

Dynamics of Soret and Dufour effects on oscillatory flow of couple stress fluid due to stretchable curved surface

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Muhammad Imran¹ , Muhammad Naveed¹ and Zaheer Abbas²

Abstract

In this theoretical study, we have discussed the impacts of Soret and Dufour on the flow of time-dependent couple stress fluid across a stretchable curved oscillatory surface with magnetic field. The to and fro motion of the surface causes the flow phenomenon. The boundary layer flow equations depicting the flow phenomena are expressed in mathematical form by means of a curvilinear coordinates system along with some suitable similarity variables. The developed flow equations in the form of partial differential equations are then solved analytically by utilizing a valuable method called the homotopy analysis method (HAM). A detailed examination is performed to evaluate the influence of various involved parameters namely, the non-dimensional radius of curvature, couple stress parameter, magnetic parameter, the relation of the parameter of the surface oscillating frequency to its stretching rate constant and Soret and Dufour numbers on velocity field, pressure distribution, concentration and temperature distribution, rate of heat transfer, coefficient of skin friction, and rate of mass transport via graphs and are explained in detail. It is confirmed that the amplitude of velocity and pressure distribution exhibits reducing behavior with higher values of the couple stress parameter. The concentration and temperature profiles also diminish with varying time instants.

Keywords

Curved oscillatory stretchable surface, magnetohydrodynamic (MHD), couple stress fluid, Soret and Dufour number, homotopy analysis method

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Introduction

Over the last few decades, owing to its abundant and noteworthy applications in many industrial and engineering processes, several researchers and engineers have considered the exploration of boundary layer stretching flows with modified velocities included linear, nonlinear, exponential, and oscillatory. These processes include wire drawing, hot rolling, artificial fibers, paper production, the distillation of towers, glass blowing, polymer extrusion process, aerodynamic extrusion of plastic sheets, and cooling of electronic chips, etc. In all such operations, the quality of the finished product relies heavily on the surface stretching velocity and

stretching rate. To our best knowledge, Crane¹ was firstly analyzed the flow problem over stretching surface by considering linear velocity and identified its solution in closed form. Many researchers expanded

¹Department of Mathematics, Division of Science and Technology, University of Education, Lahore, Lahore, Pakistan

²Department of Mathematics, The Islamia University of Bahawalpur, Bahawalpur, Pakistan

Corresponding author:

Muhammad Naveed, Department of Mathematics, Division of Science and Technology, University of Education, Jauharabad Campus, Jauharabad, Lahore 54770, Pakistan.

Email: rana.m.naveed@gmail.com



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the idea¹ in various directions over a flat and curved stretching surface for various kinds of viscous fluids. Sajid et al.² has attempted to discuss the flow of viscous fluid across the curved stretchable sheet by considering the effects of a radius of curvature in the sheet. Abbas et al.³ has examined the stretchable motion of viscous liquid toward a curved sheet in the presence of constant magnetic field. Saif et al.⁴ has considered the significance of convective heat transfer in the flow of viscous liquid toward a nonlinearly curved stretching surface. Amjad et al.⁵ has conducted numerical investigation for Williamson fluid motion toward an exponential curved stretchable surface. Wang⁶ takes into consideration the flow problem due to oscillatory stretching sheet. After Wang, Abbas et al.⁷ has considered the influences of thermal and velocity slip over the oscillating stretchable surface. The effect magnetic field on flow of thermally radiative viscous fluid motion across the convectively heated curved oscillatory surface was addressed by Abbas et al.⁸ Imran et al.⁹ has computed the consequences of heat generation and Joule heating on oscillatory flow of ferrofluid across the curved stretching surface. For a more comprehensive study, Refs.^{10–17} provide a more extensive review of the scientific literature on various flow problems on different flow geometries.

Many real-life fluids, including blood, ketchup, yogurt, mud, clay coatings, apple sauce, melted plastics, polymer melts, certain oils, and greases are classified into non-Newtonian fluids. These fluids have a complex correlation between shear stresses and strain rates which means that Newton's law of viscosity does not comply with non-Newtonian liquids. The prominent characteristics of all these fluids cannot be presented through an exclusively constitutive relationship. Many scientists and engineers have been attracted to the exploration of non-Newtonian liquids due to its broad practical utilizations in the nuclear and chemical industries; the petroleum industry, plastic foam manufacturing, polymer solutions, and bioengineering, etc. Based on different physical features, a variety of flow models for such liquids are available in the literature. Couple stress fluid is an important type of non-Newtonian liquid that Stokes¹⁸ first purposed. Simply, the concept of couple stress is the expansion of the conventional viscous idea that supports couple stress and couples in the body. Because of rotational contact between liquid particles, the stress tensor in the couple stress concept is considered anti-symmetric. The anti-symmetric part of the tensor would enhance the order of the momentum equation from two to four. Examples of couple stress liquid are lubricants, colloidal liquids, and blood which contain electro-rheological and synthetic properties. Ali et al.¹⁹ have discussed the various features of heat transfer phenomena on hydromagnetic couple stress liquid motion flow across an oscillatory stretchable

surface in the existence of a porous medium. Analytical findings of the mixed convective motion of couple stress nanoliquid above on oscillatory stretchable surface with heat generation effects have been found by Khan et al.²⁰ Albadan²¹ has evaluated the consequences of temperature dependent viscosity for bioconvection motion of couple stress nanofluid toward oscillatory sheet. Heat transfer analysis with impacts of nonlinear thermal radiation in three-dimensional couple stress fluid motion over permeable stretching surface was done by Tarakaramu et al.²²

The examination of an electrically conducting hydromagnetic flow with combined heat and mass transport has received a great deal of interest from various researchers, investigators, and engineers due to its large number of utilizations in many industries and technological operations likely in the designing purpose of cooling systems, food processing, hydromagnetic generators, cooling towers, instruments of blood flow measurement, packed sphere bed, spinning of fibers, electric motors, pumps and flow meters and many more. The hydromagnetic motion of viscoelastic liquid was investigated by Alharbi et al.²³ under the influence of energy and mass transportation on a permeable stretchable sheet. The impact of heat and mass transportation on hydromagnetic micropolar liquid flow across a curved stretched sheet was analyzed by Yasmin et al.²⁴ Whenever heat transport and mass transport occur at the same time in a flowing fluid, then the association between the driving potentials and the fluxes is often more complex. It has been distinguished that heat alteration is also induced by the concentration gradients, which is termed as diffusion-thermo (Dufour) effects. Moreover, the production of mass flux due to thermal gradient is known as the thermal-diffusion (Soret) effect. Generally, in most of the studies relating to heat and mass transport phenomenon impacts of Soret and Dufour are usually ignored because they are weaker in order of magnitude than the impacts defined by Fourier's and Fick's theories. However, some disciplines like geosciences, petrology, nuclear waste disposal, and hydrology, etc. in which such phenomena's play an important role. Zheng et al.²⁵ has analytically inspected the effects of Soret and Dufour on hydromagnetic viscous liquid motion over an oscillatory stretching surface. Dufour's and Soret impacts on the magnetohydrodynamic flow of Powell-Eyring fluid through oscillatory stretched sheet were determined by Khan et al.²⁶ Imtiaz et al.²⁷ have analyzed the impacts of Dufour and Soret on viscous fluid flow across curved stretching surface. Physical characteristics of Dufour and Soret impacts on mixed convection flow of non-Newtonian fluid over nonlinear sheet was investigated by Akbar et al.²⁸ Seid et al.²⁹ has conducted an analytical investigation to evaluate the outcomes of Soret and Dufour numbers in

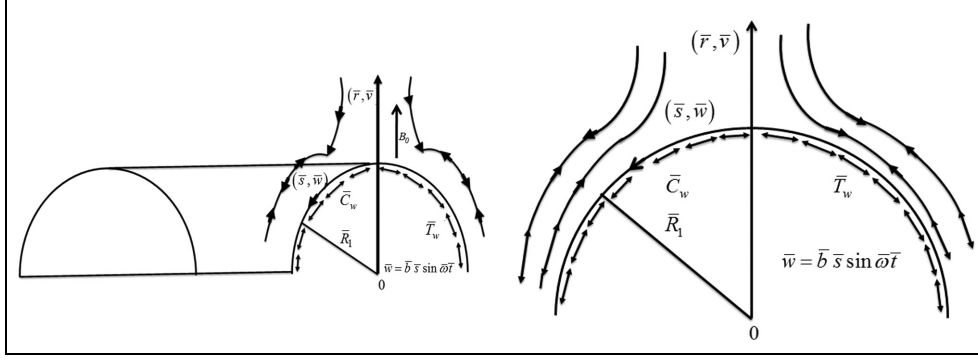


Figure 1. Flow geometry.

nanoliquid flow across a vertical stretched sheet. Siddique et al.³⁰ have explored the effects of Soret and Dufour on unsteady motion of viscoelastic nanoliquid over an exponentially stretching sheet.

An analysis of literature survey reflects that most of the researchers have paid special attention to examine the various attributes of heat and mass transfer in the flow of non-Newtonian fluid on a linear, nonlinear and oscillatory stretching flat or curved surface. But no one has yet considered to examine the influence of Soret and Dufour effects in the flow of couple stress fluid on a curved oscillatory surface. Therefore, the prime objective of the present research is to explore the heat and mass transport phenomena with Soret and Dufour impacts on a couple stress liquid across a curved oscillatory stretchable surface. The formulated flow equations in the form of PDEs are highly nonlinear in nature which are than solved analytically by employing an effective analytical method namely the homotopy analysis method (HAM). The influences of various pertinent parameters on flow, energy and concentration are shown graphically and are discussed in detail.

Problem explanation

Consider a laminar, incompressible and unsteady flow of couple stress liquid past toward a stretchable curved oscillating sheet that coiled in the form of a semi-circle having radius $\bar{R}_1$ (see Figure 1). The sheet velocity $\bar{w} = \bar{b} \bar{s} \sin \bar{\omega} \bar{t}$ is considered the cause of the flow mechanism. Here $(\bar{\omega}, \bar{b})$ denotes the oscillatory frequency and stretchable rate of the sheet, respectively. A persistent magnetic field having power B_0 is forced along the radial direction $\bar{r}$ - of the surface. The influences of convinced magnetic field are omitted by choosing very low Reynolds numbers. Let $\bar{T}_w$ be the surface temperature, $\bar{C}_w$ is the surface concentration, $\bar{T}_\infty$ is the ambient fluid temperature, and $\bar{C}_\infty$ is the concentration away from the surface with $\bar{T}_w > \bar{T}_\infty$ and $\bar{C}_w > \bar{C}_\infty$. The flow equations for the specified problem are provided by

$$\nabla \cdot \mathbf{V} = 0, \quad (1)$$

$$\left(\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} \right) = -\frac{1}{\bar{\rho}} \nabla \bar{p} + \bar{\nu} \nabla^2 \mathbf{V} - \bar{\eta} \nabla^4 \mathbf{V} + \frac{1}{\bar{\rho}} (\mathbf{J} \times \mathbf{B}), \quad (2)$$

$$\bar{\rho} \bar{c}_p \left(\frac{\partial \bar{T}}{\partial t} + (\mathbf{V} \cdot \nabla) \bar{T} \right) = \bar{k}_\beta \nabla^2 \bar{T} + \frac{\bar{D}_m \bar{k}_T}{\bar{c}_s} \nabla^2 \bar{C}, \quad (3)$$

$$\left(\frac{\partial \bar{C}}{\partial t} + (\mathbf{V} \cdot \nabla) \bar{C} \right) = \bar{D}_m \nabla^2 \bar{C} + \frac{\bar{D}_m \bar{k}_T}{\bar{T}_m} \nabla^2 \bar{T}, \quad (4)$$

In above equations $\mathbf{V}$ is the velocity vector, ∇^2 the Laplacian operator, $\bar{p}$ is the pressure, $\mathbf{J}$ is the current density, $\mathbf{B}$ is the magnetic flux vector, $\bar{T}$ is the temperature, $\bar{\rho}$ is the density, $\bar{D}_m$ the coefficient of mass diffusion, $\bar{\eta} = \bar{\mu}_1 / \bar{\rho}$ is the couple stress viscosity, and $\bar{\nu}$ is the kinematic viscosity, $\bar{c}_p$ is the specific heat capacitance, $\bar{T}_m$ is the mean temperature of the fluid, $\bar{c}_s$ is the concentration susceptibility, $\bar{k}_T$ is the ratio of thermal diffusion and $\bar{C}$ is the concentration.

The current density $\mathbf{J}$ is defined as

$$\mathbf{J} = \bar{\sigma} (\mathbf{V} \times \mathbf{B}), \quad (5)$$

For current flow problem, consider the velocity field of the form

$$\mathbf{V} = (\bar{v}(\bar{r}, \bar{s}, \bar{t}), \bar{w}(\bar{r}, \bar{s}, \bar{t}), 0), \quad (6)$$

here $(\bar{v}, \bar{w})$ are considered the fluid velocity parts along with $(\bar{r}, \bar{s})$ directions. By employing equation (6) along with boundary layer approximations equations (1)–(5) can be expressed in curvilinear coordinates system as^{8,27}

$$\frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\bar{v}}{(\bar{R}_1 + \bar{r})} + \frac{\partial \bar{w}}{\partial \bar{s}} \frac{\bar{R}_1}{(\bar{R}_1 + \bar{r})} = 0, \quad (7)$$

$$\frac{\partial \bar{p}}{\partial \bar{r}} = \frac{\bar{\rho} \bar{w}^2}{(\bar{R}_1 + \bar{r})}, \quad (8)$$

$$\begin{aligned} \frac{\partial \bar{w}}{\partial \bar{t}} + \frac{\bar{v} \bar{w}}{\bar{r} + \bar{R}_1} + \frac{\partial \bar{w}}{\partial \bar{r}} \bar{v} + \frac{\partial \bar{w}}{\partial \bar{s}} \frac{\bar{R}_1 \bar{w}}{(\bar{r} + \bar{R}_1)} = -\frac{1}{\bar{\rho}} \frac{\bar{R}_1}{(\bar{r} + \bar{R}_1)} \frac{\partial \bar{p}}{\partial \bar{s}} + \bar{v} \left(\frac{\partial^2 \bar{w}}{\partial \bar{r}^2} + \frac{\partial}{\partial \bar{r}} \left(\frac{\bar{w}}{(\bar{r} + \bar{R}_1)} \right) \right) \\ - \bar{\eta} \left(\frac{\partial^4 \bar{w}}{\partial \bar{r}^4} + \frac{3 \partial \bar{w}}{(\bar{r} + \bar{R}_1)^3 \partial \bar{r}} - \frac{3}{(\bar{r} + \bar{R}_1)^2} \frac{\partial^2 \bar{w}}{\partial \bar{r}^2} + \frac{2}{\bar{r} + \bar{R}_1} \frac{\partial^3 \bar{w}}{\partial \bar{r}^3} - \frac{3 \bar{w}}{(\bar{r} + \bar{R}_1)^4} \right) - \frac{\bar{\sigma}}{\bar{\rho}} B_0^2 \bar{w}, \end{aligned} \quad (9)$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \frac{\bar{w} \bar{R}_1 \partial \bar{T}}{(\bar{r} + \bar{R}_1) \partial \bar{s}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{r}} = \bar{\alpha} \left(\frac{\partial^2 \bar{T}}{\partial \bar{r}^2} + \frac{\partial \bar{T}}{(\bar{r} + \bar{R}_1) \partial \bar{r}} \right) + \frac{\bar{D}_m \bar{k}_T}{\bar{c}_s \bar{c}_p} \left(\frac{\partial^2 \bar{C}}{\partial \bar{r}^2} + \left(\frac{1}{\bar{r} + \bar{R}_1} \right) \frac{\partial \bar{C}}{\partial \bar{r}} \right), \quad (10)$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} + \left(\frac{\bar{R}_1 \bar{w}}{\bar{R}_1 + \bar{r}} \right) \frac{\partial \bar{C}}{\partial \bar{s}} + \bar{v} \frac{\partial \bar{C}}{\partial \bar{r}} = \bar{D}_m \left(\frac{\partial^2 \bar{C}}{\partial \bar{r}^2} + \left(\frac{1}{\bar{r} + \bar{R}_1} \right) \frac{\partial \bar{C}}{\partial \bar{r}} \right) + \frac{\bar{D}_m \bar{k}_T}{\bar{T}_m} \left(\frac{\partial^2 \bar{T}}{\partial \bar{r}^2} + \frac{1}{(\bar{r} + \bar{R}_1)} \frac{\partial \bar{T}}{\partial \bar{r}} \right), \quad (11)$$

With boundary conditions²¹

$$\left. \begin{aligned} \bar{v} = 0, \quad \bar{w} = \bar{b} \bar{s} \sin \bar{\omega} \bar{t}, \quad \frac{\partial^2 \bar{w}}{\partial \bar{r}^2} \rightarrow 0, \quad \frac{\partial^3 \bar{w}}{\partial \bar{r}^3} \rightarrow 0, \quad \bar{C} = \bar{C}_w, \quad \bar{T} = \bar{T}_w \quad \text{at } \bar{r} = 0, \\ \frac{\partial \bar{w}}{\partial \bar{r}} \rightarrow 0, \quad \bar{T} \rightarrow \bar{T}_\infty, \quad \bar{w} \rightarrow 0, \quad \bar{C} \rightarrow \bar{C}_\infty, \quad \text{as } \bar{r} \rightarrow \infty. \end{aligned} \right\} \quad (12)$$

Now introducing the following dimensionless variables as

$$\begin{aligned} \sqrt{\frac{\bar{b}}{\bar{v}}} \bar{r} = \bar{\xi}, \quad \bar{b} \bar{s} \bar{f}_{\bar{\xi}}(\bar{\xi}, \bar{\tau}) = \bar{w}, \quad \frac{-\bar{R}_1}{\bar{R}_1 + \bar{r}} \sqrt{\bar{b}} \bar{v} \bar{f}(\bar{\xi}, \bar{\tau}) = \bar{v}, \\ \bar{t} \bar{\omega} = \bar{\tau}, \quad \bar{\rho} \bar{b}^2 \bar{s}^2 \bar{P}(\bar{\xi}, \bar{\tau}) = \bar{p}, \quad \bar{\phi}(\bar{\xi}, \bar{\tau}) = \bar{C} - \bar{C}_\infty / (\bar{C}_w - \bar{C}_\infty), \\ \bar{\theta}(\bar{\xi}, \bar{\tau}) = \bar{T} - \bar{T}_\infty / (\bar{T}_w - \bar{T}_\infty). \end{aligned} \quad (13)$$

By incorporating equation (13), equation (7) is proved identically and the rest of equations (8)–(11) become

$$\frac{\bar{f}_{\bar{\xi}}^2}{C_1 + \bar{\xi}} = \bar{P}_{\bar{\xi}}, \quad (14)$$

$$\begin{aligned} \bar{P}(\bar{\xi}, \bar{\tau}) = \left(\frac{(C_1 + \bar{\xi}) \bar{f}_{\bar{\xi}} \bar{\xi} \bar{\xi}}{2C_1} - \frac{\bar{f}_{\bar{\xi}}}{2C_1(C_1 + \bar{\xi})} + \frac{\bar{f}_{\bar{\xi}} \bar{\xi}}{2C_1} \right) - \frac{(C_1 + \bar{\xi}) S \bar{f}_{\bar{\xi}} \bar{\tau}}{2C_1} \\ - \lambda \left(\frac{\bar{f}_{\bar{\xi}} \bar{\xi} \bar{\xi} \bar{\xi} (C_1 + \bar{\xi})}{2C_1} - \frac{3}{2C_1} \frac{\bar{f}_{\bar{\xi}} \bar{\xi} \bar{\xi}}{(C_1 + \bar{\xi})} + \frac{\bar{f}_{\bar{\xi}} \bar{\xi} \bar{\xi} \bar{\xi}}{C_1} + \frac{3}{2C_1} \frac{\bar{f}_{\bar{\xi}} \bar{\xi}}{(C_1 + \bar{\xi})^2} + \frac{3}{2C_1} \frac{\bar{f}_{\bar{\xi}}}{(C_1 + \bar{\xi})^3} \right) \\ + \frac{\bar{f} \bar{f}_{\bar{\xi}}}{2(C_1 + \bar{\xi})} - \frac{(C_1 + \bar{\xi}) \bar{f}_{\bar{\xi}} M^2}{2C_1} + \frac{1}{2} (\bar{f}_{\bar{\xi}} \bar{\xi} - \bar{f} \bar{f}_{\bar{\xi}}^2), \end{aligned} \quad (15)$$

$$\left(\bar{\theta}_{\bar{\xi}} + \frac{\bar{\theta}_{\bar{\xi}}}{(C_1 + \bar{\xi})} \right) + Pr Du \left(\bar{\phi}_{\bar{\xi}} + \frac{\bar{\phi}_{\bar{\xi}}}{(C_1 + \bar{\xi})} \right) + Pr \left(\frac{C_1}{(C_1 + \bar{\xi})} \bar{f} \bar{\theta}_{\bar{\xi}} - S \bar{\theta}_{\bar{\tau}} \right) = 0, \quad (16)$$

$$\left(\bar{\phi}_{\bar{\xi}} + \frac{\bar{\phi}_{\bar{\xi}}}{(C_1 + \bar{\xi})} \right) + Sc Sr \left(\bar{\theta}_{\bar{\xi}} + \frac{\bar{\theta}_{\bar{\xi}}}{(C_1 + \bar{\xi})} \right) + Sc \left(\frac{C_1}{(C_1 + \bar{\xi})} \bar{f} \bar{\phi}_{\bar{\xi}} - S \bar{\phi}_{\bar{\tau}} \right) = 0, \quad (17)$$

where, $\lambda (= \bar{\eta} \bar{b} / \bar{v}^2)$ the dimensionless couple stress parameters, $C_1 (= \bar{R}_1 \sqrt{\bar{b} / \bar{v}})$ radius of curvature variable, $S (= \bar{\omega} / \bar{b})$ the parameter of surface frequency to its stretched constant, $Pr (= \bar{v} / \bar{\alpha})$ the Prandtl number, $M^2 (= \bar{\sigma} B_0^2 / \bar{\rho} \bar{b})$ the magnetic parameter, $Du (= \bar{D}_m \bar{k}_T (\bar{C}_w - \bar{C}_\infty) / (\bar{T}_w - \bar{T}_\infty) \bar{v} \bar{c}_s \bar{c}_p)$ the Dufour number, $Sc (= \bar{v} / \bar{D}_m)$ the Schmidt number, and $Sr (= \bar{D}_m \bar{k}_T (\bar{T}_w - \bar{T}_\infty) / (\bar{C}_w - \bar{C}_\infty) \bar{v} \bar{T}_m)$ the Soret number respectively.

After elimination of the pressure expression present in equations (14) and (15), the equation for the velocity of the fluid is attained as

$$\begin{aligned}
& \bar{f}_{\bar{\xi}\bar{\xi}\bar{\xi}\bar{\xi}} + 2\left(\frac{\bar{f}_{\bar{\xi}\bar{\xi}\bar{\xi}}}{C_1 + \bar{\xi}}\right) - S\left(\bar{f}_{\bar{\xi}\bar{\xi}\bar{\tau}} + \frac{\bar{f}_{\bar{\xi}\bar{\tau}}}{(C_1 + \bar{\xi})}\right) - \frac{\bar{f}_{\bar{\xi}\bar{\xi}}}{(C_1 + \bar{\xi})^2} + \frac{\bar{f}_{\bar{\xi}}}{(C_1 + \bar{\xi})^3} + \frac{\bar{f}\bar{f}_{\bar{\xi}\bar{\xi}\bar{\xi}}C_1}{(C_1 + \bar{\xi})} \\
& - \frac{C_1}{(C_1 + \bar{\xi})}\bar{f}_{\bar{\xi}}\bar{f}_{\bar{\xi}\bar{\xi}} - \frac{C_1}{(C_1 + \bar{\xi})^2}(\bar{f}_{\bar{\xi}}^2 - \bar{f}_{\bar{\xi}\bar{\xi}}\bar{f}) - \frac{C_1\bar{f}\bar{f}}{(C_1 + \bar{\xi})^3} - \left(\bar{f}_{\bar{\xi}\bar{\xi}} + \frac{\bar{f}_{\bar{\xi}}}{(C_1 + \bar{\xi})}\right)M^2 \\
& - \lambda\left(\bar{f}_{\bar{\xi}\bar{\xi}\bar{\xi}\bar{\xi}\bar{\xi}\bar{\xi}} + 3\left(\frac{3\bar{f}_{\bar{\xi}}}{(C_1 + \bar{\xi})^5} - \frac{3\bar{f}_{\bar{\xi}\bar{\xi}}}{(C_1 + \bar{\xi})^4} + \frac{2\bar{f}_{\bar{\xi}\bar{\xi}\bar{\xi}}}{(C_1 + \bar{\xi})^3} - \frac{\bar{f}_{\bar{\xi}\bar{\xi}\bar{\xi}\bar{\xi}}}{(C_1 + \bar{\xi})^2} + \frac{\bar{f}_{\bar{\xi}\bar{\xi}\bar{\xi}\bar{\xi}\bar{\xi}}}{(C_1 + \bar{\xi})}\right)\right) = 0.
\end{aligned} \tag{18}$$

The relevant boundary conditions reduce as

$$\begin{aligned}
& \bar{f}_{\bar{\xi}\bar{\xi}\bar{\xi}}(0, \bar{\tau}) = 0, \quad \bar{\phi}(0, \bar{\tau}) = 1, \quad \bar{f}(0, \bar{\tau}) = 0, \quad \bar{\theta}(0, \bar{\tau}) = 1, \quad \bar{f}_{\bar{\xi}\bar{\xi}\bar{\xi}\bar{\xi}}(0, \bar{\tau}) = 0, \quad \bar{f}_{\bar{\xi}}(0, \bar{\tau}) = \sin\bar{\tau}, \\
& \bar{\phi}(\infty, \bar{\tau}) = 0, \quad \bar{f}_{\bar{\xi}}(\infty, \bar{\tau}) = 0, \quad \bar{f}_{\bar{\xi}\bar{\xi}}(\infty, \bar{\tau}) = 0, \quad \bar{\theta}(\infty, \bar{\tau}) = 0.
\end{aligned} \tag{19}$$

The rate of heat transport, coefficient of skin friction, as well as mass transport along the curved wall are characterized as

$$\bar{N}\bar{u}_{\bar{s}} = \frac{\bar{s}\bar{q}_{\bar{w}}}{(\bar{T}_w - \bar{T}_{\infty})\bar{k}_{\bar{\alpha}}}, \quad \bar{C}_{\bar{f}} = \frac{\tau_{\bar{r}\bar{s}}}{\bar{\rho}\bar{u}_w^2}, \quad \bar{S}\bar{h}_{\bar{s}} = \frac{\bar{s}\bar{j}_{\bar{w}}}{(\bar{C}_w - \bar{C}_{\infty})\bar{D}_m} \tag{20}$$

here, $\bar{q}_{\bar{w}}$, $\bar{j}_{\bar{w}}$ and $\tau_{\bar{r}\bar{s}}$ are depicted by

$$\begin{aligned}
& \bar{q}_{\bar{w}} = -\left(\bar{k}_{\bar{\alpha}} \frac{\partial \bar{T}}{\partial \bar{r}}\right)\Big|_{\bar{r}=0}, \quad \bar{j}_{\bar{w}} = -\left(\bar{D}_m \frac{\partial \bar{C}}{\partial \bar{r}}\right)\Big|_{\bar{r}=0}, \\
& \tau_{\bar{r}\bar{s}} = \left(\bar{\mu}\left(\frac{\partial \bar{w}}{\partial \bar{r}} - \frac{\bar{w}}{\bar{R}_1 + \bar{r}}\right) - \bar{\eta}\left(\frac{\partial^3 \bar{w}}{\partial \bar{r}^3} - \frac{1}{(\bar{R}_1 + \bar{r})^2} \frac{\partial \bar{w}}{\partial \bar{r}} + \frac{\bar{w}}{(\bar{R}_1 + \bar{r})^3}\right)\right)\Big|_{\bar{r}=0},
\end{aligned} \tag{21}$$

Making use of equation (13) in equations (20) and (21), we get

$$\begin{aligned}
& Re_s^{1/2} \bar{C}_{\bar{f}} = \left(\bar{f}_{\bar{\xi}\bar{\xi}}(0, \bar{\tau}) - \frac{\bar{f}_{\bar{\xi}}(0, \bar{\tau})}{C_1}\right) \\
& - \lambda\left(\bar{f}_{\bar{\xi}\bar{\xi}\bar{\xi}\bar{\xi}}(0, \bar{\tau}) - \frac{\bar{f}_{\bar{\xi}\bar{\xi}}(0, \bar{\tau})}{C_1^2} + \frac{\bar{f}_{\bar{\xi}}(0, \bar{\tau})}{C_1^3}\right),
\end{aligned} \tag{22}$$

$$Re_s^{-1/2} \bar{N}\bar{u}_{\bar{s}} = -\bar{\theta}_{\bar{\xi}}(0, \bar{\tau}), \tag{23}$$

$$Re_s^{-1/2} \bar{S}\bar{h}_{\bar{s}} = -\bar{\phi}_{\bar{\xi}}(0, \bar{\tau}). \tag{24}$$

Explanation of solution method

In this section, we have focused our attention on exploring the methodology by incorporating the HAM, which is utilized to determine the series solution of the developed equations (16), (17), and (18) with governing boundary conditions indicated in equation (19) by considering the following expressions:

$$\begin{aligned}
& \bar{\phi}_0(\bar{\xi}, \bar{\tau}) = \exp(-\bar{\xi}), \quad \bar{\theta}_0(\bar{\xi}, \bar{\tau}) = \exp(-\bar{\xi}), \\
& \bar{f}_0(\bar{\xi}, \bar{\tau}) = \frac{\sin\bar{\tau}\left(12 - 12\exp(-\bar{\xi}) + 6\bar{\xi}\exp(-\bar{\xi}) - \bar{\xi}^2\exp(-\bar{\xi})\right)}{6},
\end{aligned} \tag{25}$$

$$\begin{aligned}
& \bar{L}_{\bar{\phi}}(\bar{\phi}) = \frac{\partial^2 \bar{\phi}}{\partial \bar{\xi}^2} - \bar{\phi}, \quad \bar{L}_{\bar{\theta}}(\bar{\theta}) = \frac{\partial^2 \bar{\theta}}{\partial \bar{\xi}^2} - \bar{\theta}, \\
& \bar{L}_{\bar{f}}(\bar{f}) = \frac{\partial^6 \bar{f}}{\partial \bar{\xi}^6} + \frac{\partial^5 \bar{f}}{\partial \bar{\xi}^5} - 2\frac{\partial^4 \bar{f}}{\partial \bar{\xi}^4} - 2\frac{\partial^3 \bar{f}}{\partial \bar{\xi}^3} + \frac{\partial^2 \bar{f}}{\partial \bar{\xi}^2} + \frac{\partial \bar{f}}{\partial \bar{\xi}},
\end{aligned} \tag{26}$$

which hold the following outcomes

$$\begin{aligned}
\bar{L}_f [\bar{C}_2 \exp(\bar{\xi}) + \bar{C}_3 \bar{\xi} \exp(\bar{\xi}) + (\bar{C}_4 + \bar{C}_5 \bar{\xi} + \bar{C}_6 \bar{\xi}^2) \exp(-\bar{\xi}) + \bar{C}_1] &= 0, \\
\bar{L}_\theta [\bar{C}_7 \exp(\bar{\xi}) + \bar{C}_8 \exp(-\bar{\xi})] &= 0, \\
\bar{L}_\phi [\bar{C}_9 \exp(\bar{\xi}) + \bar{C}_{10} \exp(-\bar{\xi})] &= 0,
\end{aligned} \tag{27}$$

here $\bar{C}_i$ ($i = 1 - 10$) are denotes some arbitrary constants.

Let $\bar{q}_1 \in [0, 1]$ signifies an embedding parameter, $\bar{h}_f$, $\bar{h}_\phi$ and $\bar{h}_\theta$ are non- zero auxiliary parameters. By construct- ing the equation for zeroth- order deformation as

$$(1 - \bar{k}) \bar{N}_f [\bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k}) - \bar{f}_0^*(\bar{\xi}, \bar{\tau})] = \bar{k} \bar{h}_f \bar{Q}_f [\bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})], \tag{28}$$

$$(1 - \bar{k}) \bar{N}_\theta [\bar{\theta}^*(\bar{\xi}, \bar{\tau}; \bar{k}) - \bar{\theta}_0^*(\bar{\xi}, \bar{\tau})] = \bar{k} \bar{h}_\theta \bar{Q}_\theta [\bar{\theta}^*(\bar{\xi}, \bar{\tau}; \bar{k})], \tag{29}$$

$$(1 - \bar{k}) \bar{N}_\phi [\bar{\phi}^*(\bar{\xi}, \bar{\tau}; \bar{k}) - \bar{\phi}_0^*(\bar{\xi}, \bar{\tau})] = \bar{k} \bar{h}_\phi \bar{Q}_\phi [\bar{\phi}^*(\bar{\xi}, \bar{\tau}; \bar{k})], \tag{30}$$

subject to boundary conditions

$$\left. \begin{aligned}
\frac{\partial \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}} \Big|_{\bar{\xi}=0} &= \sin \bar{\tau}, \quad \bar{\theta}^*(\bar{\xi}, \bar{\tau}; \bar{k}) \Big|_{\bar{\xi}=0} = 1, \quad \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k}) \Big|_{\bar{\xi}=0} = 0, \\
\frac{\partial^3 \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}^3} \Big|_{\bar{\xi}=0} &= 0, \quad \bar{\phi}^*(\bar{\xi}, \bar{\tau}; \bar{k}) \Big|_{\bar{\xi}=0} = 1, \quad \frac{\partial^4 \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}^4} \Big|_{\bar{\xi}=0} = 0, \\
\frac{\partial \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}} \Big|_{\bar{\xi}=\infty} &= \bar{\theta}^*(\bar{\xi}, \bar{\tau}; \bar{k}) \Big|_{\bar{\xi}=\infty} = \frac{\partial^2 \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}^2} \Big|_{\bar{\xi}=\infty} = \bar{\phi}^*(\bar{\xi}, \bar{\tau}; \bar{k}) \Big|_{\bar{\xi}=\infty} = 0.
\end{aligned} \right\} \tag{31}$$

The connected non- linear operators are of the form

$$\begin{aligned}
\bar{Q}_f [\bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})] &= \frac{\partial^4 \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}^4} + \frac{2}{\bar{\xi} + C_1} \frac{\partial^3 \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}^3} - \frac{1}{(\bar{\xi} + C_1)^2} \frac{\partial^2 \hat{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}^2} \\
&+ \frac{1}{(\bar{\xi} + C_1)^3} \frac{\partial \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}} - \frac{C_1}{(\bar{\xi} + C_1)^3} \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k}) \frac{\partial \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}} \\
&+ \frac{C_1}{(\bar{\xi} + C_1)} \left[\bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k}) \frac{\partial^3 \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{q})}{\partial \bar{\xi}^3} - \frac{\partial \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}} \frac{\partial^2 \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}^2} \right] \\
&- \frac{C_1}{(\bar{\xi} + C_1)^2} \left(\frac{\partial \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}} \right)^2 - S \left[\frac{\partial^3 \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}^2 \partial \bar{\tau}} + \frac{1}{(\bar{\xi} + C_1)} \frac{\partial^2 \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi} \partial \bar{\tau}} \right]
\end{aligned} \tag{32}$$

$$\begin{aligned}
&+ \frac{C_1}{(\bar{\xi} + C_1)^2} \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{q}) \frac{\partial^2 \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}^2} - M^2 \left[\frac{\partial^2 \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}^2} + \frac{\partial \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}} \frac{1}{(\bar{\xi} + C_1)} \right] \\
&- \lambda \left[\frac{\partial^6 \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}^6} + \frac{3}{(\bar{\xi} + C_1)} \frac{\partial^5 \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}^5} - \frac{3}{(\bar{\xi} + C_1)^2} \frac{\partial^4 \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}^4} \right. \\
&\quad \left. + \frac{6}{(\bar{\xi} + C_1)^3} \frac{\partial^3 \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}^3} - \frac{9}{(\bar{\xi} + C_1)^4} \frac{\partial^2 \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}^2} + \frac{3}{(\bar{\xi} + C_1)^5} \frac{\partial \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}} \right], \\
\bar{Q}_\theta [\bar{\theta}^*(\bar{\xi}, \bar{\tau}; \bar{k}), \bar{\phi}^*(\bar{\xi}, \bar{\tau}; \bar{k}), \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})] &= \left[\frac{\partial^2 \bar{\theta}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}^2} + \frac{1}{C_1 + \bar{\xi}} \frac{\partial \bar{\theta}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}} \right] \\
&+ Pr Du \left[\frac{\partial^2 \bar{\phi}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}^2} + \frac{1}{C_1 + \bar{\xi}} \frac{\partial \bar{\phi}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}} \right] \\
&+ Pr \left[\frac{C_1}{(C_1 + \bar{\xi})} \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k}) \frac{\partial \bar{\theta}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}} - S \frac{\partial \bar{\theta}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}} \right]
\end{aligned} \tag{33}$$

$$\begin{aligned}
\bar{Q}_\phi [\bar{\phi}^*(\bar{\xi}, \bar{\tau}; \bar{k}), \bar{\theta}^*(\bar{\xi}, \bar{\tau}; \bar{k}), \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})] &= \left[\frac{\partial^2 \bar{\phi}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}^2} + \frac{1}{C_1 + \bar{\xi}} \frac{\partial \bar{\phi}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}} \right] \\
&+ ScSr \left[\frac{\partial^2 \bar{\theta}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}^2} + \frac{1}{C_1 + \bar{\xi}} \frac{\partial \bar{\theta}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}} \right] \\
&+ Sc \left[\frac{C_1}{(C_1 + \bar{\xi})} \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k}) \frac{\partial \bar{\phi}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}} - S \frac{\partial \bar{\phi}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}} \right],
\end{aligned} \tag{34}$$

Put $\bar{k} = 0$ and $\bar{k} = 1$ in equations (28), (29), and (30), we obtain following solutions respectively

$$\bar{f}^*(\bar{\xi}, \bar{\tau}; 0) = \bar{f}_0^*(\bar{\xi}, \bar{\tau}), \quad \bar{f}^*(\bar{\xi}, \bar{\tau}; 1) = \bar{f}^*(\bar{\xi}, \bar{\tau}), \tag{35}$$

And

$$\bar{\theta}^*(\bar{\xi}, \bar{\tau}; 0) = \bar{\theta}_0^*(\bar{\xi}, \bar{\tau}), \quad \bar{\theta}^*(\bar{\xi}, \bar{\tau}; 1) = \bar{\theta}^*(\bar{\xi}, \bar{\tau}), \tag{36}$$

$$\bar{\phi}^*(\bar{\xi}, \bar{\tau}; 0) = \bar{\phi}_0^*(\bar{\xi}, \bar{\tau}), \quad \bar{\phi}^*(\bar{\xi}, \bar{\tau}; 1) = \bar{\phi}^*(\bar{\xi}, \bar{\tau}), \tag{37}$$

By employing Taylor series, we can expand $\bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})$, $\bar{\theta}^*(\bar{\xi}, \bar{\tau}; \bar{k})$ and $\bar{\phi}^*(\bar{\xi}, \bar{\tau}; \bar{k})$ as

$$\bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k}) = \bar{f}_0^*(\bar{\xi}, \bar{\tau}) + \sum_{n=1}^{\infty} \bar{f}_n^*(\bar{\xi}, \bar{\tau}) \bar{k}^n, \quad \bar{f}_n^*(\bar{\xi}, \bar{\tau}) = \frac{1}{n!} \frac{\partial^n \bar{f}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{k}^n} \Big|_{\bar{k}=0}, \tag{38}$$

$$\bar{\theta}^*(\bar{\xi}, \bar{\tau}; \bar{k}) = \bar{\theta}_0^*(\bar{\xi}, \bar{\tau}) + \sum_{n=1}^{\infty} \bar{\theta}_n^*(\bar{\xi}, \bar{\tau}) \bar{k}^n, \quad \bar{\theta}_n^*(\bar{\xi}, \bar{\tau}) = \frac{1}{n!} \frac{\partial^n \bar{\theta}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{k}^n} \Big|_{\bar{k}=0}, \tag{39}$$

$$\bar{\phi}^*(\bar{\xi}, \bar{\tau}; \bar{k}) = \bar{\phi}_0^*(\bar{\xi}, \bar{\tau}) + \sum_{n=1}^{\infty} \bar{\phi}_n^*(\bar{\xi}, \bar{\tau}) \bar{k}^n, \quad \bar{\phi}_n^*(\bar{\xi}, \bar{\tau}) = \frac{1}{n!} \frac{\partial^n \bar{\phi}^*(\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{k}^n} \Big|_{\bar{k}=0}, \tag{40}$$

Inserting equations (38), (39), and (40) in (28–30) and by equating coefficient of like powers of $\bar{k}$ leads to the BVP for $\bar{f}_n^*(\bar{\xi}, \bar{\tau})$ ($n = 0, 1, 2, \dots$), $\bar{\theta}_n^*(\bar{\xi}, \bar{\tau})$ ($n = 0, 1, 2, \dots$) and $\bar{\phi}_n^*(\bar{\xi}, \bar{\tau})$ ($n = 0, 1, 2, \dots$). Note that equations (28), (29), and (30) comprises auxiliary parameters $\bar{h}_{\bar{f}}$, $\bar{h}_{\bar{\theta}}$ and $\bar{h}_{\bar{\phi}}$. The convergence of the series representing in equations (38), (39), and (40) are highly dependent upon $\bar{h}_{\bar{f}}$, $\bar{h}_{\bar{\theta}}$ and $\bar{h}_{\bar{\phi}}$. Therefore $\bar{h}_{\bar{f}}$, $\bar{h}_{\bar{\theta}}$ and $\bar{h}_{\bar{\phi}}$ should be accurately selected so that the above series (38), (39), and (40) are convergent at $\bar{k} = 1$.

Hence using equations (35)–(37) in (38), (39), and (40), we have

$$\bar{f}^*(\bar{\xi}, \bar{\tau}; 1) = \bar{f}_0^*(\bar{\xi}, \bar{\tau}) + \sum_{n=1}^{\infty} \bar{f}_n^*(\bar{\xi}, \bar{\tau}), \tag{41}$$

$$\bar{\theta}^*(\bar{\xi}, \bar{\tau}; 1) = \bar{\theta}_0^*(\bar{\xi}, \bar{\tau}) + \sum_{n=1}^{\infty} \bar{\theta}_n^*(\bar{\xi}, \bar{\tau}), \tag{42}$$

$$\bar{\phi}^*(\bar{\xi}, \bar{\tau}; 1) = \bar{\phi}_0^*(\bar{\xi}, \bar{\tau}) + \sum_{n=1}^{\infty} \bar{\phi}_n^*(\bar{\xi}, \bar{\tau}), \tag{43}$$

Differentiate equations (28)–(31) by n -times w.r.t embedding parameters $\bar{k}$, then set $\bar{k} = 0$, and at the last divided by $n!$, we have

$$\bar{N}_{\bar{f}} [\bar{f}_n^*(\bar{\xi}, \bar{\tau}) - \bar{\delta}_n \bar{f}_{n-1}^*(\bar{\xi}, \bar{\tau})] = \bar{h}_{\bar{f}} \bar{R}_n^{\bar{f}}(\bar{\xi}, \bar{\tau}), \tag{44}$$

$$\bar{N}_{\bar{\theta}} [\bar{\theta}_n^*(\bar{\xi}, \bar{\tau}) - \bar{\delta}_n \bar{\theta}_{n-1}^*(\bar{\xi}, \bar{\tau})] = \bar{h}_{\bar{\theta}} \bar{R}_n^{\bar{\theta}}(\bar{\xi}, \bar{\tau}), \tag{45}$$

$$\bar{N}_{\bar{\phi}} [\bar{\phi}_n^*(\bar{\xi}, \bar{\tau}) - \bar{\delta}_n \bar{\phi}_{n-1}^*(\bar{\xi}, \bar{\tau})] = \bar{h}_{\bar{\phi}} \bar{R}_n^{\bar{\phi}}(\bar{\xi}, \bar{\tau}), \tag{46}$$

$$\left. \begin{aligned} \left. \begin{aligned} \frac{\partial \bar{f}^* (\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}} \bigg|_{\bar{\xi}=0} &= \bar{\theta}^* (\bar{\xi}, \bar{\tau}; \bar{k}) \big|_{\bar{\xi}=0} = \bar{f}^* (\bar{\xi}, \bar{\tau}; \bar{k}) \big|_{\bar{\xi}=0} = 0, \\ \frac{\partial^3 \bar{f}^* (\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}^3} \bigg|_{\bar{\xi}=0} &= 0, \quad \bar{\phi}^* (\bar{\xi}, \bar{\tau}; \bar{k}) \big|_{\bar{\xi}=0} = \frac{\partial^4 \bar{f}^* (\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}^4} \bigg|_{\bar{\xi}=0} = 0, \end{aligned} \right\} \\ \frac{\partial \bar{f}^* (\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}} \bigg|_{\bar{\xi}=\infty} &= \bar{\theta}^* (\bar{\xi}, \bar{\tau}; \bar{k}) \big|_{\bar{\xi}=\infty} = \frac{\partial^2 \bar{f}^* (\bar{\xi}, \bar{\tau}; \bar{k})}{\partial \bar{\xi}^2} \bigg|_{\bar{\xi}=\infty} = \bar{\phi}^* (\bar{\xi}, \bar{\tau}; \bar{k}) \big|_{\bar{\xi}=\infty} = 0. \end{aligned} \right\} \quad (47)$$

Where

$$\begin{aligned} R_n^* &= \frac{\partial^4 \bar{f}_{n-1}^*}{\partial \bar{\xi}^4} + \frac{2}{\bar{\xi} + C_1} \frac{\partial^3 \bar{f}_{n-1}^*}{\partial \bar{\xi}^3} - \frac{1}{(\bar{\xi} + C_1)^2} \frac{\partial^2 \bar{f}_{n-1}^*}{\partial \bar{\xi}^2} + \frac{1}{(\bar{\xi} + C_1)^3} \frac{\partial \bar{f}_{n-1}^*}{\partial \bar{\xi}} \\ &+ \sum_{i=0}^{n-1} \left[\frac{C_1}{\bar{\xi} + C_1} \left(\bar{f}_{n-i-1}^* \frac{\partial^3 \bar{f}_i^*}{\partial \bar{\xi}^3} - \frac{\partial \bar{f}_{n-i-1}^*}{\partial \bar{\xi}} \frac{\partial^2 \bar{f}_i^*}{\partial \bar{\xi}^2} \right) \right. \\ &\quad \left. + \frac{C_1}{(\bar{\xi} + C_1)^2} \left(\bar{f}_{n-i-1}^* \frac{\partial^2 \bar{f}_i^*}{\partial \bar{\xi}^2} - \frac{\partial \bar{f}_{n-i-1}^*}{\partial \bar{\xi}} \frac{\partial \bar{f}_i^*}{\partial \bar{\xi}} \right) - \frac{C_1}{(\bar{\xi} + C_1)^3} \bar{f}_{n-i-1}^* \frac{\partial \bar{f}_i^*}{\partial \bar{\xi}} \right] \\ &- \lambda \left[\frac{\partial^6 \bar{f}_{n-1}^*}{\partial \bar{\xi}^6} + \frac{3}{(\bar{\xi} + C_1)} \frac{\partial^5 \bar{f}_{n-1}^*}{\partial \bar{\xi}^5} - \frac{3}{(\bar{\xi} + C_1)^2} \frac{\partial^4 \bar{f}_{n-1}^*}{\partial \bar{\xi}^4} \right. \\ &\quad \left. + \frac{6}{(\bar{\xi} + C_1)^3} \frac{\partial^3 \bar{f}_{n-1}^*}{\partial \bar{\xi}^3} - \frac{9}{(\bar{\xi} + C_1)^4} \frac{\partial^2 \bar{f}_{n-1}^*}{\partial \bar{\xi}^2} + \frac{3}{(\bar{\xi} + C_1)^5} \frac{\partial \bar{f}_{n-1}^*}{\partial \bar{\xi}} \right], \end{aligned} \quad (48)$$

$$\begin{aligned} R_n^{\bar{\theta}} &= \left[\frac{\partial^2 \bar{\theta}_{n-1}^*}{\partial \bar{\xi}^2} + \frac{1}{\bar{\xi} + C_1} \frac{\partial \bar{\theta}_{n-1}^*}{\partial \bar{\xi}} \right] + Pr Du \left[\frac{\partial^2 \bar{\phi}_{n-1}^*}{\partial \bar{\xi}^2} + \frac{1}{\bar{\xi} + C_1} \frac{\partial \bar{\phi}_{n-1}^*}{\partial \bar{\xi}} \right] \\ &+ Pr \left[\frac{C_1}{\bar{\xi} + C_1} \sum_{i=0}^{n-1} \bar{f}_{n-i-1}^* \frac{\partial \bar{\theta}_i^*}{\partial \bar{\xi}} - \frac{\partial \bar{\theta}_{n-1}^*}{\partial \bar{\tau}} S \right], \end{aligned} \quad (49)$$

$$\begin{aligned} R_n^{\bar{\phi}} &= \left[\frac{\partial^2 \bar{\phi}_{n-1}^*}{\partial \bar{\xi}^2} + \frac{1}{\bar{\xi} + C_1} \frac{\partial \bar{\phi}_{n-1}^*}{\partial \bar{\xi}} \right] + Sc Sr \left[\frac{\partial^2 \bar{\theta}_{n-1}^*}{\partial \bar{\xi}^2} + \frac{1}{\bar{\xi} + C_1} \frac{\partial \bar{\theta}_{n-1}^*}{\partial \bar{\xi}} \right] \\ &+ Sc \left[\frac{C_1}{\bar{\xi} + C_1} \sum_{i=0}^{n-1} \bar{f}_{n-i-1}^* \frac{\partial \bar{\phi}_i^*}{\partial \bar{\xi}} - \frac{\partial \bar{\phi}_{n-1}^*}{\partial \bar{\tau}} S \right], \end{aligned} \quad (50)$$

And

$$\bar{\delta}_n = \begin{cases} 0 & n \leq 1, \\ 1 & n > 1. \end{cases} \quad (51)$$

The general solution is given as

$$\begin{aligned} \bar{f}_n^* (\bar{\xi}, \bar{\tau}) &= \hat{\bar{f}}_n (\bar{\xi}, \bar{\tau}) + \bar{C}_1 + \bar{C}_2 \exp(\bar{\xi}) + \bar{C}_3 \bar{\xi} \exp(\bar{\xi}) + \bar{C}_4 \exp(-\bar{\xi}) \\ &+ \bar{C}_5 \bar{\xi} \exp(-\bar{\xi}) + \bar{C}_6 \bar{\xi}^2 \exp(-\bar{\xi}), \end{aligned} \quad (52)$$

$$\bar{\theta}_n^* (\bar{\xi}, \bar{\tau}) = \hat{\bar{\theta}}_n (\bar{\xi}, \bar{\tau}) + \bar{C}_7 \exp(\bar{\xi}) + \bar{C}_8 \exp(-\bar{\xi}), \quad (53)$$

$$\bar{\phi}_n^* (\bar{\xi}, \bar{\tau}) = \hat{\bar{\phi}}_n (\bar{\xi}, \bar{\tau}) + \bar{C}_9 \exp(\bar{\xi}) + \bar{C}_{10} \exp(-\bar{\xi}), \quad (54)$$

where $\hat{\bar{f}}_n (\bar{\xi}, \bar{\tau})$, $\hat{\bar{\theta}}_n (\bar{\xi}, \bar{\tau})$ and $\hat{\bar{\phi}}_n (\bar{\xi}, \bar{\tau})$ represent the particular solution. For the boundary conditions given in equation (47), the constants $\bar{C}_i$, ($i = 1, 2, 3, 4, 5, 6$) are formulated as

$$\begin{aligned} \bar{C}_1 &= -\hat{\bar{f}}_n (0, \bar{\tau}) - \frac{\partial \hat{\bar{f}}_n (0, \bar{\tau})}{\partial \bar{\xi}}, \quad \bar{C}_4 = \frac{\partial \hat{\bar{f}}_n (0, \bar{\tau})}{\partial \bar{\xi}}, \\ \bar{C}_8 &= -\hat{\bar{\theta}}_n (0, \bar{\tau}), \quad \bar{C}_{10} = -\hat{\bar{\phi}}_n (0, \bar{\tau}), \quad \bar{C}_2 = \bar{C}_3 = \bar{C}_5 = \bar{C}_7 = \bar{C}_{10} = 0, \end{aligned} \quad (55)$$

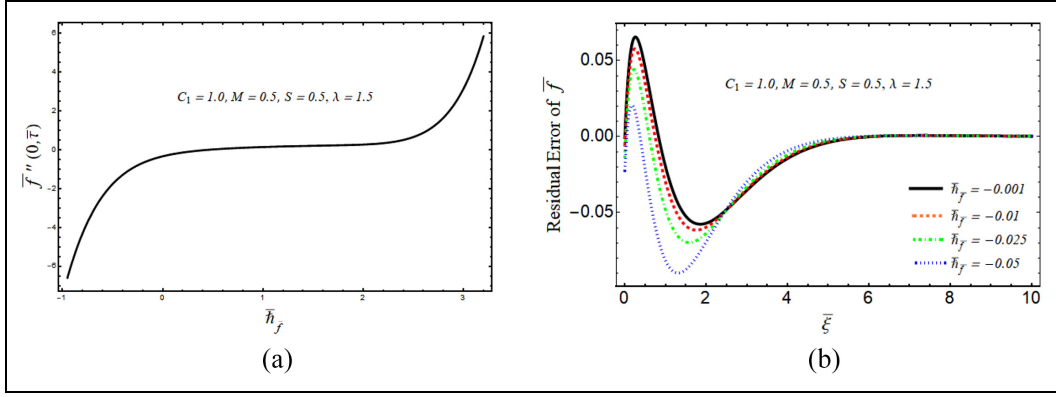


Figure 2. h-curve and residual error for velocity at seventh iteration.

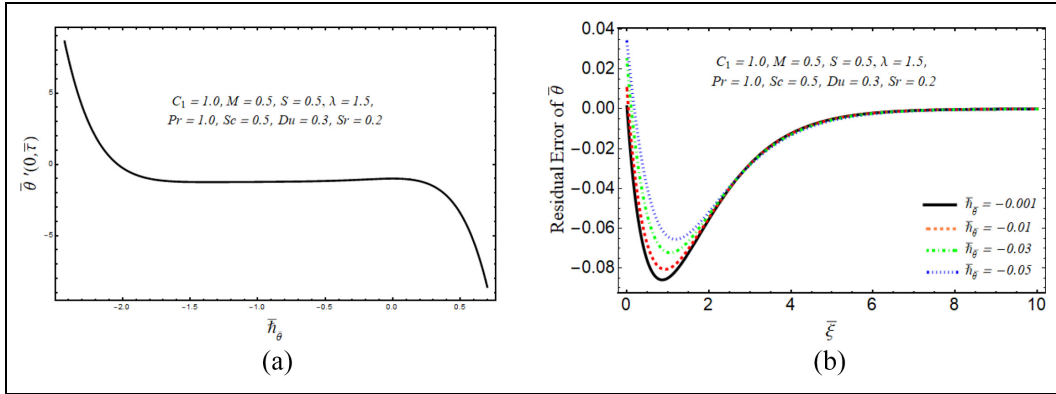


Figure 3. h-curve and residual error for temperature at seventh iteration.

To confirm the convergence of the acquired series using HAM via $\bar{h}$ – curves, the appropriate choice of the non-zero auxiliary variables $\bar{h}_f, \bar{h}_\theta$ and $\bar{h}_\phi$ are very imperative to guarantee the convergence of the solution. Figures 2(a), 3(a) and 4(a) present such curves showing the feasible region of $\bar{h}_f, \bar{h}_\theta$ and $\bar{h}_\phi$ for a given set of parameters. From these figures, it is clear that the convergent solution can be achieved for $-1.0 \leq \bar{h}_f \leq 2.0, -2.0 \leq \bar{h}_\theta \leq 0.2$ and $-2.0 \leq \bar{h}_\phi \leq 0.0$. Moreover, Figures 2(b), 3(b) and 4(b) are designs to display the residual errors of $\bar{f}, \bar{\theta}$ and $\bar{\phi}$ for different values of auxiliary parameters $\bar{h}_f, \bar{h}_\theta$ and $\bar{h}_\phi$ by keeping other parameters fixed. These figures reveals that optimal values for minimum residual errors of $\bar{f}, \bar{\theta}$ and $\bar{\phi}$ are $\bar{h}_f = -0.001, \bar{h}_\theta = -0.001$ and $\bar{h}_\phi = -0.001$ respectively.

Result and discussions

In this segment, the graphical outcomes of divergent involved flow parameters such as couple stress parameter λ , relation parameter of surface frequency oscillation parameter to its stretched constant parameter S , Soret number Sr and magnetic parameter M , Dufour number Du and Prandtl number Pr , dimensionless radius of curvature C_1 , and Schmidt number Sc on the fluid flow velocity field, the pressure field, the

concentration, and temperature distribution, the coefficient of skin friction, the local Sherwood and Nusselt and numbers are analyzed and explained in detail respectively.

Table 1 is constructed to scrutinize the validity of the present analytical outcomes with the existing published literature by taking the couple stress parameter $(\delta) = 0$ and the radius of curvature parameter $C_1 \rightarrow \infty$ at different time instants, and they are found to be in good agreement.

Figure 5(a) to (d) illustrates the three-dimensional physical behavior of the axial and the normal parts of the velocity by letting $(\bar{r} \rightarrow 0)$ and $(\bar{r} \rightarrow \infty)$.

The response of the streamlines of the axial and normal velocity parts at various time interval is demonstrated through Figure 6(a) to (c) and Figure 7(a) to (c). Figures 6(a) and 7(a) portray the symmetrical response of streamlines. While Figures 6(b), (c), 7(b) and (c) show the oscillatory behavior of streamlines.

(a) $\bar{t} \in (-\pi/2, \pi/2)$ (c) $\bar{t} \in (0, 2\pi)$ (b) $\bar{t} \in (0, 10\pi)$.

Figure 8(a) to (d) interpreted the influence of diverse variables likely C_1, S, λ and M on the profile of flow velocity $\bar{f}'(\xi, \bar{\tau})$ at a fixed location $\xi = 0.1$ from the wall of the surface for $\bar{\tau} \in (0, 10\pi)$. This figure depicts that the fluid velocity amplitude $\bar{f}'(\xi, \bar{\tau})$ diminishes for upward values of radius of curvature

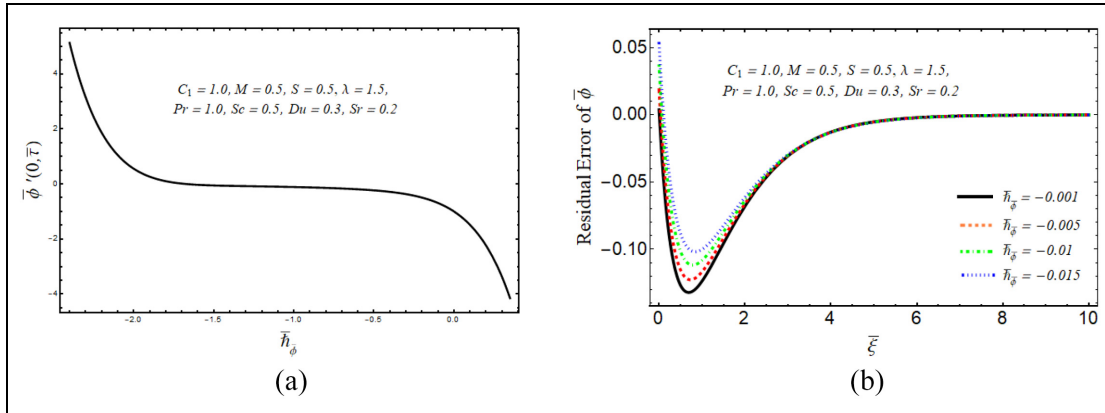


Figure 4. h-curve and residual error for concentration at seventh iteration.

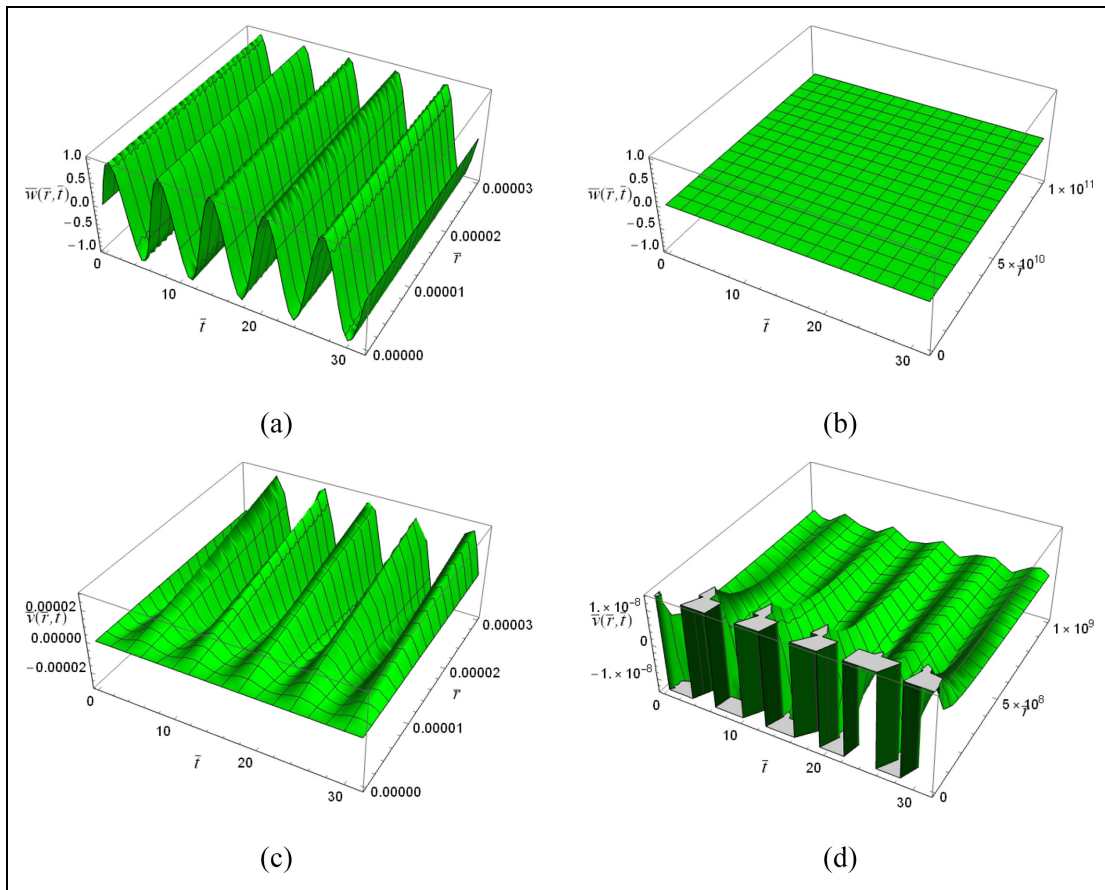


Figure 5. 3-Dimensional view of velocity parts (For $\bar{r} = 3 \times 10^{-5}$ and $\bar{r} = 1 \times 10^{11}$).

C_1 , couple stress parameter λ and M , whereas it grows with parameter S .

The influences of parameters C_1 , λ , M and S on fluid velocity description $\bar{f}'(\bar{\xi}, \bar{\tau})$ at some fixed time instant $\bar{\tau} = 0.5\pi$ are depicted through Figure 9(a) to (d) respectively. The velocity profile $\bar{f}'(\bar{\xi}, \bar{\tau})$ shows the diminishing response with parametric values of C_1 , λ , M and S .

Figure 10(a) to (d) is plotted to demonstrate the outcomes of unlike concerning parameters C_1 , λ , M and

S on pressure distribution $\bar{P}(\bar{\xi}, \bar{\tau})$ for $\bar{\tau} \in (0, 10\pi)$. It is detected that $\bar{P}(\bar{\xi}, \bar{\tau})$ amplitude is decreased by an enlarge in C_1 , λ and (S). While it enhances with magnetic parameter M .

Figure 11(a) to (d) presents the consequences of various parameters like C_1 , Du , Pr and S on $\bar{\theta}(\bar{\xi}, \bar{\tau})$ at $\bar{\tau} = 0.5\pi$. It is confirmed from Figure 11(a) that temperature profile is enhanced for varying (C_1). Figure 11(b) shows that for upward values of Du , the

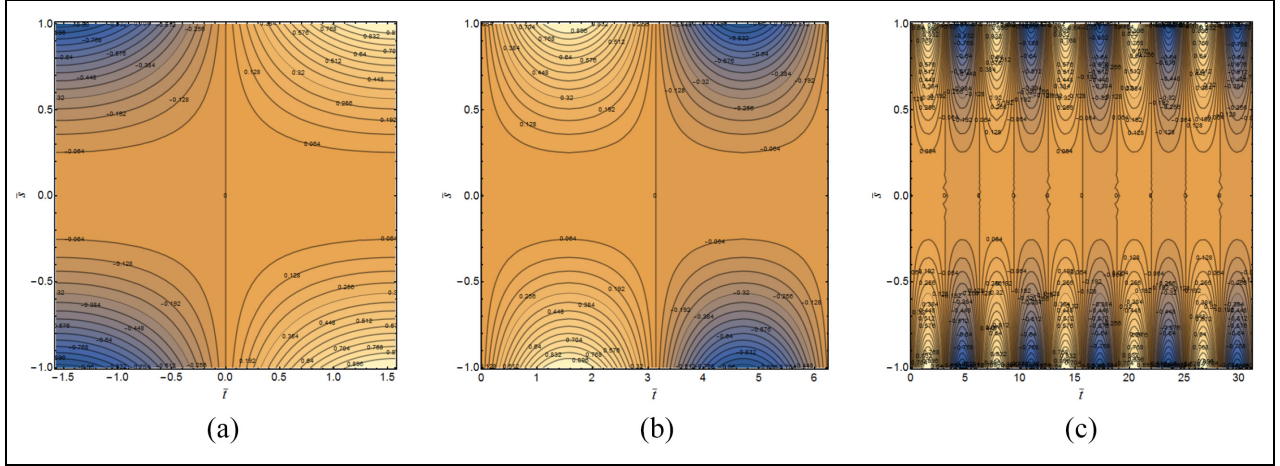


Figure 6. Alterations in streamlines of the axial velocity $\bar{w}(\bar{r}, \bar{t})$ with $C_1 = 1.0$, $\lambda = 1.0$, $M = 1.0$ and $S = 2.0$. (a) $\bar{t} \in (-\pi/2, \pi/2)$ (b) $\bar{t} \in (0, 2\pi)$ (c) $\bar{t} \in (0, 10\pi)$.

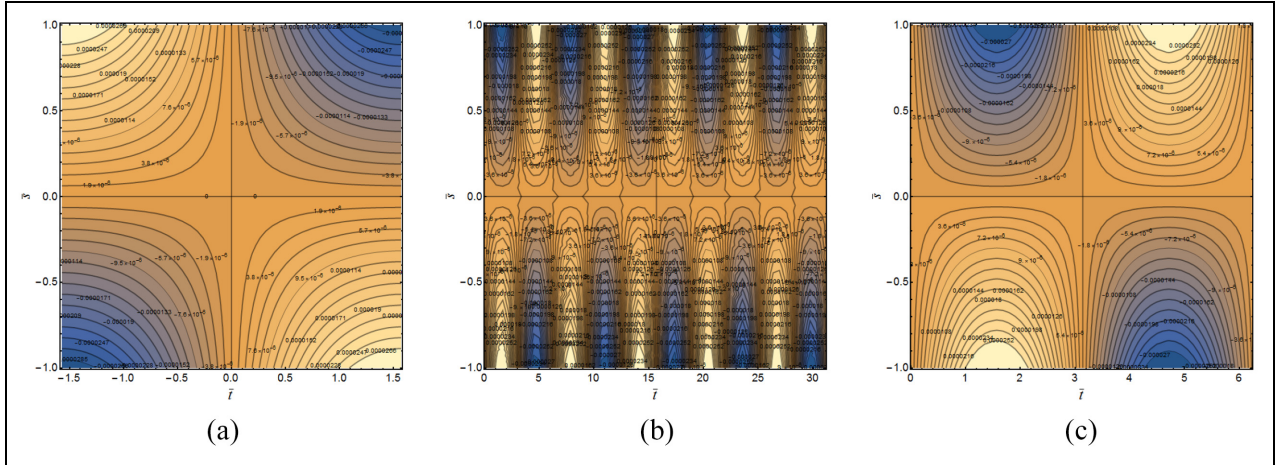


Figure 7. Variations in streamlines of normal velocity part with $\lambda = 1.0$, $C_1 = 1.0$, $S = 2.0$ and $M = 1.0$. (a) $\bar{t} \in (-\pi/2, \pi/2)$ (b) $\bar{t} \in (0, 10\pi)$ (c) $\bar{t} \in (0, 2\pi)$.

thermal boundary layer thickness and temperature field are boosted. It is due to an increase in Dufour number that causes the enhancement in energy flux because of the concentration gradient which is responsible for enlarging the temperature. While, it is diminished due to Pr and S (see Figure 11(c) and (d)). This is due to the fact that the Prandtl number and thermal diffusivity have an inverse relationship. When the Prandtl number rises, the fluid thermal diffusivity decreases, which causes the fluid temperature to decline.

The variation in concentration distribution profile $\bar{\phi}(\bar{\xi}, \bar{\tau})$ under the influences of different parameters namely C_1 , S , Sc and Sr at $\bar{\tau} = 0.5\pi$ is discussed via Figure 12(a) to (d). From Figure 12(a), (c) and (d) we have seen that the concentration profile shows growing behavior for improving values of C_1 , Sc and Sr . Large

values of Sr cause an enhancement in temperature flux, so the concentration profile enlarges. However, from Figure 12(b) it is clear that concentration profile is decreased by increasing S .

Figure 13(a) to (d) demonstrates the impacts of diverse time intervals $\bar{\tau} = 3\pi/2, \pi, \pi/2$ and $\bar{\tau} = \pi/2, \pi/3, \pi/4, \pi/6$ on profiles of velocity, temperature, and concentration respectively. Figure 13(a) elucidates that the limit of $\bar{f}'(\bar{\xi}, \bar{\tau})$ lies between $(-1$ and $1)$, this is due to the fact that the surface is considered to be oscillatory. It can be seen from Figure 13(b) to (d) that uplifting values of $\bar{\tau}$, the fluid flow velocity shows a growing response while temperature and concentration are diminished.

Figure 14(a) to (d) interpreted the influences of dissimilar parameters such as C_1 , M , λ and S on

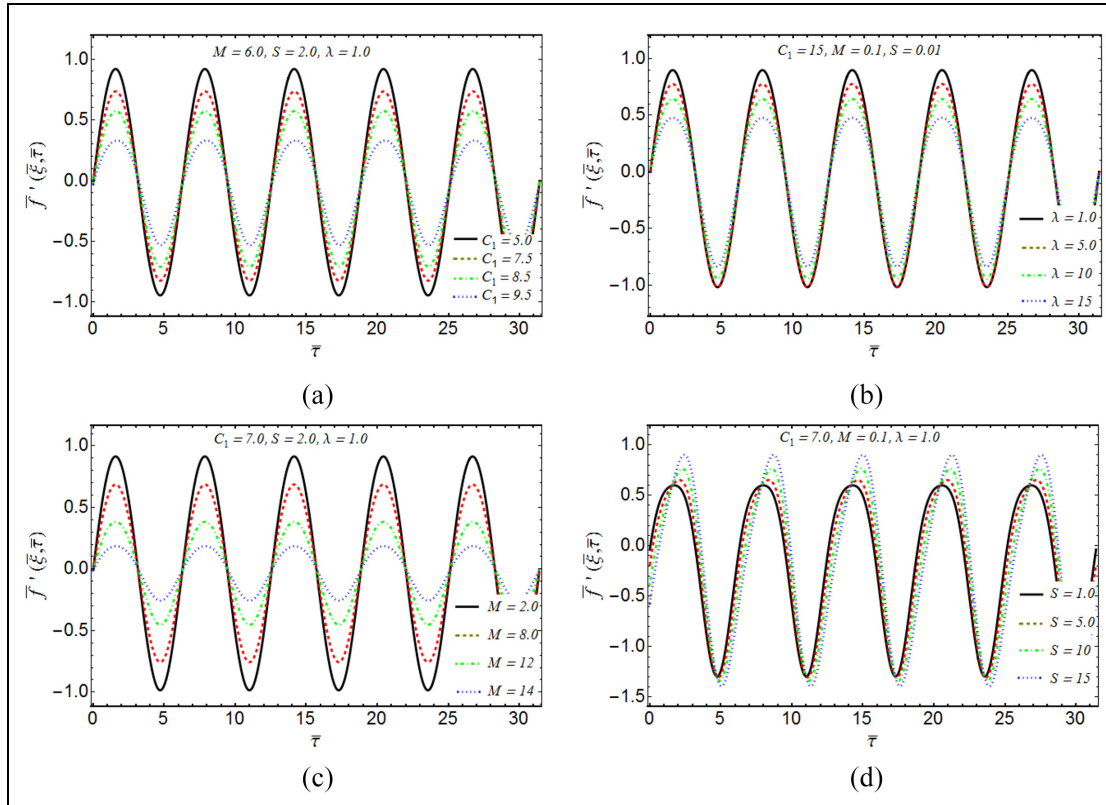


Figure 8. Deviation in velocity field $\bar{f}'(\xi, \bar{\tau})$ when $\bar{\tau} = (0, 10\pi)$.

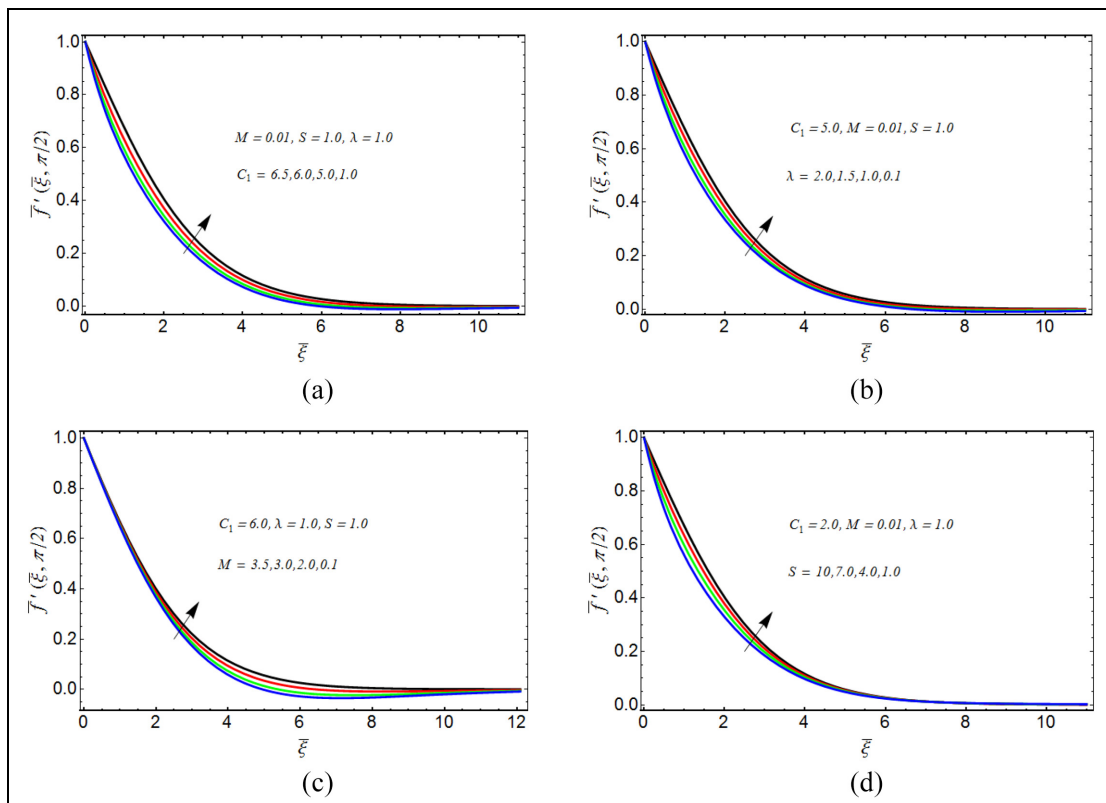


Figure 9. Deviation in velocity field $\bar{f}'(\xi, \bar{\tau})$ when $\bar{\tau} = \pi/2$.

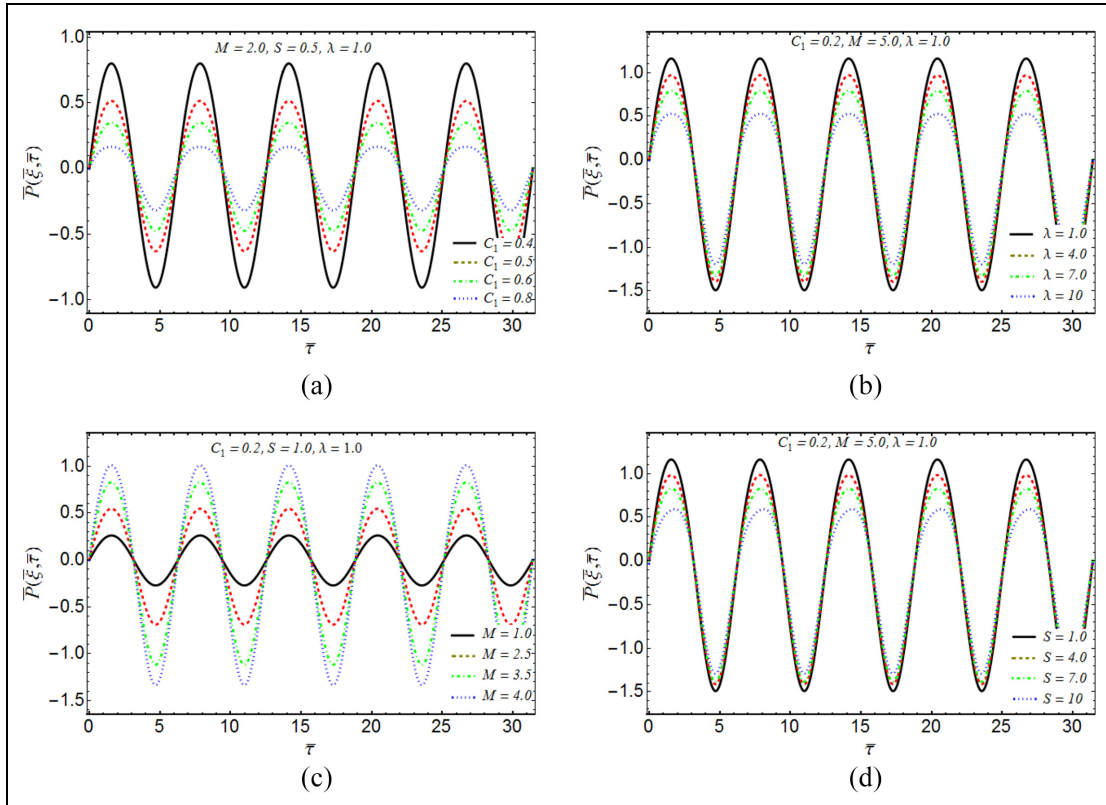


Figure 10. Alteration in pressure profile $\bar{P}(\xi, \bar{\tau})$.

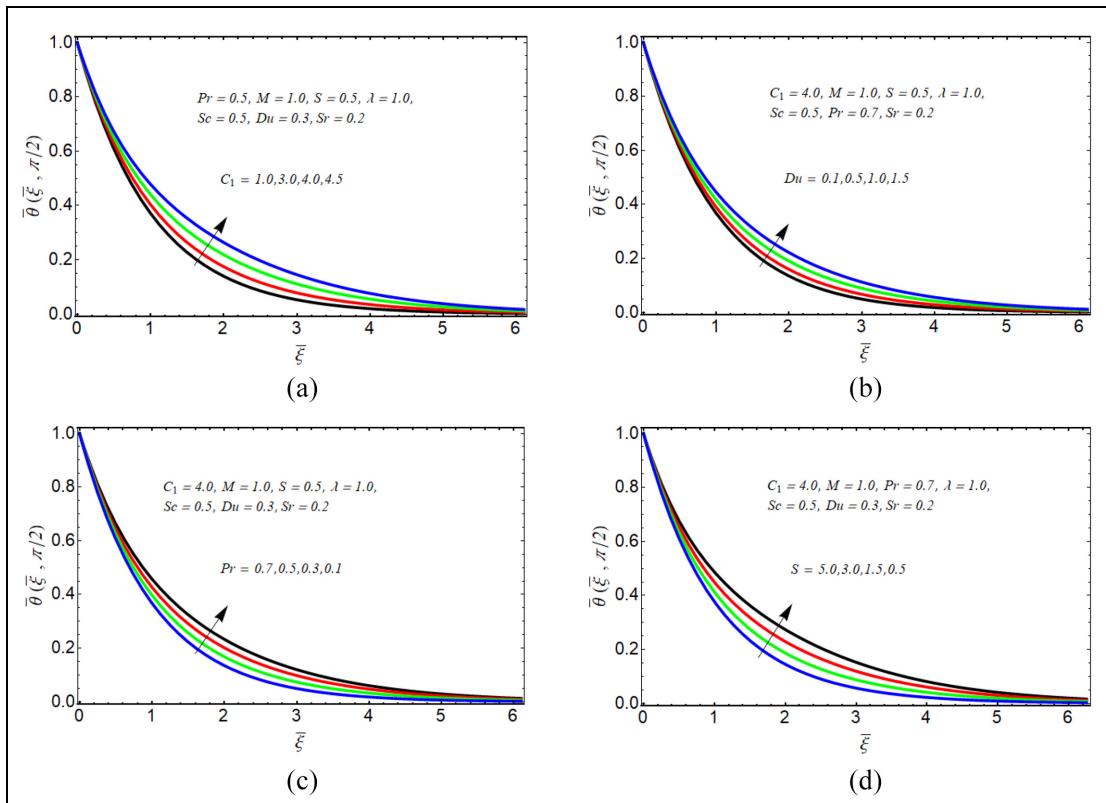


Figure 11. Deviation in temperature profile $\bar{\theta}(\xi, \bar{\tau})$.

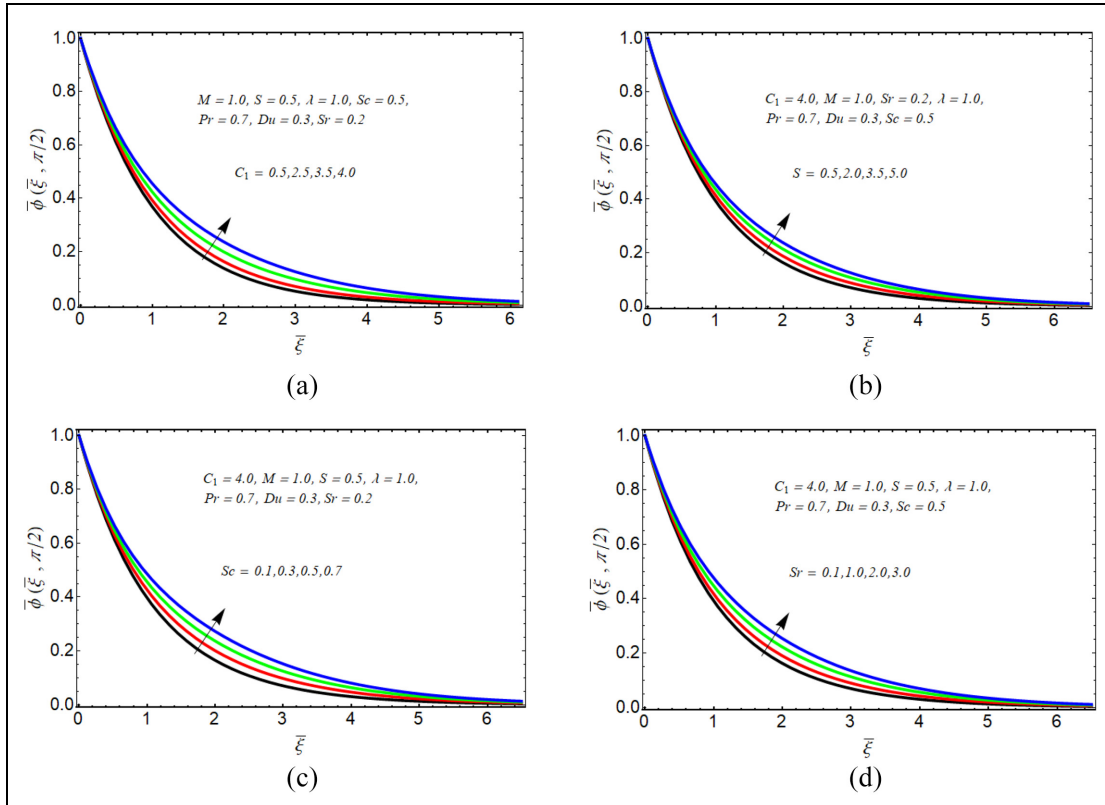


Figure 12. Deviation in concentration profile $\bar{\phi}(\bar{\xi}, \bar{\tau})$.

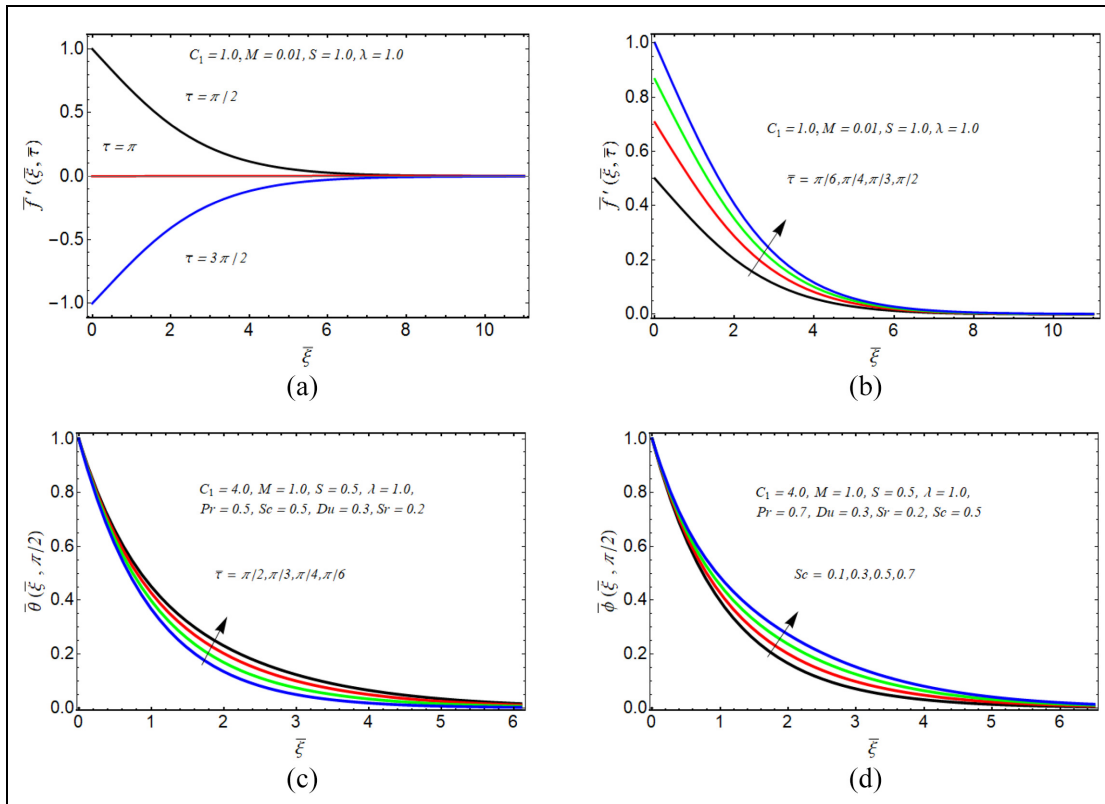


Figure 13. Influences of dissimilar values of $\bar{\tau}$ (a and b) on profile $\bar{f}'(\bar{\xi}, \bar{\tau})$ (c) on temperature profile $\bar{\theta}(\bar{\xi}, \bar{\tau})$ (d) on the fluid concentration profile $\bar{\phi}(\bar{\xi}, \bar{\tau})$.

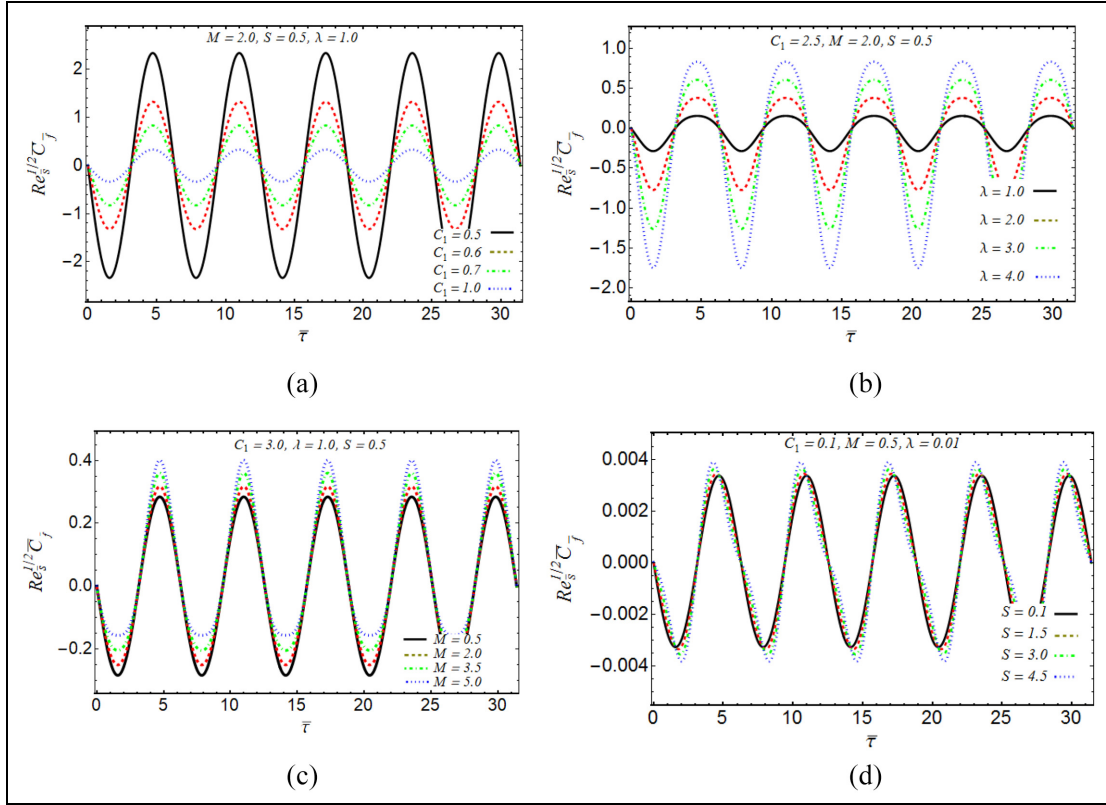


Figure 14. Change in skin friction coefficient $\left(\text{Re}_s^{1/2} \bar{C}_f\right)$.

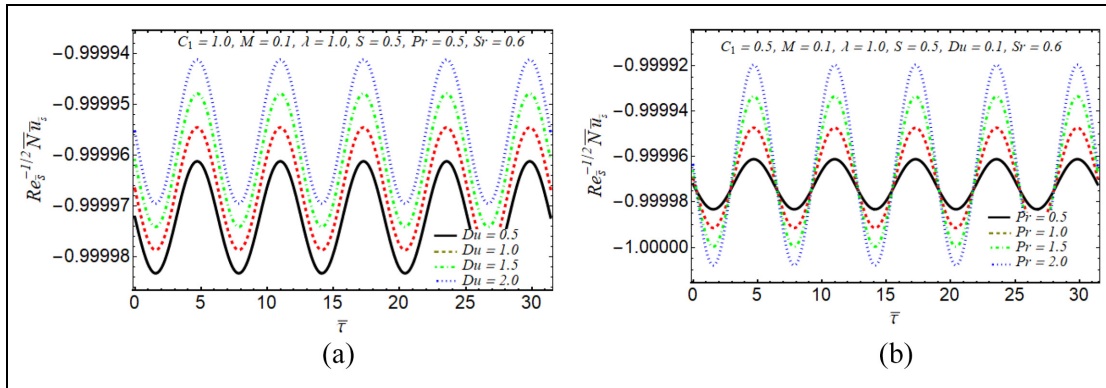


Figure 15. Variation in Nusselt number $\left(\text{Re}_s^{-1/2} \bar{N} \bar{u}_s\right)$.

$\left(\text{Re}_s^{1/2} \bar{C}_f\right)$ for $\bar{\tau} \in (0, 10\pi)$. It is clearly seen from this figure that $\left(\text{Re}_s^{1/2} \bar{C}_f\right)$ amplitude is diminished with C_1 and is enlarged with uplifting λ , S and M .

Figure 15(a) and (b) illustrates the outcomes of Dufour number Du and Prandtl number Pr for time intervals $\bar{\tau} \in (0, 10\pi)$ on $\left(\text{Re}_s^{-1/2} \bar{N} \bar{u}_s\right)$. This Figure clarifies that with improving the values of Du the amplitude of local Nusselt number is decreased and for varying Pr it is enhanced.

Figure 16(a) and (b) portrays the impacts of the Schmidt number Sc and Sr on Sherwood number $\left(\text{Re}_s^{-1/2} \bar{S} \bar{h}_s\right)$ for time $\bar{\tau} \in (0, 10\pi)$. One can be observed from this figure that the amplitude of $\left(\text{Re}_s^{-1/2} \bar{S} \bar{h}_s\right)$ growing due to Sr . Whilst, it declines for Sc .

Concluding remarks

In this investigation, the impacts of Dufour and Soret on time-dependent boundary layer motion of Couple

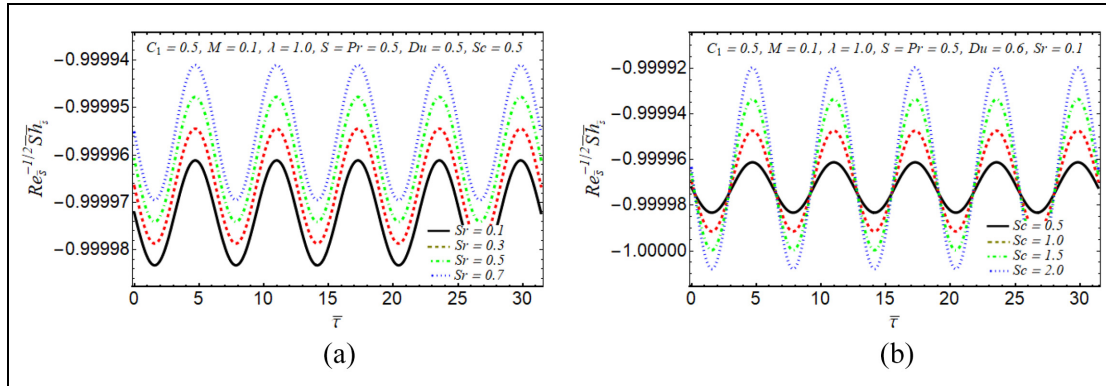


Figure 16. Alteration in Sherwood number $\left(\text{Re}_s^{-1/2} \bar{S} \bar{h}_s\right)$.

Table 1. Comparison of the numerical results of $\bar{f}''(0, \bar{\tau})$ with existing results for divergent values of $\bar{\tau}$ when $S = 1.0, M = 12, \lambda = 0.0$ and $C_1 \rightarrow \infty$.

$\bar{\tau}$	Shah et al. ¹⁶	Tarakaramu et al. ²²	Present results
$\bar{\tau} = 1.5\pi$	11.678656	11.678565	11.678656
$\bar{\tau} = 5.5\pi$	11.678707	11.678706	11.678706
$\bar{\tau} = 9.5\pi$	11.678656	11.678656	11.678656

Stress liquid across curved oscillatory stretchable surface have been analyzed. Also, the consequences of the magnetic field are taken into account in the momentum equation. Here, the key findings of the current analysis are presented as follows

- The amplitude of $\bar{f}'(\bar{\xi}, \bar{\tau})$ is declined for increasing values of C_1 , λ and M . whilst, it improved for S .
- At a time $\bar{\tau} = 0.5\pi$, for varying values of C_1 , λ , M and parameter S the fluid velocity is decreased.
- The pressure distribution amplitude is decreasing function of C_1, λ and S . However, it increases for M .
- The improvement in temperature distribution is noted with an increment in C_1 and Du . However, it shows the opposite response for Pr and S .
- For growing values of C_1 and Sr the concentration, distribution depicts growing response, while it declines for Schmidt number Sc and S .
- The surface drag force amplitude is diminished with C_1 . While for increasing values of λ , M and S the surface drag force amplitude is increased.
- The amplitude of heat transport rate coefficient declines by mounting the Du and it rises for uplifting values of Prandtl number Pr .
- The amplitude of local Sherwood number shows decreasing manner with increasing Sr and for growing Schmidt number Sc , it is amplified.

- The profile of velocity enhances with varying instants of time $\bar{\tau} = \pi/2, \pi/3, \pi/4, \pi/6$ whereas, concentration and temperature profiles show diminishing behavior.

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ORCID iDs

Muhammad Imran  <https://orcid.org/0000-0002-2760-3595>
 Muhammad Naveed  <https://orcid.org/0000-0002-9616-3087>
 Zaheer Abbas  <https://orcid.org/0000-0002-1300-7098>

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Appendix

Notation

$\bar{b}$	stretching rate (s^{-1})
B_0	strength of magnetic field
$\bar{c}_p$	specific heat capacity (J/KgK)
$\bar{c}_s$	concentration susceptibility
$\bar{C}_f$	dimensionless Coefficient of skin friction
$\bar{C}$	concentration of the fluid (mol/V^3)
C_1	dimensionless radius of curvature

$\bar{C}_w$	concentration at surface (mol/V^3)
$\bar{C}_\infty$	Ambient concentration (mol/V^3)
$\bar{D}_m$	concentration diffusion coefficient (m^2/s)
Du	non-dimensional Dufour number
$\bar{f}_\xi$	non-dimensional velocity
$\bar{k}_\alpha$	thermal conductivity ($W/m^{-2}K$)
$\bar{k}_T$	ratio of thermal diffusion
M	magnetic parameter
$\bar{N}u_s$	non-dimensional local Nusselt number
$\bar{p}$	pressure of the fluid
$\bar{P}$	dimensionless pressure
Pr	Prandtl number
$\bar{R}_1$	radius of curvature
$\bar{s}$	axial directional coordinate of the curved surface
S	Relation of oscillatory frequency of the surface to stretching rate constant
Sc	Schmidt number
Sr	Soret number
$\bar{t}$	time (s)
$\bar{T}$	temperature of the fluid (K)
$\bar{T}_m$	mean temperature of the fluid (K)

$\bar{T}_w$	temperature of the oscillatory surface (K)
$\bar{T}_\infty$	ambient fluid temperature (K)
Sc	Schmidt number
$\bar{S} \bar{h}_s$	dimensionless Sherwood number
$\bar{s}$	normal direction to the curved surface (m)
$\bar{v}$	component of velocity along radial $\bar{r}$ -direction (m/s)
$\bar{w}$	component of velocity along axial s -direction (m/s)

Greek letter

$\bar{\alpha}$	thermal diffusivity
λ	Couple stress parameter
$\bar{\rho}$	density of the fluid
$\bar{\xi}$	dimensionless variable
$\bar{\nu}$	Kinematic viscosity
$\bar{\sigma}$	electrical conductivity (Sm^{-1})
$\bar{\omega}$	angular frequency (s^{-1})
$\bar{\theta}$	dimensionless fluid temperature
$\bar{\phi}$	dimensionless fluid concentration