left(\frac{\tau}{\kappa T^2} \right) \right]. \tag{54}$$

The truncated version of Eq. (53) is obtained in the absence of last term

$$\dot{q}\tau = - \left(Xq + \frac{\kappa}{Y} (XT)' \right). \tag{55}$$

4. Regularity and irregularity in the energy density

Now, we analyze the irregular behavior of the system under $f(G, T)$ gravity and observe that the obtained results are different from GR and other modified gravity, such differences appear due to the coupling between matter and curvature. To highlight the effects of modified corrections, we study a specific type of $f(\mathcal{G}, T)$ model taken as, $f(\mathcal{G}, T) = f_1(\mathcal{G}) + f_2(T)$. In comparison with $f(\mathcal{G})$ gravity, we use the linear version $f_2 = \lambda T$ and obtained distinct conclusions based on a nontrivial coupling. This linear term corresponds to GR for proper rescaling of λ . By using this form of the model, we explore interesting results for

various forms of fluids. This section is devoted to the major goal of our efforts, in which we will use various fluid properties in the following subsections to better understand the functions of various components in the existence and departure of irregularity of the spherically symmetric charged fluids.

4.1. Non-dissipative pressureless charged fluid

To begin, we consider charged non-dissipative geodesic pressureless fluid with $P_\perp = \tilde{q} = P_r = 0$ and $X = 1$ in an increasing order. As a result, Equations (51) and (52) have the form

$$\begin{aligned} & \left[2\mathcal{E} - \left(\mu(1+\lambda) - \frac{(\mathcal{G} + \lambda T + \alpha \mathcal{G}^2)}{2} + 2 \left(\frac{\chi_1}{2} + \chi_1 \alpha \mathcal{G} + \alpha \chi_1^* \right. \right. \right. \\ & \quad \left. \left. - \frac{\chi_2}{2Y^2} - \frac{\chi_2 \alpha \mathcal{G}}{Y^2} - \frac{\alpha \chi_2^*}{Y^2} + \frac{Z_2}{2C^2} + \frac{Z_2 \alpha \mathcal{G}}{C^2} + \frac{\alpha Z_2^*}{C^2} + 3\pi E^2 \right) \right) \Big] \\ & + \frac{3\dot{C}}{C} \left[2\mathcal{E} - \mu \right. \\ & \quad \left. (1+\lambda) - 2 \left(\frac{\chi_1}{2} + \chi_1 \alpha \mathcal{G} + \alpha \chi_1^* + \frac{Z_2}{2C^2} + \frac{Z_2 \alpha \mathcal{G}}{C^2} + \frac{\alpha Z_2^*}{C^2} + 2\pi E^2 \right) \right] \\ & + \frac{6\alpha C' Z_1}{Y^2 C} + 24\pi^2 E^2 \left(\frac{3\dot{C}}{C} - 4 \right) = 0, \end{aligned} \quad (56)$$

$$\begin{aligned} & \left[2\mathcal{E} - \left(\mu(1+\lambda) - \frac{(\mathcal{G} + \lambda T + \alpha \mathcal{G}^2)}{2} + 2 \left(\frac{\chi_1}{2} + \chi_1 \alpha \mathcal{G} \right. \right. \right. \\ & \quad \left. \left. + \alpha \chi_1^* - \frac{\chi_2}{2Y^2} - \frac{\chi_2 \alpha \mathcal{G}}{Y^2} - \frac{\alpha \chi_2^*}{Y^2} + \frac{Z_2}{2C^2} + \frac{Z_2 \alpha \mathcal{G}}{C^2} + \frac{\alpha Z_2^*}{C^2} + 3\pi E^2 \right) \right) \Big]' = \\ & - \frac{3C'}{C} \left[2 \left(\frac{\chi_2}{2Y^2} + \frac{\chi_2 \alpha \mathcal{G}}{Y^2} + \frac{\alpha \chi_2^*}{Y^2} - \frac{Z_2}{2C^2} - \frac{Z_2 \alpha \mathcal{G}}{C^2} - \frac{\alpha Z_2^*}{C^2} - 2\pi E^2 \right) + 2\mathcal{E} \right] \\ & + \frac{6\alpha \dot{C} Z_1}{C} - 24\pi^2 E^2 \left(\frac{3C'}{C} + 4 \right). \end{aligned} \quad (57)$$

The Weyl curvature, $f(\mathcal{G}, T)$ terms, and charge are all responsible for energy density inhomogeneity, according to Eq. (57). Using Eqs. (14) and (44) in Eq. (56), we now have

$$2\mathcal{E} + \mathcal{E} \frac{6\dot{C}}{C} = -\sigma\mu + q_0 + [\alpha \xi_0 + \lambda(\xi_0^* - \sigma\mu)], \quad (58)$$

where

$$\begin{aligned} \xi_0 &= \left[2 \left(\frac{\chi_1}{2\alpha} + \chi_1 \mathcal{G} + \chi_1^* - \frac{\chi_2}{2Y^2} - \frac{\chi_2 \mathcal{G}}{Y^2} - \frac{\chi_2^*}{Y^2} + \frac{Z_2}{2\alpha C^2} \right. \right. \\ & \quad \left. \left. + \frac{Z_2 \mathcal{G}}{C^2} + \frac{Z_2^*}{C^2} \right) \right. \\ & \quad \left. - \frac{\mathcal{G}}{2\alpha} - \frac{\mathcal{G}^2}{2} + \frac{6\dot{C}}{C} \left(\frac{\chi_1}{2\alpha} + \chi_1 \mathcal{G} + \chi_1^* + \frac{Z_2}{2\alpha C^2} + \frac{Z_2 \mathcal{G}}{C^2} + \frac{Z_2^*}{C^2} \right) \right. \\ & \quad \left. - \frac{2Z_1 C'}{Y^2 C} \right], \\ \xi_0^* &= \left[\frac{\frac{T}{2}(1+\lambda)}{1+3\lambda} - \frac{l_0(1+\frac{1}{\lambda})}{1+3\lambda} \frac{\dot{T}}{2} \right], \quad q_0 = \frac{l_0^*(1+\lambda)}{1+3\lambda} \\ & \quad - 24\pi^2 E^2 \left(\frac{3\dot{C}}{C} \frac{\dot{E}}{2\pi E} - \frac{\dot{C}}{2\pi C} - 4 \right). \end{aligned}$$

Due to modified components, Eq. (58) demonstrates the fascinating fact that shear free condition and conformal flatness do not entail each other. The solution of Eq. (58), where $\mathcal{E}(0, r) = 0$ is the specified integration function, has the form

$$\begin{aligned} \mathcal{E} &= -\frac{\int_0^t \sigma \mu C^3 dt}{2C^3} + \frac{\int_0^t q_0 C^3 dt}{2C^3} \\ & \quad + \frac{\int_0^t [\alpha \xi_0 + \lambda(\xi_0^* - \sigma\mu)] C^3 dt}{2C^3}. \end{aligned} \quad (59)$$

In Eq. (59), the terms on the right side describe the consequences of GR [47], effects of charge, role of $f(\mathcal{G})$ terms [73], and $f(T)$ constituents, respectively. The contribution of modified terms in the density inhomogeneity can be recognized by assuming the role of parameters α and λ . Now, we would like to discuss this result in detail as

If we take $\lambda = 0$, then Eq. (59) will describe the consequences of GR and $f(\mathcal{G})$ terms ($\alpha \xi_0$) along with the impact of the charge. Thereby, the influence of $f(T)$ terms would vanish in this scenario. On the other hand, if we choose $\alpha = 0$, then the significant role of $f(T)$ corrections has been observed explicitly. In this case, the factor $\lambda(\xi_0^* - \sigma\mu) > 0$ is regarded as the additional source to induce inhomogeneity of the system by mean of $f(T)$ influence. Therefore, it is concluded that the Weyl scalar including the effects of charge on the grounds of $f(\mathcal{G}, T)$ gravity increases the inhomogeneity of the system. This result could be recovered in GR [47] when $\alpha = 0 = \lambda$ and in the absence of charge. Consequently, the system will be deformed under shearing motion only.

4.2. Non-radiating charged fluid

In this scenario, we are dealing with a locally isotropic charged fluid, which is non-radiating in nature. For this, we shall use $\Pi = \tilde{q} = 0, P_r = P = P_\perp$. Equations (51) and (52)

$$\begin{aligned}
& \left[2\mathcal{E} - \left(\mu(1+\lambda) - \lambda P - \frac{(\mathcal{G} + \lambda T + \alpha \mathcal{G}^2)}{2} \right. \right. \\
& \quad + 2 \left(\frac{\chi_1}{2X^2} + \frac{\chi_1 \alpha \mathcal{G}}{X^2} + \frac{\alpha \chi_1^*}{X^2} - \frac{\chi_2}{2Y^2} \right. \\
& \quad \left. \left. - \frac{\chi_2 \alpha \mathcal{G}}{Y^2} - \frac{\alpha \chi_2^*}{Y^2} + \frac{Z_2}{2C^2} + \frac{Z_2 \alpha \mathcal{G}}{C^2} + \frac{\alpha Z_2^*}{C^2} + 3\pi E^2 \right) \right] \\
& \quad + \frac{3\dot{C}}{C} \left[2\mathcal{E} - (\mu + P) \right. \\
& \quad \left. (1+\lambda) - 2 \left(\frac{\chi_1}{2X^2} + \frac{\chi_1 \alpha \mathcal{G}}{X^2} + \frac{\alpha \chi_1^*}{X^2} + \frac{Z_2}{2C^2} + \frac{Z_2 \alpha \mathcal{G}}{C^2} + \frac{\alpha Z_2^*}{C^2} + 2\pi E^2 \right) \right] \\
& \quad (60)
\end{aligned}$$

$$\begin{aligned}
& \left[2\mathcal{E} - \left(\mu(1+\lambda) - \lambda P - \frac{(\mathcal{G} + \lambda T + \alpha \mathcal{G}^2)}{2} \right. \right. \\
& \quad + 2 \left(\frac{\chi_1}{2X^2} + \frac{\chi_1 \alpha \mathcal{G}}{X^2} + \frac{\alpha \chi_1^*}{X^2} - \frac{\chi_2}{2Y^2} \right. \\
& \quad \left. \left. - \frac{\chi_2 \alpha \mathcal{G}}{Y^2} - \frac{\alpha \chi_2^*}{Y^2} + \frac{Z_2}{2C^2} + \frac{Z_2 \alpha \mathcal{G}}{C^2} + \frac{\alpha Z_2^*}{C^2} + 3\pi E^2 \right) \right] \\
& \quad = -\frac{3C'}{C} \left[2 \left(\frac{\chi_2}{2Y^2} + \right. \right. \\
& \quad \left. \left. \frac{\chi_2 \alpha \mathcal{G}}{Y^2} + \frac{\alpha \chi_2^*}{Y^2} - \frac{Z_2}{2C^2} - \frac{Z_2 \alpha \mathcal{G}}{C^2} - \frac{\alpha Z_2^*}{C^2} - 2\pi E^2 \right) + 2\mathcal{E} \right] \\
& \quad + \frac{6\alpha \dot{C} Z_1}{X^2 C} - 24\pi^2 E^2 \\
& \quad \left(\frac{3C'}{C} + 4 \right). \\
& \quad (61)
\end{aligned}$$

As in the preceding situation, these equations indicate that energy-density inhomogeneity is not caused solely by the Weyl tensor. We get the following equation by combining Eqs. (14) and (44) in Eq. (60)

$$2\mathcal{E} + \mathcal{E} \frac{6\dot{C}}{C} = -\sigma X(\mu + P) + q_0 + [\alpha \xi_1 + \lambda(\xi_1^* - \sigma X(\mu + P))], \quad (62)$$

where

$$\begin{aligned}
\xi_1 &= \left[2 \left(\frac{\chi_1}{2X^2 \alpha} + \frac{\chi_1 \mathcal{G}}{X^2} + \frac{\chi_1^*}{X^2} - \frac{\chi_2}{2Y^2} - \frac{\chi_2 \mathcal{G}}{Y^2} - \frac{\chi_2^*}{Y^2} + \frac{Z_2}{2\alpha C^2} \right. \right. \\
& \quad \left. \left. + \frac{Z_2 \mathcal{G}}{C^2} + \frac{Z_2^*}{C^2} \right) \right. \\
& \quad \left. - \frac{\mathcal{G}}{2\alpha} - \frac{\mathcal{G}^2}{2} + \frac{6\dot{C}}{C} \left(\frac{\chi_1}{2X^2 \alpha} + \frac{\chi_1 \mathcal{G}}{X^2} + \frac{\chi_1^*}{X^2} + \frac{Z_2}{2\alpha C^2} + \frac{Z_2 \mathcal{G}}{C^2} \right. \right. \\
& \quad \left. \left. + \frac{Z_2^*}{C^2} \right) - \frac{2Z_1 C'}{Y^2 C} \right], \\
\xi_1^* &= \left[\frac{\dot{T}}{2} (1+\lambda) - \frac{l_0(1+\frac{1}{\lambda})}{1+3\lambda} \frac{\dot{T}}{2} + 2\dot{P} \left(1+\lambda - \frac{1}{2} \right) - \frac{\dot{T}}{2} \right].
\end{aligned}$$

whose solution has following form

$$\begin{aligned}
\mathcal{E} &= -\frac{\int_0^t \sigma X(\mu + P) C^3}{2C^3} dt + \frac{\int_0^t q_0 C^3}{2C^3} dt \\
& \quad + \frac{\int_0^t [\alpha \xi_1 + \lambda(\xi_1^* - \sigma X(\mu + P))] C^3}{2C^3} dt. \\
& \quad (63)
\end{aligned}$$

The integration function is specified as $\mathcal{E}(0, r) = 0$. We take the shear-free situation without taking into account conformal flatness since neither the shear-free condition implies to conformal flatness nor the vanishing of the Weyl curvature corresponds shear-free. As a result we get

$$\mathcal{E} = \frac{f(r)}{C^3} + \frac{\int_0^t q_0 C^3}{2C^3} dt + \frac{\int_0^t [\alpha \xi_1 + \lambda \xi_1^*] C^3}{2C^3} dt. \quad (64)$$

where $F(r)$ is chosen at random to fulfill $F(0) = 0$. All the above-mentioned consequences would also be observed in this scenario of isotropic non-radiating fluid. Also, the conformal flatness is observed under shear-free and charge-free configurations along with the zero contribution of the dark source influences as presented in Eq. (64). However, just $\sigma = 0$ does not encourage the vanishing of the Weyl scalar as the correction terms and charge have a remarkable influence to induce the irregularity in the system. This demonstrates the significance of $f(\mathcal{G}, T)$ gravity. Thus, the Weyl tensor does not vanish at any time t for a given initial homogeneous configuration, i.e., $\mathcal{E}(0, r) = 0$, according to Eq. (64).

4.3. Non-radiating locally anisotropic charged fluid

Now we look at the role of pressure anisotropy in a non-radiating locally anisotropic charged fluid, for which ($\Pi \neq 0, \bar{q} = 0$). Equations (51) and (52) can then be written as

$$\begin{aligned}
& \left[2\mathcal{E} - \left((\mu - \Pi) + \lambda(\mu - \Pi - P) - \frac{(\mathcal{G} + \lambda T + \alpha \mathcal{G}^2)}{2} \right. \right. \\
& \quad + 2 \left(\frac{\chi_1}{2X^2} + \frac{\chi_1 \alpha \mathcal{G}}{X^2} + \right. \\
& \quad \left. \frac{\alpha \chi_1^*}{X^2} - \frac{\chi_2}{2Y^2} - \frac{\chi_2 \alpha \mathcal{G}}{Y^2} - \frac{\alpha \chi_2^*}{Y^2} + \frac{Z_2}{2C^2} + \frac{Z_2 \alpha \mathcal{G}}{C^2} \right. \\
& \quad \left. \left. + \frac{\alpha Z_2^*}{C^2} + 3\pi E^2 \right) \right] + \frac{3\dot{C}}{C} \left[2\mathcal{E} \right. \\
& \quad \left. - (\mu + P_\perp)(1+\lambda) - 2 \left(\frac{\chi_1}{2X^2} + \frac{\chi_1 \alpha \mathcal{G}}{X^2} + \frac{\alpha \chi_1^*}{X^2} + \frac{Z_2}{2C^2} \right. \right. \\
& \quad \left. \left. + \frac{Z_2 \alpha \mathcal{G}}{C^2} + \frac{\alpha Z_2^*}{C^2} \right) \right] \\
& \quad (65)
\end{aligned}$$

$$\begin{aligned}
& \left[2\mathcal{E} - \left((\mu - \Pi) + \lambda(\mu - \Pi - P) \right. \right. \\
& \quad \left. \left. - \frac{(\mathcal{G} + \lambda T + \alpha \mathcal{G}^2)}{2} + 2 \left(\frac{\chi_1}{2X^2} + \frac{\chi_1 \alpha \mathcal{G}}{X^2} + \right. \right. \right. \\
& \quad \left. \left. \frac{\alpha \chi_1^*}{X^2} - \frac{\chi_2}{2Y^2} - \frac{\chi_2 \alpha \mathcal{G}}{Y^2} - \frac{\alpha \chi_2^*}{Y^2} + \frac{Z_2}{2C^2} + \frac{Z_2 \alpha \mathcal{G}}{C^2} \right. \right. \\
& \quad \left. \left. \left. + \frac{\alpha Z_2^*}{C^2} + 3\pi E^2 \right) \right) \right]' = -\frac{3C'}{C} \\
& \left[\Pi(1 + \lambda) + 2 \left(+\frac{\chi_2}{2Y^2} + \frac{\chi_2 \alpha \mathcal{G}}{Y^2} + \frac{\alpha \chi_2^*}{Y^2} - \frac{Z_2}{2C^2} - \frac{Z_2 \alpha \mathcal{G}}{C^2} \right. \right. \\
& \quad \left. \left. - \frac{\alpha Z_2^*}{C^2} - 2\pi E^2 \right) + \right. \\
& \left. 2\mathcal{E} \right] + \frac{6\alpha \dot{C} Z_1}{X^2 C} - 24\pi^2 E^2 \left(\frac{3C'}{C} + 4 \right). \quad (66)
\end{aligned}$$

Equations (14) and (44) in Eq. (65) are used to obtain the evolution equation, which is the equation for irregularity-causing elements. Thus we obtain

$$\begin{aligned}
& [\Pi + 2\mathcal{E}] + \frac{[\Pi + 6\mathcal{E}]\dot{C}}{C} = -(\mu + P_r)X\sigma + q_0 \\
& + \left[\alpha \xi_1 + \lambda \left(\xi_1^* - (\mu + P_r)X\sigma - \right. \right. \\
& \left. \left. \left(\dot{\Pi} + \frac{\Pi \dot{C}}{C} \right) \right) \right]. \quad (67)
\end{aligned}$$

The emerging quantity $[\Pi + 2\mathcal{E}]$ in Eq. (67) is referred as structure scalar [74]. It was defined by following tensor

$$X_{\delta\beta} = {}^* R_{\delta\nu\beta\eta}^* V^\nu V^\eta = \frac{1}{2} \eta_{\delta\nu}^{\epsilon\zeta} R_{\epsilon\zeta\beta\eta}^* V^\nu V^\eta, \quad (68)$$

where $R_{\delta\beta\nu\alpha}^* = \frac{1}{2} \eta_{\epsilon\zeta\nu\alpha} R_{\delta\beta}^{\epsilon\zeta}$ and $\eta_{\epsilon\zeta\nu\alpha}$ indicates the Levi-Civita tensor. Equation (68) turns out to be

$$X_{\delta\beta} = \frac{1}{3} X_T h_{\delta\beta} + X_{TF} \left(\chi_{\delta\beta} \chi_{\beta} - \frac{1}{3} h_{\delta\beta} \right), \quad (69)$$

The trace and trace-free segment of Eq. (68) are contained in Eq. (69). The required form of structure scalar is given by applying modified equations and Eqs. (46), (47) and Eq. (49), we obtain

$$X_{TF} = -\Pi - \mathcal{E}. \quad (70)$$

In terms of X_{TF} , the evolution equation (67) is represented as

$$\begin{aligned}
& X_{TF} + 6X_{TF} \frac{\dot{C}}{C} = -\frac{5\Pi\dot{C}}{C} - \dot{\Pi} + (\mu + P_r)X\sigma - q_0 \\
& - \left[\alpha \xi_1 + \lambda \left[\xi_1^* - (\mu + P_r)X\sigma - \right. \right. \\
& \left. \left. \left(\dot{\Pi} + \frac{\Pi \dot{C}}{C} \right) \right] \right], \quad (71)
\end{aligned}$$

The associated solution is written as

$$\begin{aligned}
& X_{TF} = -\frac{\int_0^t [5\Pi\dot{C} + \dot{\Pi}C - (\mu + P_r)X\sigma C] C^2 dt}{2C^3} \\
& - \frac{\int_0^t q_0 C^3 dt}{2C^3} - \\
& \frac{\int_0^t [\alpha \xi_1 + \lambda [\xi_1^* - (\mu + P_r)X\sigma - (\dot{\Pi} + \frac{\Pi \dot{C}}{C})]] C^3 dt}{2C^3}. \quad (72)
\end{aligned}$$

In this case, the inhomogeneity of the system is determined through the scalar X_{TF} as presented in Eq. (72). It is inferred that the first term in the aforesaid equation corresponds to the consequences of GR, demonstrating the influence of anisotropy on the inhomogeneity under shearing motion. If we choose the parameter $\lambda = 0$, the corresponding outcome would be reduced in $f(\mathcal{G})$ gravity. This reveals the impact of $\alpha \xi_1$ term, however, the system's inhomogeneity would be decreased. Similarly, we may analyze the impact of $f(T)$ terms explicitly by selecting $\alpha = 0$. Hence, the term $\lambda [\xi_1^* - (\mu + P_r)X\sigma - (\dot{\Pi} + \frac{\Pi \dot{C}}{C})] > 0$ reduce the system irregularity under $f(T)$ scenario. Thereby, the system's initial regular behavior has been affected by anisotropy and $f(\mathcal{G})$ and $f(T)$ terms explicitly.

4.4. Dissipative charged pressureless fluid

Finally, we will investigate the impact of dissipation in the growth of irregularity by using geodesic radiating charged dust. As a result, in this situation, we consider $X = 1$ and $P_\perp = 0 = P_r$. Thus Eqs. (51) and (52) have the form

$$\begin{aligned}
& \left[2\mathcal{E} - \left(\tilde{\mu}(1 + \lambda) - \frac{(\mathcal{G} + \lambda T + \alpha \mathcal{G}^2)}{2} \right. \right. \\
& + 2 \left(\frac{\chi_1}{2} + \chi_1 \alpha \mathcal{G} + \alpha \chi_1^* - \frac{\chi_2}{2Y^2} \right. \\
& \left. \left. - \frac{\chi_2 \alpha \mathcal{G}}{Y^2} - \frac{\alpha \chi_2^*}{Y^2} + \frac{Z_2}{2C^2} + \frac{Z_2 \alpha \mathcal{G}}{C^2} + \frac{\alpha Z_2^*}{C^2} + 3\pi E^2 \right) \right) \right] \\
& + \frac{3\dot{C}}{C} \left[2\mathcal{E} - \tilde{\mu} \right. \\
& \left. (1 + \lambda) - 2 \left(\frac{\chi_1}{2} + \chi_1 \alpha \mathcal{G} + \alpha \chi_1^* + \frac{Z_2}{2C^2} \right. \right. \\
& \left. \left. + \frac{Z_2 \alpha \mathcal{G}}{C^2} + \frac{\alpha Z_2^*}{C^2} + 2\pi E^2 \right) \right] \quad (73)
\end{aligned}$$

$$\begin{aligned}
& -3\left(\tilde{q}(1+\lambda) - \frac{2\alpha Z_1}{Y}\right) \frac{C'}{YC} + 24\pi^2 E^2 \left(\frac{3\dot{C}}{C} - 4\right) = 0, \\
& \left[2\mathcal{E} - \left(\tilde{\mu}(1+\lambda) - \frac{(\mathcal{G} + \lambda T + \alpha \mathcal{G}^2)}{2}\right.\right. \\
& \quad \left.\left.+ 2\left(\frac{\chi_1}{2} + \chi_1 \alpha \mathcal{G} + \alpha \chi_1^* - \frac{\chi_2}{2Y^2} - \frac{\chi_2 \alpha \mathcal{G}}{Y^2} - \frac{\alpha \chi_2^*}{Y^2} + \frac{Z_2}{2C^2} + \frac{Z_2 \alpha \mathcal{G}}{C^2} + \frac{\alpha Z_2^*}{C^2} + 3\pi E^2\right)\right)\right]' \\
& = -\frac{3C'}{C} \left[2\left(\frac{\chi_2}{2Y^2} + \frac{\chi_2 \alpha \mathcal{G}}{Y^2} + \frac{\alpha \chi_2^*}{Y^2} - \frac{Z_2}{2C^2} - \frac{Z_2 \alpha \mathcal{G}}{C^2} - \frac{\alpha Z_2^*}{C^2} - 2\pi E^2\right) + 2\mathcal{E}\right] \\
& - 3\left(\tilde{q}(1+\lambda) - \frac{2\alpha Z_1}{Y}\right) \frac{Y\dot{C}}{C} - 24\pi^2 E^2 \left(\frac{3C'}{C} + 4\right).
\end{aligned} \tag{74}$$

Equation (74) may be written as

$$\begin{aligned}
\Psi \equiv 2\mathcal{E} + \frac{3 \int_0^r \tilde{q} Y \dot{C} C^2}{C^3} dr - \frac{\int_0^r q_0^* C^3}{C^3} dr \\
- \frac{\int_0^r [\alpha \xi_2 - \lambda(\frac{T'}{2} + \frac{3\tilde{q} Y \dot{C}}{C})] C^3}{C^3} dr,
\end{aligned} \tag{75}$$

where

$$\begin{aligned}
\xi_2 = & \left[2\left(\frac{\chi_1}{2\alpha} + \chi_1 \mathcal{G} + \chi_1^* - \frac{\chi_2}{2Y^2} - \frac{\chi_2 \mathcal{G}}{Y^2} - \frac{\chi_2^*}{Y^2} + \frac{Z_2}{2\alpha C^2} + \frac{Z_2 \mathcal{G}}{C^2} + \frac{Z_2^*}{C^2}\right) \right. \\
& - \frac{\mathcal{G}'}{2\alpha} - \frac{\mathcal{G}^2}{2} + \frac{6C'}{C} \left(\frac{\chi_1}{2\alpha} + \chi_1 \mathcal{G} + \chi_1^* + \frac{Z_2}{2\alpha C^2} + \frac{Z_2 \mathcal{G}}{C^2} + \frac{Z_2^*}{C^2}\right) - \frac{2Z_1 \dot{C}}{Y^2 C} \Big], \\
q_0^* = & 24\pi^2 E^2 \left[\frac{E'}{2\pi E} + \frac{C'}{2\pi C} - \frac{3C'}{C} - 4\right].
\end{aligned}$$

Here, $\tilde{\mu}' = 0$ if and only if $\Psi = 0$, according to equation (75). Equation (73) yields an evolution equation for Ψ by combining Eqs. (14) and (44), we get

$$\begin{aligned}
\dot{\Psi} + \left[\sigma \tilde{\mu} + \frac{1}{Y} \tilde{q}' - \frac{C'}{YC} \tilde{q}\right] &= \frac{\dot{\Omega}}{C^3} - \frac{3\Psi \dot{C}}{C} + q_0 \\
&+ \left[\alpha \xi_0 + \lambda \left(\xi_0^* - \left(\tilde{\mu} \sigma + \frac{\tilde{q}'}{Y} - \frac{\tilde{q} C'}{YC}\right)\right)\right],
\end{aligned} \tag{76}$$

with $\Omega = 3 \int_0^r \tilde{q} Y \dot{C} C^2 dr - \int_0^r q_0^* C^3 dr - \int_0^r [\alpha \xi_2 - \lambda(\frac{T'}{2} + \frac{3\tilde{q} Y \dot{C}}{C})] C^3 dr$. The general solution of the above equation has the form

$$\begin{aligned}
\Psi = & \frac{\int_0^t \dot{\Omega} - C^3(\sigma \tilde{\mu} - \frac{\tilde{q} C'}{YC} + \frac{\tilde{q}'}{Y})}{C^3} dt + \frac{\int_0^t q_0 C^3}{C^3} dt \\
& + \frac{\int_0^t [\alpha \xi_0 + \lambda(\xi_0^* - (\tilde{\mu} \sigma + \frac{\tilde{q}'}{Y} - \frac{\tilde{q} C'}{YC}))] C^3}{C^3} dt.
\end{aligned} \tag{77}$$

The inhomogeneity in this case is determined by the factor Ψ as shown in Eq. (77). This factor includes the influence of dissipation, charge as well as $f(\mathcal{G}, T)$ constituents under the shearing motion of the system. The combined effects of $f(\mathcal{G})$ and $f(T)$ corrections are the additional sources that increase the irregularity. The same behavior would be observed for $f(\mathcal{G})$ and $f(T)$ gravity provided $\lambda = 0$ and $\alpha = 0$. Subsequently, this outcome reveals the significance of the parameters α and λ under $f(\mathcal{G}, T)$ scenario.

The above expression can be further modified by expressing Eq. (45) as

$$\tilde{q} = \frac{\phi(r)}{Y^2 C^2} - \frac{\int_0^t \xi_2^*}{C^2 Y^2} dt,$$

rephrased as

$$\tilde{q} = \frac{\Phi(r)}{Y^2 C^2}, \tag{78}$$

where

$$\xi_2^* = \frac{(l_1 + l_1^* + \frac{\lambda T' X}{2Y})}{1 + 3\lambda}.$$

The irregular energy density of charged radiating pressureless fluid is dependent on radiating terms, correction terms, shear, and electromagnetic field, as shown by Eq. (77). Taking $\sigma = 0$, for which Eq. (14) implies that $C = rY$, to study the impact of these quantities. Then using Eqs. (78) and (77) becomes

$$\begin{aligned}
\Psi = & \frac{\int_0^t [\int_0^r [\Phi(r) \dot{\Theta} r - q_0^* C^3 - \alpha \xi_2 C^3 + \lambda(\frac{T' C^3}{2} + \Phi(r) \dot{\Theta} r)] - \frac{\Phi'(r) r^3}{C^2} + \frac{\Phi(r) r^2}{C^3} (\frac{5C' r}{2} - C)]}{C^3} dt \\
& + \frac{\int_0^t q_0 C^3}{C^3} dt + \frac{\int_0^t [\alpha \xi_0 C^3 + \lambda[\xi_0^* C^3 - \frac{\Phi'(r) r^3}{C^2} + \frac{\Phi(r) r^2}{C^3} (\frac{5C' r}{2} - C)]]}{C^3} dt,
\end{aligned} \tag{79}$$

Now we study how τ has influenced the evolution of Ψ . In the diffusion approximation ($\epsilon = 0$), we have $\tilde{q} = q$ and $\tilde{\mu} = \mu$. As a result, Eq. (45) can be written as

$$\dot{q} + 4 \frac{\Theta q}{3} = -\zeta_2^*. \quad (80)$$

With Eq. (55), we obtain

$$q = \frac{(\zeta_2^* \tau - \kappa T' r)}{\left(-4 \frac{\tau}{3} \Theta + 1\right) C}. \quad (81)$$

Now by considering Eq. (78), we get

$$\Phi(r) = \frac{\tau \zeta_2^* C^3}{r^2 \left(-4 \frac{\tau}{3} \Theta + 1\right)} - \frac{\kappa C^3 T'}{r \left(-4 \frac{\tau}{3} \Theta + 1\right)}. \quad (82)$$

If we insert Eq. (82) into Eq. (79), we can see the effect of τ on Ψ in Maxwell- $f(\mathcal{G}, T)$ theory. The collapse and development of the self-gravitating fluid are greatly influenced by the charge. Thus, we can consider it a vital ingredient in the structure formation whose emergence influence the dynamics of the considered system. From the four proposed cases, it can be seen easily that charge plays a crucial role in density inhomogeneity. In the first case, the positive values of charge are increasing the inhomogeneity of the system. The next case depicts the same behavior of the charge. For the third case, Eq. (75) reveals that the density of the system is decreasing under the positive effects of charge. Lastly, we can observe the same nature of charge as in the first two cases. All of the above results reduce to GR in the limit of $f(\mathcal{G}, T) = 0$ [47].

5. Conclusions

GR is thought to be the best choice for describing the phenomena of gravity. In the universe, it may explain a wide range of gravitational events, from microscopic to large structures. Further, it successfully satisfies the usual solar system tests. Despite multiple important advancements in astronomical research, GR remains insufficient to reflect the evolving process of our cosmos. Thus, a number of open problems motivate researchers to extend it. It is possible to consider $f(R)$ and $f(\mathcal{G})$ gravity, which exclusively discusses curvature and matter, as intriguing gravitational models for the systematic explanation of DE and relativistic fluid. These theories can be rendered ghost-free, making them suitable for understanding a variety of

phenomena, including inflation, and the accelerating dynamic phase of the cosmos. It might be beneficial to take $f(\mathcal{G}, T)$ into consideration to comprehend the mutual effects of matter curvature. The acceleration of the cosmos under $f(\mathcal{G}, T)$ gravity is indicated by the non-geodesic motion. Additionally, it addresses a number of astrophysical difficulties like inflation and late-time expansion in a more precise way. In this paper, we investigated the collapse process in the context of Maxwell- $f(\mathcal{G}, T)$ theory. By systematic formulations, we found that the formation of inhomogeneity is strongly influenced by $f(\mathcal{G}, T)$, which can be viewed as a replacement for DE. Thus, in our system of study, the contribution of $f(\mathcal{G}, T)$ theory is significant as through this we can observe the role of DM and DE on the evolution of a spherical star. It explains that mutual effects increase the inhomogeneity in the system, which leads the system to collapse. Thus, we may say that the construction of the considered system under this theory is more impacting than other hypotheses.

In a self-gravitating fluid, the density distribution can be utilized to characterize the collapsing process. Moreover, the collapse process becomes more intriguing under the impact of the charge. Hence, it is needed to point out the factors that cause irregularity. The impact of Maxwell- $f(\mathcal{G}, T)$ gravity on the relativistic fluid which describes the dynamics of the considered spherical system is examined in this study. We used the nonzero divergence of the stress-energy tensor to calculate the corresponding dynamical expressions. The Ellis equations are developed to specify the link between charge, Weyl curvature, and matter variables by adding extra factors that correspond to dark source terms. This type of study is indeed important to understand the instabilities in relativistic stars which is the cause of the gravitational collapse.

We have considered different forms of fluid such as isotropic, anisotropic, and pressureless fluids, to draw conclusions under various circumstances. These cases are taken to understand the emergence of inhomogeneity for different configurations of fluid under Maxwell- $f(\mathcal{G}, T)$ theory by considering the model $f(\mathcal{G}, T) = \mathcal{G} + \lambda T + \alpha \mathcal{G}^2$. In this model, the term T referred to the trace of the stress-energy tensor, and its value is found to be $T = -\mu + P_\perp - P_r$. This term behaves differently under various fluid cases for the case of dust and isotropy it turns out to be $T = -\mu$, for anisotropic fluid, it remains the same. It shows that T does not remain the same when the pressure of the fluid or gas changes, its expression changes accordingly. In our calculations, we have used the symbol

T but it does not mean that it is the same for all the cases, it changes its form. We use it to make it easier for us to understand the results and compare them with [47] in a more effective way. After examining specific fluid cases, we obtained our intended factors for inhomogeneity. These cases revealed that modified terms and matter variables influence inhomogeneity. The following points summarize our results

1. It has been observed in the case of pressureless non-dissipative fluid and non-radiating locally isotropic fluid that irregularity is controlled by conformal tensor, $f(\mathcal{G}, T)$ terms, and charge. Charge and modified terms from the $f(\mathcal{G}, T)$ theory affect conformal flatness and the vanishing of the Weyl curvature.
2. In the case of anisotropic charged fluid, we have discovered the quantity (known as structure scalar) plays its role in inhomogeneity along with other factors. After solving the corresponding evolution equation, it is noticed that pressure anisotropy, charge, and shear also affect the regularity of the spherical relativistic fluids.
3. Finally, we looked at how an electromagnetic field affects dissipative pressureless charged fluid. For the quantities responsible for inhomogeneity, we investigated the role of charge and the evolution equation. The relaxation effects are also revealed in the evolution of Ψ (which indicates the presence of energy-density irregularity) in this case.

We can see the fact that few variables explored in this paper have an impact on the existence of energy-density irregularity, but a single specific form of these variables is still unknown. In fact, in order to obtain the stability of homogeneity, we can consider scenarios in which any of the variables listed above, as well as their combinations, vanish. We focused our attention in this paper on inhomogeneity in the arena of $f(\mathcal{G}, T)$ gravity. Inhomogeneity is the key quantity in structure formation, where structure development is an effort to simulate how these structures are formed by gravitational instability of small early variations in the density. Pressure anisotropy and inhomogeneity influence the collapse process, demonstrating that these parameters have a significant impact on the formation of the universe.

Gravitational collapse is the most exciting process, in which new stellar bodies such as clusters, stellar black holes, galaxies, and quasars form as a result of pressure deficiency in the burning star. In a self-gravitating fluid, the density distribution can be utilized to characterize the collapsing process. Pressure anisotropy and inhomogeneity contribute to this phenomenon. The process of gravitational collapse is thought to be highly dissipative [75]. As a result, inhomogeneity has an effect on the universe dynamics, which becomes more obvious as structure formation develops. In this work, we studied the phenomenon of collapse in the arena of $f(\mathcal{G}, T)$ theory and electromagnetic field. This hypothesis is based on the non-conservative energy-momentum tensor and is an essential aspect of the cosmos' dynamics. In our study using systematic formulations, we discovered that $f(\mathcal{G}, T)$, which can be considered as a substitute for DE, has a considerable impact on the emergence of inhomogeneity. The collapse and development of the self-gravitating fluid are greatly influenced by the electromagnetic field. Since inhomogeneity and anisotropy are extremely appealing in the collapse process, therefore, we may conclude that inhomogeneity, charge, curvature terms, and anisotropy all have a significant impact on the formation of the universe's structure. In the limit of $f(\mathcal{G}, T) = 0$ our results reduce to GR [47].

One may assert that this theory is not the standard gravity, as a higher-order gravity. But this assertion is rather naive because of the fact that the same argument could be made for the explanation of stellar structures in Newtonian gravity (NG) and the (further) general relativistic one. If someone halts on the basic concept of NG, then the data describes that many astronomical systems cannot be aptly explained by NG. Further, GR, for which the NG is just a weak field approximation, eloquently analyze various interesting phenomenon, like medium mass compact stars, etc. Therefore, an extension of NG, i.e., GR, is what actually diligently explains medium-mass neutron stars. Thus, a GR extension, namely $f(\mathcal{G}, T)$ theory, may be a more applicable hypothesis accurately describing an inhomogeneous state of compact objects and accurately describing their evolution for a wide range of parametric regions. In the case of $f(\mathcal{G}, T)$ description of compact objects, gravity still describes the hydrostatic equilibrium of spherical matter content, in contrast to the strange star hypothesis.

Appendix

The estimated values of χ_1 and χ_2^* in Eq. (30) are written as

$$\begin{aligned}\chi_1 = & \frac{-8}{XYC} \left(-\frac{XC'^2}{YC} - \frac{2XC'C'\dot{Y}}{CY^2} - \frac{2\dot{C}'\dot{C}X'}{YC} - \frac{2C'\dot{C}X'\dot{Y}}{CY^2} \right. \\ & - \frac{\dot{C}^2X'^2}{CXY} - \frac{\dot{Y}^2XC'^2}{CY^3} - \frac{C''C'X'X^2}{Y^3C} - \frac{C''\dot{C}X}{YC} + \frac{C^2X\ddot{Y}}{C2Y^2} \\ & + \frac{\frac{3}{2}C^2Y'X'X^2}{CY^3} + \frac{Y'XC'\ddot{C}}{Y^2C} + \frac{C'Y'\dot{C}\ddot{X}}{CY^2} + \frac{\ddot{C}\dot{C}\ddot{Y}}{CX} \\ & + \frac{\frac{3}{2}\dot{C}^2\ddot{Y}\ddot{X}}{CX^2} + \frac{C'X'\dot{C}\ddot{X}}{CYX} + \frac{C'X'\dot{C}\ddot{Y}}{CY^2} + \frac{C''X\ddot{C}}{CY} + \frac{\dot{Y}^2\ddot{X}\ddot{C}}{Y2X^2} \\ & - \frac{\ddot{Y}X}{C2} - \frac{X''X^2}{C2Y} + \frac{\dot{Y}\ddot{X}}{2C} \\ & - \frac{Y'X'X^2}{C2Y^2} - \frac{C'^2X''X^2}{C2Y^3} - \frac{C'^2\dot{Y}\ddot{X}}{C2Y^2} - \frac{\dot{C}^2\ddot{Y}}{C2X} \\ & \left. + \frac{\dot{C}^2X''}{C2Y} - \frac{\dot{C}^2Y'X'}{C2Y^2} - \frac{X''XCX'Y'}{Y^4} - \frac{\dot{Y}^2CX^2}{X^32Y} \right), \\ \chi_1^* = & \frac{-3X^2}{Y^2} \left(-\frac{4G''}{3C^2} + \frac{4\dot{Y}Y\dot{G}}{3X^2C^2} + \frac{4Y'G'}{3C^2Y} - \frac{8C''\dot{C}\dot{G}}{3X^2C^2} + \frac{8C''C'G'}{3C^2Y^2} \right. \\ & + \frac{4C'^2G''}{3Y^2C^2} - \frac{4C'^2X^2Y'G'}{Y^3X^2C^2} - \frac{8C'Y'\dot{C}\dot{G}}{3YX^2C^2} - \frac{4\dot{C}^2G''}{3X^2C^2} + \frac{4\dot{C}^2Y'G'}{3C^2X^2Y} \\ & - \frac{8\dot{C}^2C'G'}{3X^2C^3} + \frac{4\dot{C}^2Y\dot{Y}\dot{G}}{C^2X^4} \\ & \left. - \frac{4\dot{C}^2G'^2}{3X^2C^2} - \frac{8\dot{C}\dot{Y}C'G'}{3C^2X^2Y} \right).\end{aligned}$$

χ_2 and χ_2^* in Eq. (31) has the following form

$$\begin{aligned}\chi_2 = & \frac{-4}{C^2} \left(\frac{2\dot{C}^2}{X^2} - \frac{4\dot{C}'C'\dot{Y}}{X^2Y} - \frac{8\dot{C}'\dot{C}X'}{X^3} + \frac{2C'^2\dot{Y}^2}{X^2Y^2} + \frac{2C'\dot{C}X'\dot{Y}}{X^3Y} \right. \\ & + \frac{2\dot{C}^2X'^2}{X^4} + \frac{\dot{C}^2Y'X'}{X^3Y} + \frac{\ddot{Y}Y}{X^2} - \frac{X''}{X} + \frac{\dot{Y}\ddot{X}Y}{X^3} + \frac{Y'X'}{XYC^2} \\ & - \frac{C'^2\ddot{Y}}{X^2Y} + \frac{C'^2X''}{XY^2} - \frac{3C'^2X'Y'}{XY^3} \\ & + \frac{C'^2\dot{Y}\ddot{X}}{YX^3} + \frac{2C''C'X'}{Y^2X} - \frac{2\ddot{C}C''}{X^2} + \frac{2\ddot{C}C'Y'}{X^2Y} + \frac{2\ddot{C}\dot{C}\ddot{Y}Y}{X^4} \\ & + \frac{\dot{C}^2\ddot{Y}Y}{X^4} - \frac{3\dot{C}^2\dot{Y}\ddot{X}Y}{X^5} \\ & \left. - \frac{\dot{C}^2X''}{X^3} + \frac{2C''\dot{C}\ddot{X}}{X^3} + \frac{\dot{Y}^2C^2\ddot{X}^2}{X^6} \right), \\ \chi_2^* = & \frac{-8Y^2}{C^2} \left(-\frac{\ddot{G}}{2X^2} + \frac{\dot{X}\ddot{G}}{2X^3} + \frac{X'G'}{2Y^2X} + \frac{C'^2\ddot{G}}{2Y^2X^2} - \frac{C'^2\dot{X}\ddot{G}}{2Y^2X^3} \right. \\ & + \frac{\frac{3}{2}C^2X'G'}{Y^4X} + \frac{C'X'\dot{C}\ddot{G}}{Y^2X^3} \\ & + \frac{\ddot{C}C'G}{Y^2X^2} - \frac{\ddot{C}\dot{C}\ddot{G}}{X^4} + \frac{\frac{3}{2}\dot{C}^2\ddot{X}\ddot{G}}{X^5} - \frac{\dot{C}^2\ddot{G}}{2X^4} + \frac{\dot{C}^2X'G'}{2Y^2X^3} - \frac{\dot{C}X'C'G'}{Y^2X^3} \left. \right).\end{aligned}$$

The term Z_1 in Eq. (32) is given by

$$\begin{aligned}Z_1 = & \frac{-8}{C^2Y^2} \left(-\frac{\dot{C}'Y^2\dot{C}\ddot{G}}{X^2} + \dot{C}'C'G' + \frac{C'Y'\dot{C}\dot{Y}\ddot{G}}{X^2} - \frac{C'^2\dot{Y}G'}{Y} \right. \\ & + \frac{\dot{C}^2X'Y^2\ddot{G}}{X^3} - \frac{C'^2\dot{Y}G'}{2Y} \\ & - \frac{\dot{G}Y^2G'}{2} + \frac{C'^2\dot{G}G'}{2} - \frac{\dot{C}^2Y^2\dot{G}G'}{2X^2} - \frac{Y^2\dot{G}'}{2} + \frac{C'^2\dot{G}'}{2} \\ & - \frac{\dot{C}^2Y^2\dot{G}'}{2X^2} + \frac{X'Y^2\dot{G}}{2X} - \frac{C'^2X'\dot{G}}{2X} \\ & \left. - \frac{\dot{C}X'C'G'}{X} + \frac{Y^2\dot{C}^2X'\ddot{G}}{2X^3} + \frac{\dot{C}^2Y\dot{Y}G'}{2X^2} \right).\end{aligned}$$

In Eqs. (33)–(34) Z_2 and Z_2^* are of the form

$$\begin{aligned}Z_2 = & \frac{-4}{X^2} \left(\frac{2\ddot{C}C'Y'}{Y^3} + \frac{X''XC'^2}{Y^4} + \frac{X''\dot{C}^2}{Y^2X} \right. \\ & - \frac{XX''}{Y^2} + \frac{2C''XC'X'}{Y^4} - \frac{3C'^2X'XY'}{Y^5} \\ & + \frac{X'Y'\dot{C}^2}{Y^3X} + \frac{X'XY'}{Y^3} - \frac{2\ddot{C}C''}{Y^2} - \frac{\ddot{Y}C'^2}{Y^3} + \frac{\ddot{Y}\dot{C}^2}{YX^2} \\ & + \frac{\ddot{Y}}{Y} + \frac{2\ddot{C}\dot{C}\ddot{Y}}{YX^2} - \frac{2\dot{C}X'C'Y'}{XY^3} \\ & - \frac{2\dot{C}^2\dot{Y}\ddot{X}}{YX^3} + \frac{2C''\dot{C}\ddot{X}}{XY^2} - \frac{2\dot{C}X'CX'Y'}{XY^3} + \frac{C'^2\dot{X}\ddot{Y}}{XY^3} \\ & - \frac{\dot{X}\ddot{Y}}{XY} + \frac{2\dot{C}^2}{Y^2} + \frac{2C'^2Y^2}{Y^4} + \frac{2X'^2\dot{C}^2}{X^2Y^2} - \frac{4C'\dot{C}Y'}{Y^3} - \frac{4X'\dot{C}\dot{C}'}{XY^2} \left. \right), \\ Z_2^* = & \frac{-8}{Y^2} \left(\frac{C''C\ddot{G}}{2X^2} - \frac{C''CX\ddot{G}}{2X^3} - \frac{C''CX'G'}{2X^2Y^2} + \frac{\ddot{C}CG''}{2X^2} \right. \\ & - \frac{\ddot{C}C\dot{Y}Y\ddot{G}}{2X^4} - \frac{\ddot{C}CY'G'}{2YX^2} \\ & + \frac{2\dot{C}Y^2\dot{C}\ddot{G}}{X^4} + \frac{2\ddot{C}C'G'}{X^2} - \frac{C'Y'C\ddot{G}}{2YX^2} + \frac{C'Y'CX'G'}{2XY^3} \\ & - \frac{C'X'CG''}{2Y^2X} + \frac{C'X'CY\ddot{G}}{2YX^3} + \frac{C'X'CY'G'}{2Y^3X} - \frac{\dot{C}Y'Y\ddot{C}\ddot{G}}{2X^4} + \frac{\dot{C}Y'Y\dot{C}\ddot{X}\ddot{G}}{2X^5} \\ & + \frac{\dot{Y}\dot{C}CX'G'}{2X^3Y} - \frac{\dot{C}X'CG''}{2X^3} + \frac{\dot{C}YX'CY\ddot{G}}{2X^5} + \frac{\dot{C}X'CY'G'}{2YX^3} - \frac{\dot{C}^2\dot{X}Y^2\ddot{G}}{X^5} \\ & + \frac{\dot{C}X'CG'}{X^3} + \frac{X''\dot{C}C\ddot{G}}{2X^3} - \frac{X''CC'G'}{2Y^2} - \frac{X'Y'CC\ddot{G}}{2X^3Y} + \frac{X'Y'CC'G'}{2XY^3} \\ & - \frac{\ddot{Y}CC'Y\ddot{G}}{2X^4} + \frac{\ddot{Y}C'CG'}{2YX^2} + \frac{\dot{X}\dot{Y}CY^2\ddot{G}}{2X^5Y} - \frac{\dot{X}\dot{Y}C'G'}{2X^3Y} \\ & + \frac{C'^2\dot{C}\ddot{G}}{CX^2} - \frac{C'^3G'}{CY^2} + \frac{\dot{R}^2Y^2\ddot{X}\ddot{G}}{X^5} - \frac{\dot{C}X'CG'}{X^3} - \frac{C'^2\dot{C}Y^2\ddot{G}}{CX^4} \\ & + \frac{C'^3G'}{CX^2} - \frac{\dot{C}'C\dot{G}G'}{X^2} \\ & - \frac{\dot{C}'CG'}{X^2} + \frac{X'CC'\ddot{G}}{X^3} + \frac{\dot{C}'CYG'}{X^2Y} + \frac{C'Y'G'\ddot{G}}{X^2Y} + \frac{C'Y'CG'}{X^2Y} \\ & + \frac{CCX'G'}{X^3} - \frac{C'Y'CX'G'}{X^3Y} \\ & \left. - \frac{C'CY^2G'}{X^2Y^2} + \frac{\dot{C}CX'\dot{G}G'}{X^3} - \frac{CCX^2\ddot{G}}{X^4} - \frac{\dot{C}CX'Y\ddot{G}}{YX^3} \right).\end{aligned}$$

The evaluated values of l_0 , l_0^* , l_1 and l_1^* in Eqs. (44) and (45) are expressed as

$$\begin{aligned}
l_0 &= -\frac{(\mathcal{G} + \lambda\dot{T} + 2\mathcal{G}\alpha\mathcal{G})}{2} + (2\alpha\mathcal{G} + 1) \\
&\quad \left(\chi_1 + \chi_1 \frac{2\dot{C}}{C} + \chi_1 \frac{\dot{Y}}{Y} + 2\frac{Z_2 X^2}{C^2} + \chi_2 \frac{\dot{Y}}{Y} \right) \\
&\quad + 2\alpha \left(\chi_1 \mathcal{G} + \chi_1^* - \frac{Z_1'}{Y^2} + 2\frac{\chi_1^* \dot{X}}{X} - 2\frac{Z_1 X'}{XY^2} \right. \\
&\quad + X' \frac{Z_1}{XY^2} + Y' \frac{Z_1}{Y^3} + \chi_2^* \frac{\dot{Y}}{Y} - 2\frac{Z_1 C'}{CY^2} \\
&\quad \left. + 2\frac{Z_2^* X^2}{C^2} + \chi_1^* \frac{\dot{Y}}{Y} + 2\chi_1^* \right), \\
l_0^* &= \frac{4\pi E^2}{X^2} \left(2\frac{\dot{C}}{C} + \frac{\dot{E}}{E} \right), \\
l_1 &= \frac{X}{2Y} (\mathcal{G}' + \lambda T' + 2\mathcal{G}'\alpha\mathcal{G}) + (2\alpha\mathcal{G} + 1) \\
&\quad \left(X' \frac{\chi_1}{Y} + X' \frac{\chi_2}{Y} + X \frac{2\chi_2 C'}{CY} + X \frac{\chi_2'}{Y} - X \frac{2Z_2 C'}{CY} \right) \\
&\quad + 2\alpha \left(\frac{-Z_1'}{XY} + Z_1 \frac{\dot{Y}}{XY^2} + X' \frac{\chi_2^*}{Y} + Z_1 \frac{\dot{X}}{X^2 Y} \right. \\
&\quad + X' \frac{\chi_1^*}{Y} - Z_1 \frac{2\alpha \dot{Y}}{XY^2} - \frac{Z_1 \dot{Y}}{XY^2} \\
&\quad \left. + \frac{\chi_2 X \mathcal{G}'}{Y} + X \frac{\chi_2^*}{Y} - 2\frac{\alpha Z_1 \dot{C}}{XYC} - 2\frac{\alpha Z_2^* \dot{C} X}{CY} + 2\frac{\alpha \chi_2^* X C'}{CY} \right), \\
l_1^* &= -\frac{4\pi E^2}{Y^2} \left(\frac{C'}{C} + \frac{E'}{E} \right).
\end{aligned}$$

Data availability All data generated or analyzed during this study are included in this published article.

References

- [1] A De Felice and S Tsujikawa *Living Rev. Relativ.* **13** 1 (2010)
- [2] S Capozziello and M De Laurentis *Phys. Rep.* **509** 167 (2011)
- [3] S Nojiri and S D Odintsov *Phys. Rep.* **505** 59 (2011)
- [4] E Berti et al *Class. Quantum Grav.* **32** 243001 (2015)
- [5] A Azadi, D Momeni and M Nouri-Zonoz *Phys. Lett. B* **670** 210 (2008)
- [6] A V Astashenok, S Capozziello and S D Odintsov *Phys. Rev. D* **89** 103509 (2014)
- [7] S Nojiri, S D Odintsov and V Oikonomou *Phys. Rep.* **692** 1 (2017)
- [8] S D Odintsov, V K Oikonomou and T Paul *Class. Quantum Grav.* **37** 235005 (2020)
- [9] G J Olmo, D Rubiera-Garcia and A Wojnar *Phys. Rep.* **876** 1 (2020)
- [10] J Santos, J S Alcaniz, M J Reboucas and F C Carvalho *Phys. Rev. D* **76** 083513 (2007)
- [11] M Sharif and Z Yousaf *Mon. Not. R. Astron. Soc.* **432** 264 (2013)
- [12] M Sharif and Z Yousaf *Astrophys. Space Sci.* **352** 321 (2014)
- [13] M Sharif and Z Yousaf *Gen. Relativ. Gravit.* **47** 48 (2015)
- [14] G J Olmo and D Rubiera-Garcia *Universe* **1** 173 (2015)
- [15] Z Yousaf *Phys. Scr.* **95** 075307 (2020)
- [16] S Nojiri and S D Odintsov *Phys. Lett. B* **631** 1 (2005)
- [17] A De Felice and S Tsujikawa *Phys. Lett. B* **675** 1 (2009)
- [18] R Myrzakulov, D Sáez-Gómez and A Tureanu *Gen. Relativ. Gravit.* **43** 1671 (2011)
- [19] M J S Houndjo, M E Rodrigues, D Momeni and R Myrzakulov *Can. J. Phys.* **92** 1528 (2014)
- [20] K Bamba, M Ilyas, M Z Bhatti and Z Yousaf *Gen. Relativ. Gravit.* **49** 112 (2017)
- [21] M V de S Silva and M E Rodrigues, *Eur. Phys. J. C* **78** 638 (2018)
- [22] M Z Bhatti, Z Yousaf and A Khadim *Phys. Rev. D* **101** 104029 (2020)
- [23] G Mustafa, X Tie-Cheng and M F Shamir *Ann. Phys.* **413** 168059 (2020)
- [24] P Kanti, R Gannouji and N Dadhich *Phys. Rev. D* **92** 041302 (2015)
- [25] V K Oikonomou *Phys. Rev. D* **92** 124027 (2015)
- [26] Z Yousaf, M Z Bhatti, S Khan and P K Sahoo *Phys. Dark Universe* **36** 101015 (2022)
- [27] Z Yousaf, M Z Bhatti and S Khan *Eur. Phys. J. Plus* **137** 322 (2022)
- [28] P Kanti, B Kleihaus and J Kunz *Phys. Rev. Lett.* **107** 271101 (2011)
- [29] G Antoniou, A Bakopoulos and P Kanti *Phys. Rev. Lett.* **120** 131102 (2018)
- [30] A De, T H Loo, R Solanki and P K Sahoo **69** 2100088 (2021)
- [31] G Antoniou, A Bakopoulos and P Kanti *Phys. Rev. D* **97** 084037 (2018)
- [32] M Sharif and A Ikram *Eur. Phys. J. C* **76** 1 (2016)
- [33] T Harko, F S N Lobo, S Nojiri and S D Odintsov *Phys. Rev. D* **84** 024020 (2011)
- [34] M F Shamir and M Ahmad *Europ. Phys. J. C* **77** 1 (2017)
- [35] Z Yousaf *Astrophys. Space Sci.* **363** 226 (2018)
- [36] Z Yousaf *Europ. Phys. J. Plus* **134** 245 (2019)
- [37] M Z Bhatti, M Sharif, Z Yousaf and M Ilyas *Int. J. Mod. Phys. D* **27** 1850044 (2018)
- [38] M Ilyas, A R Athar and B Masud *Int. J. Geom. Meth. Mod. Phys.* **18** 2150152 (2021)
- [39] S K Maurya, K N Singh and R Nag *Chin. J. Phys.* **74** 313 (2021)
- [40] Z Yousaf, M Z Bhatti and K Hassan *Eur. Phys. J. Plus* **135** 397 (2020)
- [41] T Clifton, P Dunsby, R Goswami and A M Nzioki *Phys. Rev. D* **87** 063517 (2013)
- [42] R Goswami, A M Nzioki, S D Maharaj and S G Ghosh *Phys. Rev. D* **90** 084011 (2014)
- [43] L Herrera *Gen. Relativ. Gravit.* **35** 437 (2003)
- [44] L Herrera, G Le Denmat and N O Santos *Phys. Rev. D* **79** 087505 (2009)
- [45] S Thirukkanesh and S D Maharaj *J. Math. Phys.* **50** 022502 (2009)
- [46] L Herrera, A Di Prisco and J Ibáñez *Phys. Rev. D* **84** 064036 (2011)
- [47] L Herrera *Int. J. Mod. Phys. D* **20** 1689 (2011)
- [48] R Sharma and R Tikekar *Gen. Relativ. Gravit.* **44** 2503 (2012)
- [49] M Govender, K P Reddy and S D Maharaj *Int. J. Mod. Phys. D* **23** 1450013 (2014)
- [50] L Herrera, A Di Prisco and J Ospino *Phys. Rev. D* **91** 124015 (2015)
- [51] Z Yousaf, K Bamba and M Z Bhatti *Phys. Rev. D* **93** 124048 (2016)
- [52] Z Yousaf, K Bamba and M Z Bhatti *Phys. Rev. D* **93** 064059 (2016)
- [53] Z Yousaf, K Bamba and M Z Bhatti *Phys. Rev. D* **95** 024024 (2017)
- [54] Z Yousaf *Eur. Phys. J. Plus* **136** 281 (2021)
- [55] S Rosseland *Mon. Not. R. Astron. Soc.* **84** 720 (1924)
- [56] D G Boulware *Phys. Rev. D* **8** 2363 (1973)

- [57] E Olson and M Bailyn *Phys. Rev. D* **13** 2204 (1976)
- [58] J Bally and E R Harrison *Astrophys. J.* **220** 743 (1978)
- [59] L Herrera, A Di Prisco and J Ibanez *Phys. Rev. D* **84** 107501 (2011)
- [60] G Pinheiro and R Chan *Gen. Relativ. Gravit.* **45** 243 (2013)
- [61] A Di Prisco, L Herrera, G Le Denmat, M A H MacCallum and N O Santos *Phys. Rev. D* **76** 064017 (2007)
- [62] Y Nyonyi, S D Maharaj and K S Govinder *Eur. Phys. J. C* **73** 2637 (2013)
- [63] M Sharif and Z Yousaf *Eur. Phys. J. C* **75** 194 (2015)
- [64] Z Yousaf *Universe* **8** 131 (2022)
- [65] D Langlois. arXiv preprint [arXiv:1707.03625](https://arxiv.org/abs/1707.03625) (2017)
- [66] T Kokubu, H Maeda and T Harada *Class. Quantum Grav.* **32** 235021 (2015)
- [67] C W Misner and D H Sharp *Phys. Rev.* **136** B571 (1964)
- [68] L Herrera, N O Santos and J Carot *J. Math. Phys.* **47** 052502 (2006)
- [69] G F Ellis, R Maartens and M A H MacCallum *Relativistic cosmology*. Cambridge University Press (2012)
- [70] L Herrera, A Di Prisco, J Martin, J Ospino, N O Santos and O Troconis *Phys. Rev. D* **69** 084026 (2004)
- [71] J Truelove, *J. of Heat Transfer (Transactions of the ASME (American Society of Mechanical Engineers), Series C);(United States)*, **109** 4 (1987)
- [72] G C Rau, M S Andersen and R I Acworth *Water Resour. Res.* **48** 3 (2012)
- [73] Z Yousaf, M Z Bhatti and A Farhat Various phases of irregular energy density in charged spheres *Ann. Phys.* **442** 168935 (2022)
- [74] L Herrera, A Di Prisco and J Ospino *Gen. Relativ. Gravit.* **44** 2645 (2012)
- [75] L Herrera and N O Santos *Phys. Rev. D* **70** 084004 (2004)

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