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Decision-making model for failure modes and effect analysis based on rough fuzzy integrated clouds

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ABSTRACT

Risk assessment is one of the key topical subjects to identify and eradicate potential failures from system, design and structure at the early stage of production process. As different kinds of uncertainties such as diversity, randomness, vagueness coexist, single uncertainty based decision models are not sufficient to evaluate risk priority of failure modes. However, the existing mathematical approaches usually utilize single models ignoring the coexistence of intrapersonal and interpersonal uncertainties in experts' judgements. The weights computed from relative importance of risk factors are essential information to capture the relation of various factors in decision system. Most of the current references do not consider the objective and subjective weights simultaneously which effects the rationality of results. In the present study, a novel linguistic assessment model , namely rough fuzzy integrated clouds (RFICs), is developed to handle uncertain information by integrating rough fuzzy numbers with cloud model theory. Then a significant decision support model for the analysis of failure modes is constructed based on RFICs, hybrid weighting method and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method. The positive and negative ideal solutions are determined using statistical deviations and arithmetic operations of clouds to rank the failure modes. The proposed model is implemented on a real-world application to detect the failures of steam valve system in a power generation plant. The simulation and sensitivity of the proposed model is also analyzed for different variations of control parameters. The proposed approach is compared with four existing approaches to show its effectiveness, out-performance and improved accuracy.

1. Introduction

FMEA (Failure modes and effects analysis) has always been an effective approach to identify the failures, causes and their effects on the performance of processing of a system or machine. It is usually evaluated by a team of cross functional decision-makers with the purpose of helping risk managers to pay attention to failure modes that should be identified for the improvement of the production process or system [1]. FMEA approaches are step by step procedures that analyze and investigate all types of bottom to top levels in the system [2]. Due to their efficiency and reliability, FMEA techniques have been widely used in various real-world problems such as health care processes [3], power system of a fishing vessel [4], and manufacturing system [5], etc.

In general, the failure modes are ranked using RPN (risk priority number) by combining (by taking product) the risk factors, detection (D), occurrence (O) and severity (S). The greater value of RPN implies the higher risk of failure mode to effect the system reliability. However, RPN method has been widely used due to its easy operations and convenience, it is still criticized by many experts due to the following deficiencies:

1. All the risk factors are given equal relative importance.
2. Different evaluation of risk factors for two different failure modes can give the same RPN, however their potential risk may not be same.
3. Exact evaluations are used by the detects to assess failure modes, while it is not easy to get crisp assessments due to uncertain human knowledge and incomplete information.
4. A slight difference of risk factor evaluations may lead to a larger difference in RPN values depending on other risk factors assessments.

To deal with deficiencies of classical RPN method, a lot of modified and improved FMEA techniques have been designed, including MCDM (multi-criteria decision making) methods, AI (artificial intelligence) approaches, MP (mathematical programming) and the hybrid integrated models. In the past, the methods

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based on fuzzy sets and their extensions are widely used by experts to handle uncertainty in decision-makers (DMs) evaluations. However, the approaches based on fuzzy numbers and fuzzy sets also meet some limitations of pre-defined membership function, large number of fuzzy rules, data distribution and considering no randomness during risk assessment. Most of the methods based on averaging operators and fuzzy sets are used to handle uncertainty and subjectivity manipulation, but much of the vagueness such as selection of a membership function, cognitive bias among DMs and determination of averaging operators is ignored. Rough set theory first studied by Pawlak [6] has been effectively and widely used in the DM (decision making) problems. The concept of a rough number is a powerful mathematical technique proposed by Song et al. [7] to identify failure modes in power generation plants using upper and lower approximation limits. Compared with existing approaches, rough numbers are objective mathematical models constructed from original data without using any pre-defined parameters. Due to the objectivity of rough numbers in information assessment, most of the other models such as fuzzy numbers, interval numbers and cloud model theory are integrated with rough numbers to handle information aggregation in DM problems. FR (fuzzy rough) numbers generated from TF (triangular fuzzy) numbers and rough approximations proposed by Zhu et al. [8] to further enhance the information evaluation in FMEA approaches. Cloud theory introduced by Li et al. [9] is another approach to study randomness in experts' evaluations based on fuzzy sets and probability theory. Rough number combined with cloud theory can effectively deal with vagueness and randomness in data to manipulate the limitations of exiting fuzzy based techniques.

1.1. Knowledge gaps and objectives of the proposed study

To enhance the characterization of uncertainty and overcome the shortcomings entailed in fuzzy based FMEA and rough FMEA approaches, this article proposes a hybrid cloud theory based RF (rough fuzzy) FMEA approach known as RF integrated cloud (RFIC) TOPSIS method. This method deals with randomness and uncertainty simultaneously for assessment aggregation in group DM problems. The main characteristics of proposed method are as follows:

1. Most of the existing FMEA approaches based on fuzzy numbers are used to deal with vagueness and uncertain evaluations of experts. But, in these methods, much of the vagueness such that the selection of pre-defined functions and the rational bias between experts is ignored. Rough number is an objective mathematical model which is computed on the given assessment data-set without any additional information, suppositions and pre-defined parameters. Due to objective benefits of rough numbers, upper and lower approximations of fuzzy numbers are computed to determine RF numbers for aggregating experts' evaluations in DM problems.
2. Rough numbers have been successfully implemented to manipulate and study interpersonal uncertainty due to its strength for information aggregation in experts' evaluations into intervals. However, it cannot deal with intrapersonal linguistic uncertainty, while the evaluations are usually given in linguistic terms. To manipulate the concepts of intrapersonal and interpersonal uncertainties, simultaneously, fuzzy numbers are integrated with upper and lower approximation limits to compute a hybrid technique called RF numbers.
3. Due to different experience and knowledge of DMs, the randomness in evaluations or judgements of experts is not considered in fuzzy based and rough number based FMEA approaches. Cloud theory based on probability theory and fuzzy sets is an AI approach to reflect both randomness and fuzziness in human knowledge concepts. It handles the issue that degrees of membership in fuzzy sets are accurate by allowing a stochastic disturbance of membership values. In the proposed approach, cloud theory is integrated with RF numbers to describe the randomness and uncertainty in the judgements of DMs.
4. A steam valve system is requisite for operating steam turbine in power plants as it is a medium of water flow with high pressure and temperature in complex working environment. Steam valve system is set to control the volume of steam in a low pressure cylinder. A huge amount of water, steam and large volume of pipelines may cause the speeding of turbine rotor in the process of load rejection. The excessive speed of rotor can cause the accident. To ensure the reliability and safety of turbine system, it is necessary for steam valve to automatically close in bad working conditions and stops the steam to enter the cylinder. the problem is to identify the risks failure modes in steam valve to prevent the accidental hazards in the turbine system. The proposed method can be effectively used to identify the most risky failure modes in turbine systems using cloud evaluations and RF numbers.

1.2. Literature review

The traditional RNP often generates unreliable and inadequate results, especially when DMs experience vague information for making decisions of real-world problems. In the past few decades, researchers have developed various methods on different theories to overcome the downside of RNP and to improve the reliability and efficiency in FMEA approaches. To handle these shortcomings, scholars use fuzzy sets and fuzzy logic based approaches to construct fuzzy rules and membership functions to handle qualitative data of experts' evaluations for risk assessment in engineering problems. Song et al. [10] combined fuzzy rules with TOPSIS method and built a weighted based fuzzy TOPSIS structure for failure modes' evaluations. Liu et al. [11] studied weighted based fuzzy VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) method to identify the risks of failures in anesthesia process. Zhao et al. [12] integrated IV (interval-valued) intuitionistic fuzzy sets with MULTIMOORA (Multi-Objective Optimization by Ratio Analysis plus the Full Multiplicative From) to handle vagueness in experts' evaluations using intervals of uncertainty of membership and non-membership values to get more efficient results in FMEA approaches and studied the proposed methodology with a DM problem. Jiang et al. [13] studied more efficient and reliable FMEA approach using Dempster Shafer evidence theory and proved the validity of the proposed structure with case study on micro-electro mechanical systems. Kerk et al. [14] proposed an inference system based on IV fuzzy rules to solve the issues of traditional RPN by transforming non-monotone and incomplete fuzzy rules into IV fuzzy rules.

Most of the fuzzy FMEA approaches are based on T1F (type-1 fuzzy) sets whose membership values range from 0 to 1, however some of the uncertainty is compromised. T2F (type-2 fuzzy) sets and interval T2F sets are used to improve the techniques of T1F sets and represent the degrees of membership that are themselves fuzzy in nature and express the uncertainty in the form of intervals [15]. Bozdag et al. studied a new FMEA method based on interval T2F sets by transforming the original assessments into interval T2F numbers. Wu et al. [16] proposed a new interval T2F TOPSIS method for large scale group DM problems and checked the reliability of the method in social networking problems. Akyuz and Celik [17] initiated an interval T2F numbers based FMEA approach and discussed its reliability in oil spill problem. All these methods based on fuzzy sets and their extensions are established using additional suppositions and redefined parameters. Moreover, in all the analysis, the randomness of the intervals is ignored which can be effectively figured out using cloud theory. Jianxing et al. [18] analyzed failure modes in submarine pipelines using extended VIKOR approach under cloud model theory. Huang and Xiao [19] proposed hybrid hierarchical TOPSIS method based on synthetic weighting by integrating interval valued IF sets with cloud theory. Gong et al. [20] studied multi object selection model by introducing risk appetite parameters, acceptance and rejection functions under cloud model theory.

Cloud model (CM) theory and rough set theory can be used to overcome the limitations entailed in fuzzy sets and their extensions. Rough numbers can be used to extract imprecise information from original data without using pre-defined functions and additional information. CM theory is used to cope with randomness, fuzziness and their association relationship to solve the problems of T1F (type-1 fuzzy) and T2F (type-2 fuzzy) sets. Rough numbers have been effectively implemented by many scholars to deal with FMEA approaches. Song et al. [7] introduced the notion of rough numbers aiming to increase the reliability of FMEA techniques and demonstrated the potential of proposed rough TOPSIS method with an application in turbine system. Wan et al. [21] improved Song's [7] rough TOPSIS method by adding a risk factor of environmental effects and studied its application in intelligent manufacturing. Wang et al. [22] presented rough numbers based VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) approach using house of reliability to check the dependency among failure modes. Lu et al. [23] studied rough (fuzzy elimination and choice translation reality) (ELECTRE) approach for supplier selection in product manufacturing. Sarwar et al. [24] proposed rough ELECTRE II approach using four risk factor for the detection of failures in automatic robots. Liao et al. [25] identified the failures of power transformer using cloud FMEA approach. Integrating rough numbers and CM theory, Li et al. [26,27] introduced rough cloud technique for order preference by similarity to ideal solution (TOPSIS) and modified rough cloud TOPSIS, and studied their applications in DM problems. Chen et al. [28] proposed RF DEMATEL decision-making trial and evaluation laboratory (DEMATEL)-TOPSIS by combining RF numbers, DEMATEL and TOPSIS approaches for supplier selection. Pamučar et al. [29] proposed interval rough (interval rough) DEMATEL (decision-making trial and evaluation laboratory) method using interval rough numbers for bidder evaluation. Chaterjee et al. [30] integrated MAIRCA (multi-attributive ideal-real comparative analysis) and DEMATEL for green supply chain management. Wang et al. [31] studied group DM problems by integrations interval rough numbers with ELECTRE III approach. Ali et al. [32,33] established two novel MCDM models, namely, q-rung orthopair fuzzy bipolar soft sets and fuzzy bipolar soft expert sets, and explored their different daily-life applications. Deveci et al. [34] proposed interval type-2 HF sets and applied on WASPAS (weighted aggregated sum product assessment) approach for aircraft type selection. Zhu et al. [35] applied FR numbers to analytical hierarchy process (AHP)-VIKOR approach for risk assessment of the check valves.

Besides all the benefits of rough numbers and interval rough numbers, these models cannot deal with intrapersonal linguistic uncertainty, while the evaluations are given in the form of linguistic terms. Chen et al. [36] introduced RF numbers and proposed RF DEMATEL-ANP (analytical network process) method to study subjective, objective and intrapersonal uncertainties at the same time. Mei and Chen [37] proposed RF BWM-DEA (best worst method–data envelopment analysis) approach and proved its significance in engineering problems. Zhu et al. [8] introduced FR numbers and proposed an FMEA approach for design concept evaluation. Pamučar et al. [38] introduced IVFR numbers based FMEA approach as a best worst method. Jia et al. [39] used intuitionistic FR numbers to solve MCDM problems. Huang et al. [40] introduced CR (cloud rough) numbers to study engineering characterizes of complex products using TOPSIS approach. Xiao et al. [41] introduced IR integrated clouds by computing cloud values from interval rough numbers and implemented this approach on TOPSIS method. Huang et al. [42] discussed remanufacturing tools for machines based dual IR integrated cloud model as hybrid approach of interval numbers, rough numbers and interval clouds. The researchers have integrated TOPSIS method with different models to study vagueness in linguistic data such as: m–polar fuzzy hesitant TOPSIS method [43], Pythagorean fuzzy TOPSIS [44], rough D-TOPSIS approach [45], extended cubic intuitionistic fuzzy TOPSIS [46], q-rung orthopair fuzzy TOPSIS [47], hesitant fuzzy ELECTRE II (ELimination Et Choice Translating REality II), decision making under granular uncertainty [48], fuzzy rough attribute reduction [49], joint replacement and production control [50], hesitant fuzzy ELECTRE II [51], FMEA with type-2 fuzzy sets [52], concept selection with ANP-PROMETHEE II [53]. We have extended the FMEA approaches based on modified TOPSIS method by integrating RF numbers with CM theory and introduced RFIC TOPSIS approach.

1.3. Innovative contributions

The proposed approach is based on the integration of different theories to deal with subjective and objective uncertainties, and randomness in experts' judgements. The main contributions of the present approach are described as follows:

1. The hybridization of RF numbers and CM theory is creatively proposed to obtain a novel RFIC model to effectively upgrade the performance of existing RF numbers and CM theory. The new RFIC model overcomes the limitations of CM theory in dealing with intrapersonal and interpersonal uncertainties and the classical calculation method of cloud values. It also improves the performance of RF numbers in coping and reflecting with randomness in initial assessment information of experts.
2. The theory of TF (triangular fuzzy) numbers, rough approximation and cloud evaluations is combined with modified TOPSIS approach to identify the risks of failure modes in engineering problems. To discuss uncertainty in experts' assessments, the linguistic information is converted into RF numbers to handle both objective and subjective uncertainties. The randomness in the data is then studied by computing cloud evaluations using statistical dispersion and mathematical expectation.
3. Most of the existing FMEA approaches are dependent on the choice of experts. In RFIC modified TOPSIS approach, the dependency is minimized by computing the weights from given set of data. The objective, subjective and their combined weights are computed using cloud distances and statistical variance. A standard modified iterative procedure is used to recognize the risks of all failure modes.
4. The significance and importance of the proposed approach is studied with a case study of failure modes in automatic turbine system. The proposed method is then compared with existing approaches to show its reliability and effectiveness.

Table 1
Set of notations used in this research paper.

Abbreviation	Description
RF	Rough fuzzy
CM	Cloud model
RFIC	Rough fuzzy integrated cloud
FR	Fuzzy rough
TF	Triangular fuzzy
DM	Decision maker\Decision-making
SV	Steam valve
IF	Intuitionistic fuzzy
HF	Hesitant fuzzy

1.4. Framework of the paper

The paper is organized as follows:

1. The preliminaries and basic concepts of rough set theory, rough numbers, RF numbers and CM theory are described in Section 2.
2. In Section 3, the calculation method of novel RFIC model is illustrated in detail.
3. In Section 4, the proposed RFIC TOPSIS approach is developed by integrating RFIC model with TOPSIS method. The objective and subjective weights of cloud evaluations are computed for determining the PI (positive ideal) solution, NI (negative ideal) solution and closeness degree of each failure mode from the risk factors.
4. In Section 5, the proposed method is then applied to a problem of failures in steam valve system to detect the possible failures that can reflect the working of automatic turbine system. In Section 5.1, an sensitivity analysis is given to demonstrate the effects of ranking results for different variations of parameters.
5. In Section 6, a theoretical and numerical comparison analysis of proposed RFIC FMEA approach with rough FMEA, conventional FMEA, fuzzy FMEA and rough integrated cloud FMEA is given to show the effectiveness and feasibility of the proposed method. The weighting schemes of proposed method and other existing approaches are also compared using interval numbers. A simulation analysis based on result deviation method in Section 6.2 is also given. The merits, limitations, theoretical significance and practical implications of the proposed study are illustrated in Section 6.3
6. Section 7 contains conclusion, detailed summary of the paper and future directions.

The list of abbreviations used in this research paper are explained in Table 1.

2. Preliminaries

In this section, the basic concepts, including TF numbers, RF numbers and CM theory are elaborated in detail.

2.1. Rough sets integrated with fuzzy numbers

Zadeh [54] introduced the concept of fuzzy set to simulate vagueness and imprecision of human knowledge. A fuzzy set indicates that each object has certain degree of belongingness in the fuzzy set with respect to a given characteristics. A TF number is basically a fuzzy set on the real line, denoted by the triplet $\widetilde{\phi} = (\phi^l, \phi^m, \phi^u)$, $\phi^l < \phi^m < \phi^u \in \mathbb{R}$, with membership function as defined in Eq. (1).

$$\widetilde{\phi}(z) = \begin{cases} \dfrac{z-\phi^l}{\phi^m-\phi^l} & , \ z \in [\phi^l, \phi^m] \\ \dfrac{\phi^u - z}{\phi^u-\phi^m} & , \ z \in [\phi^m, \phi^u] \\ 0 & , \ \text{otherwise} \end{cases} \tag{1}$$

Rough set [6] is nowadays a powerful and mostly used mathematical tool to handle uncertain information in group DM problems. TF numbers combined with rough sets [36] to deal with intrapersonal and interpersonal imprecision or uncertainties simultaneously. Let J be the judgement set containing TF numbers provided by s DMs, that is, $J = \{\varphi^k = (\varphi^{kl}, \varphi^{km}, \varphi^{ku}) \mid k = 1, 2, \ldots, s\}$. The lower and upper approximations of the TF numbers can be computed in Eqs. (2) and (3).

$$\underline{apr}(\varphi^k) = \cup \{\varphi \in J \mid \varphi \leq \varphi^k\} \tag{2}$$

$$\overline{apr}(\varphi^k) = \cup \{\varphi \in J \mid \varphi \geq \varphi^k\} \tag{3}$$

The lower and upper limits $\underline{\varphi}^k$ and $\overline{\varphi}^k$ of φ^k are defined in Eqs. (4) and (5).

$$\underline{\varphi}^k = (\underline{\varphi}^{kl}, \underline{\varphi}^{km}, \underline{\varphi}^{ku}) = \frac{1}{n^{kL}} \sum_{\varphi \in \underline{apr}(\varphi^k)} (\varphi^l, \varphi^m, \varphi^u) \tag{4}$$

$$\overline{\varphi}^k = (\overline{\varphi}^{kl}, \overline{\varphi}^{km}, \overline{\varphi}^{ku}) = \frac{1}{n^{kU}} \sum_{\vartheta \in \overline{apr}(\varphi^k)} (\vartheta^l, \vartheta^m, \vartheta^u) \tag{5}$$

where φ^u, φ^m and φ^l are the elements of lower approximation of upper, medium and lower boundaries of TF number φ^k. ϑ^u, ϑ^m and ϑ^l are the elements of upper approximation of upper, medium and lower boundaries of TF number φ^k. n^{kU} and n^{kL} are the number of elements in the upper and lower approximation of TF number φ^k, respectively. The RF number of TF number φ^k is an RF interval $RF(\varphi^k)$ of φ^k described in Eq. (6).

$$RF(\varphi^k) = [\underline{\varphi}^k, \overline{\varphi}^k] = \left[(\underline{\varphi}^{kl}, \underline{\varphi}^{km}, \underline{\varphi}^{ku}), (\overline{\varphi}^{kl}, \overline{\varphi}^{km}, \overline{\varphi}^{ku})\right]. \tag{6}$$

where $\overline{\varphi}^k$ and $\underline{\varphi}^k$ are the upper and lower limits of $RF(\tilde{a}_i)$. $\overline{\varphi}^{kl}$ and $\underline{\varphi}^{kl}$ are the upper and lower limits of rough number $RN(a_i^l) = [\underline{\varphi}^{kl}, \overline{\varphi}^{kl}]$; $\overline{\varphi}^{km}$ and $\underline{\varphi}^{km}$ are the upper and lower limits of rough number $RN(a_i^m) = [\underline{\varphi}^{km}, \overline{\varphi}^{km}]$; $\overline{\varphi}^{ku}$ and $\underline{\varphi}^{ku}$ are the upper and lower limits of rough number $RN(a_i^u) = [\underline{\varphi}^{ku}, \overline{\varphi}^{ku}]$.

2.2. Cloud model theory

Cloud model CM theory [9] based on probability theory and fuzzy sets is a knowledge engineering approach to deal with vagueness like randomness and fuzziness of human concepts. In a fuzzy set, each object is given a membership value to represent its degree of uncertainty. However, it is against the spirit of fuzzy set theory, where the uncertainty of an object in a given fuzzy set is precise and certain. CM theory deals with this issue allowing a stochastic disturbance of membership degree around a determined central value.

Assume that Q is a qualitative concept defined over the universe Y and $z \in Y$. $\mu_Q(z) \in [0, 1]$ is the membership degree of z which is a random number rather than a fixed number and follows a probability distribution. The distribution on z in Y is called a *cloud* and z is known as *cloud drop*. It is clear that the hybrid RF set theory can deal with uncertainty and ambiguities in experts' judgements but it cannot handle the randomness in fuzzy values and evaluations of DMs. CM theory can efficiently deal with the randomness in judgement intervals. Generally, a cloud is represented as (Ex, En, HEn), where

(1) The expectation Ex represents the qualitative concept of the element in the domain.

Table 2
Qualitative meanings of TF number scales.

Linguistic variable	Abbreviation	Ranking	TF number
Extremely low	EL	1	(0, 0, 1)
Strongly low	SL	2	(0, 1, 2)
Very low	VL	3	(1, 2, 3)
Low	L	4	(2, 3, 4)
Somewhat moderate	SM	5	(3, 4, 5)
Moderate	M	6	(4, 5, 6)
Somewhat high	SH	7	(5, 6, 7)
High	H	8	(6, 7, 8)
Very high	VH	9	(7, 8, 9)
Extremely high	EH	10	(8, 9, 1)

(2) The entropy En describes the randomness of the concept. It measures the range and dispersion extent accepted by that qualitative concept.

(3) The hyper entropy HEn, entropy of En, shows the uncertainty of the membership degrees.

Let $C = (Ex, En, HEn)$ and $D = (Ex', En', HEn')$ be two clouds, then the addition, multiplication, division and distance between them are defined as:

1. $C + D = \left(Ex + Ex', \sqrt{En^2 + En'^2}, \sqrt{HEn^2 + HEn'^2}\right)$
2. $C \times D = \left(Ex \times Ex', |ExEx'| \times \sqrt{\left(\frac{En}{Ex}\right)^2 + \left(\frac{En'}{Ex'}\right)^2}, |\frac{Ex}{Ex'}| \times\sqrt{\left(\frac{HEn}{Ex}\right)^2 + \left(\frac{HEn'}{Ex'}\right)^2}\right)$
3. $\frac{C}{D} = \left(\frac{Ex}{Ex'}, |\frac{Ex}{Ex'}| \times \sqrt{\left(\frac{En}{Ex}\right)^2 + \left(\frac{En'}{Ex'}\right)^2}, |\frac{Ex}{Ex'}| \times \sqrt{\left(\frac{HEn}{Ex}\right)^2 + \left(\frac{HEn'}{Ex'}\right)^2}\right)$
4. $\alpha C = (\alpha Ex, \sqrt{\alpha} En, \sqrt{\alpha} HEn)$
5. $d(C, D) = \left|\left(1 - \frac{En + HEn}{Ex}\right) Ex - \left(1 - \frac{En' + HEn'}{Ex'}\right) Ex'\right|.$

3. The construction of RFICs

This section integrates RF numbers with CM theory to develop a novel manipulation model, that is, rough fuzzy integrated clouds abbreviated as RFICs. The conversion from qualitative data into quantitative values is important in the risk assessment process. The linguistic data is first converted into RF numbers and then cloud values. Suppose $L = \{\alpha_k \mid 1 \le k \le s\}$ is the linguistic data given by s experts. If $l_\alpha = l_\beta$, then the linguistic value $[l_\alpha, l_\beta]_k$ can be written as $l_{\alpha k}$. The linguistic values can be transformed into RFICs on the basis of following steps:

1. Convert the qualitative linguistic data l_k, $1 \le k \le s$ into TF numbers $\varphi^k = (\varphi^{kl}, \varphi^{km}, \varphi^{ku})$, $1 \le k \le s$ using TF number scale evaluations given in Table 2.
2. Compute the RF numbers $\left[\underline{\varphi}^k, \overline{\varphi}^k\right] = \left[(\underline{\varphi}^{kl}, \underline{\varphi}^{km}, \underline{\varphi}^{ku}), (\overline{\varphi}^{kl}, \overline{\varphi}^{km}, \overline{\varphi}^{ku})\right]$, $1 \le k \le s$ of TF numbers using Section 2.1, where $\overline{\varphi}^k = (\overline{\varphi}^{kl}, \overline{\varphi}^{km}, \overline{\varphi}^{ku})$ and $\underline{\varphi}^k = (\underline{\varphi}^{kl}, \underline{\varphi}^{km}, \underline{\varphi}^{ku})$ are the upper and lower limits of RF number $\left[\underline{\varphi}^k, \overline{\varphi}^k\right]$. It is clear to check that the upper and lower limits of RF number are TF numbers.
3. Convert each RF number evaluations $\left[(\underline{\varphi}^{kl}, \underline{\varphi}^{km}, \underline{\varphi}^{ku}), (\overline{\varphi}^{kl}, \overline{\varphi}^{km}, \overline{\varphi}^{ku})\right]$ into cloud values $C_k = (Ex_k, En_k, HEn_k)$, where the expectation Ex_k, entropy En_k and hyper entropy HEn_k terms of the cloud (Ex_k, En_k, HEn_k) are illustrated in Equations (7)–(9).

$$Ex_{ji}^k = \frac{1}{6}\left(\underline{\varphi}_{ji}^{kl} + \overline{\varphi}_{ji}^{kl} + \underline{\varphi}_{ji}^{km} + \overline{\varphi}_{ji}^{km} + \underline{\varphi}_{ji}^{ku} + \overline{\varphi}_{ji}^{ku}\right), \tag{7}$$

$$En_{ji}^k = \frac{1}{18}\left(\overline{\varphi}_{ji}^{kl} - \underline{\varphi}_{ji}^{kl} + \overline{\varphi}_{ji}^{km} - \underline{\varphi}_{ji}^{km} + \overline{\varphi}_{ji}^{ku} - \underline{\varphi}_{ji}^{ku}\right), \tag{8}$$

$$HEn_{ji}^k = \mu. \tag{9}$$

μ is a constant to be computed for the degree of uncertainty. Hyper entropy HEn represents the uncertainty of entropy En, computed from En. According to the theory presented by Zhang et al. [55], the cloud droplet $C = (Ex, En, HEn)$ follows a non-strict normal distribution if $3HEn < En$, otherwise it is fogged if $3HEn > En$. Hence, for any cloud C_k, HE_k can be computed using the relation $3HEn < En$.

4. The proposed RFIC TOPSIS approach

In this section, an extended TOPSIS approach based on RF numbers and CM theory is proposed to obtain the assessment of risk factors and their effects analysis. RF set theory can efficiently reflect the interpersonal and intrapersonal uncertainties, knowledge limitations and experts' vagueness in decision-making processes using fuzziness and approximation techniques simultaneously. CM theory can handle randomness and uncertainty at the same time in decision-makers' judgement intervals. Thus, the proposed method investigates both types of uncertainties in the risk evaluation process. Additionally, both objective and subjective weights are computed to perform an effective and comprehensive assessment of failure modes and their effects. The framework of proposed RFIC FMEA approach depicted in Fig. 1 includes three stages:

(1) Assess the performance of failures and importance of risk factors using CM theory integrated with RF numbers to deal with vagueness in the judgement of experts.

(2) Considering subjective and objective aspects, compute the weights of risk factors.

(3) Apply the RFIC TOPSIS approach to rank the failure modes.

4.1. Assessment of failure modes and risk factors

Determine the evaluations of failure modes corresponding to the judgement of each expert using qualitative values and TF numbers. Convert the given data into RF number clouds and aggregate these values from different decision-makers.

4.1.1. Construct the initial fuzzy relation matrix based on TF numbers

In the past, the decision-making processes were based on the experience of a single person. Due to the increased complexity of decision-making methods and the knowledge limitation of one person, a team of experts is needed to obtain the accuracy in the process. The experts give importance to failure modes against the risk factors using linguistic variables. The linguistic relative importance of risk factors provided by the decision-makers is illustrated in Eq. (10).

$$\lambda_j = \left(\lambda_j^1, \lambda_j^2, \ldots, \lambda_j^k, \ldots, \lambda_j^s\right) \tag{10}$$

where λ_j is the linguistic importance of jth failure mode given by s experts. λ_j^k is the linguistic importance of jth failure mode provided by kth expert.

If there are r failure modes, then the linguistic relation matrix given by kth expert is given as a matrix in Eq. (11).

$$.L^k = \begin{bmatrix} \lambda_{11}^k & \lambda_{12}^k & \ldots & \lambda_{1m}^k \\ \lambda_{21}^k & \lambda_{22}^k & \ldots & \lambda_{2m}^k \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{r1}^k & \lambda_{r2}^k & \ldots & \lambda_{rm}^k \end{bmatrix} \tag{11}$$

for $k = 1, 2, \ldots, s$. Here λ_{ji}^k is the linguistic evaluation of kth expert for jth failure mode under ith risk factor. Convert the

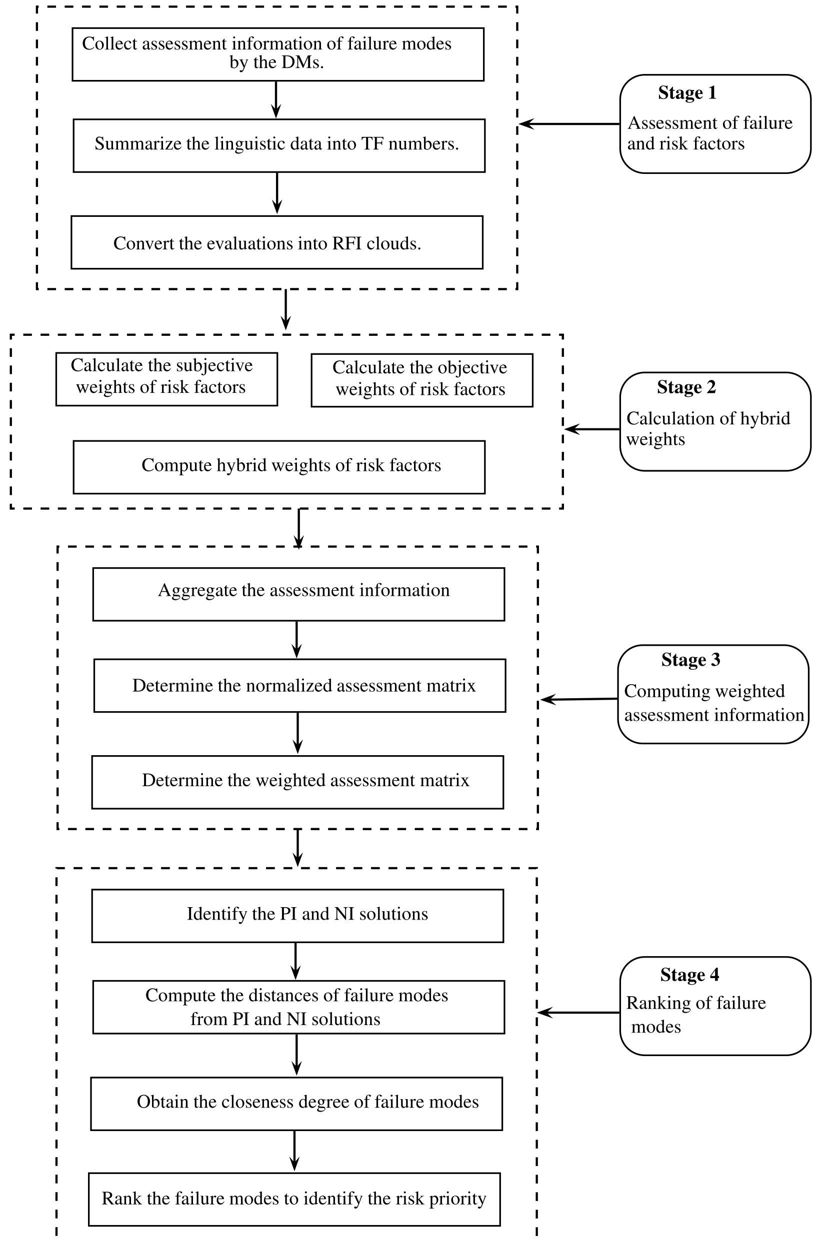

Fig. 1. Process diagram for the proposed RFIC TOPSIS approach.

linguistic relation matrix into TF relation matrix using the TF number scale evaluations given in Table 2. Using the TF number scales of linguistic values, the TF relation matrix obtained using scales of Table 2 is presented as a matrix in Eq. (12).

$$TF^k = \begin{bmatrix} (\xi_{11}^{kl}, \xi_{11}^{km}, \xi_{11}^{ku}) & (\xi_{12}^{kl}, \xi_{12}^{km}, \xi_{12}^{ku}) & \dots & (\xi_{1m}^{kl}, \xi_{1m}^{km}, \xi_{1m}^{ku}) \\ (\xi_{21}^{kl}, \xi_{21}^{km}, \xi_{21}^{ku}) & (\xi_{22}^{kl}, \xi_{22}^{km}, \xi_{22}^{ku}) & \dots & (\xi_{2m}^{kl}, \xi_{2m}^{km}, \xi_{2m}^{ku}) \\ \vdots & \vdots & \ddots & \vdots \\ (\xi_{r1}^{kl}, \xi_{r1}^{km}, \xi_{r1}^{ku}) & (\xi_{r2}^{kl}, \xi_{r2}^{km}, \xi_{r2}^{ku}) & \dots & (\xi_{rm}^{kl}, \xi_{rm}^{km}, \xi_{rm}^{ku}) \end{bmatrix} \tag{12}$$

where $TF(\lambda_{ji}^k) = \varphi_{ji}^k = (\xi_{ji}^{kl}, \xi_{ji}^{km}, \xi_{ji}^{ku})$ represents the TF number evaluation of jth failure mode under ith risk component given by kth expert, for each $1 \leq j \leq r$, $1 \leq i \leq m$, $1 \leq k \leq s$. The importance of risk factors in linguistic terms using Table 2 is given as a vector in Eq. (13).

$$\varphi_i = \left(\varphi_i^1, \varphi_i^2, \dots, \varphi_i^k, \dots, \varphi_i^s\right) \tag{13}$$

where $\varphi_i^k = TF(\lambda_i^k) = (\varphi_i^{kl}, \varphi_i^{km}, \varphi_i^{ku})$ is the TF number evaluation of ith risk factor given by kth DM, for each $1 \leq i \leq m$, $1 \leq k \leq s$.

4.1.2. Obtain RF direct relation matrix by converting the TF number evaluations into RF numbers

The evaluations based on experts' knowledge and experience contain uncertain and imprecise information as all the experts may have different thinking and experiences. Rough numbers and RF numbers have been successfully utilized to study uncertainty and vagueness in DM problems. Chen et al. [36] computed lower and upper approximations of TF numbers to compute RF numbers to discuss interpersonal and intrapersonal vagueness. Let J be the judgement set containing TF numbers provided by s DMs, that is, $J = \left\{\varphi_{ji}^k = (\varphi_{ji}^{kl}, \varphi_{ji}^{km}, \varphi_{ji}^{ku}) \mid k = 1, 2, \dots, s\right\}$. The lower and upper approximations of the TF numbers can be computed via Eqs. (14) and (15).

$$RF^k = \begin{bmatrix} \left[(\underline{\varphi}_{11}^{kl}, \underline{\varphi}_{11}^{km}, \underline{\varphi}_{11}^{ku}), (\overline{\varphi}_{11}^{kl}, \overline{\varphi}_{11}^{km}, \overline{\varphi}_{11}^{ku})\right] & \left[(\underline{\varphi}_{12}^{kl}, \underline{\varphi}_{12}^{km}, \underline{\varphi}_{12}^{ku}), (\overline{\varphi}_{12}^{kl}, \overline{\varphi}_{12}^{km}, \overline{\varphi}_{12}^{ku})\right] & \cdots & \left[(\underline{\varphi}_{1m}^{kl}, \underline{\varphi}_{1m}^{km}, \underline{\varphi}_{1m}^{ku}), (\overline{\varphi}_{1m}^{kl}, \overline{\varphi}_{1m}^{km}, \overline{\varphi}_{1m}^{ku})\right] \\ \left[(\underline{\varphi}_{21}^{kl}, \underline{\varphi}_{21}^{km}, \underline{\varphi}_{21}^{ku}), (\overline{\varphi}_{21}^{kl}, \overline{\varphi}_{21}^{km}, \overline{\varphi}_{21}^{ku})\right] & \left[(\underline{\varphi}_{22}^{kl}, \underline{\varphi}_{22}^{km}, \underline{\varphi}_{22}^{ku}), (\overline{\varphi}_{22}^{kl}, \overline{\varphi}_{22}^{km}, \overline{\varphi}_{22}^{ku})\right] & \cdots & \left[(\underline{\varphi}_{2m}^{kl}, \underline{\varphi}_{2m}^{km}, \underline{\varphi}_{2m}^{ku}), (\overline{\varphi}_{2m}^{kl}, \overline{\varphi}_{2m}^{km}, \overline{\varphi}_{2m}^{ku})\right] \\ \vdots & \vdots & \ddots & \vdots \\ \left[(\underline{\varphi}_{r1}^{kl}, \underline{\varphi}_{r1}^{km}, \underline{\varphi}_{r1}^{ku}), (\overline{\varphi}_{r1}^{kl}, \overline{\varphi}_{r1}^{km}, \overline{\varphi}_{r1}^{ku})\right] & \left[(\underline{\varphi}_{r2}^{kl}, \underline{\varphi}_{r2}^{km}, \underline{\varphi}_{r2}^{ku}), (\overline{\varphi}_{r2}^{kl}, \overline{\varphi}_{r2}^{km}, \overline{\varphi}_{r2}^{ku})\right] & \cdots & \left[(\underline{\varphi}_{rm}^{kl}, \underline{\varphi}_{rm}^{km}, \underline{\varphi}_{rm}^{ku}), (\overline{\varphi}_{rm}^{kl}, \overline{\varphi}_{rm}^{km}, \overline{\varphi}_{rm}^{ku})\right] \end{bmatrix} \quad (19)$$

Box I.

$$\underline{apr}(\varphi_{ji}^k) = \cup \left\{ \varphi \in J \mid \varphi \leq \varphi_{ji}^k \right\}, \quad (14)$$

$$\overline{apr}(\varphi_{ji}^k) = \cup \left\{ \varphi \in J \mid \varphi \geq \varphi_{ji}^k \right\}. \quad (15)$$

The lower and upper limits $\underline{\varphi}_{ji}^k$ and $\overline{\varphi}_{ji}^k$ of φ_{ji}^k are defined in Eqs. (16) and (17), respectively.

$$\underline{\varphi}_{ji}^k = (\underline{\varphi}_{ji}^{kl}, \underline{\varphi}_{ji}^{km}, \underline{\varphi}_{ji}^{ku}) = \frac{1}{n_{ji}^{kL}} \sum_{\varphi \in \underline{apr}(\varphi_{ji}^k)} (\varphi^l, \varphi^m, \varphi^u), \quad (16)$$

$$\overline{\varphi}_{ji}^k = (\overline{\varphi}_{ji}^{kl}, \overline{\varphi}_{ji}^{km}, \overline{\varphi}_{ji}^{ku}) = \frac{1}{n_{ji}^{kU}} \sum_{\vartheta \in \overline{apr}(\varphi_{ji}^k)} (\vartheta^l, \vartheta^m, \vartheta^u). \quad (17)$$

where φ^u, φ^m and φ^l are the elements of lower approximation of upper, medium and lower boundaries of TF number φ_{ji}^k. ϑ^u, ϑ^m and ϑ^l are the elements of upper approximation of upper, medium and lower boundaries of TF number φ_{ji}^k. n_{ji}^{kU} and n_{ji}^{kL} are the number of elements in the upper and lower approximation of TF number φ_{ji}^k. The RF number of TF number φ_{ji}^k is an RF interval $RF(\varphi_{ji}^k)$ of φ_{ji}^k described in Eq. (18).

$$RF(\varphi_{ji}^k) = [\underline{\varphi}_{ji}^k, \overline{\varphi}_{ji}^k] = \left[(\underline{\varphi}_{ji}^{kl}, \underline{\varphi}_{ji}^{km}, \underline{\varphi}_{ji}^{ku}), (\overline{\varphi}_{ji}^{kl}, \overline{\varphi}_{ji}^{km}, \overline{\varphi}_{ji}^{ku})\right]. \quad (18)$$

where $\overline{\varphi}_{ji}^k$ and $\underline{\varphi}_{ji}^k$ are the upper and lower limits of $RF(\tilde{a}_i)$. $\overline{\varphi}_{ji}^{kl}$ and $\underline{\varphi}_{ji}^{kl}$ are the upper and lower limits of rough number $RN(a_i^l) = [\underline{\varphi}_{ji}^{kl}, \overline{\varphi}_{ji}^{kl}]$; $\overline{\varphi}_{ji}^{km}$ and $\underline{\varphi}_{ji}^{km}$ are the upper and lower limits of rough number $RN(a_i^m) = [\underline{\varphi}_{ji}^{km}, \overline{\varphi}_{ji}^{km}]$; $\overline{\varphi}_{ji}^{ku}$ and $\underline{\varphi}_{ji}^{ku}$ are the upper and lower limits of rough number $RN(a_i^u) = [\underline{\varphi}_{ji}^{ku}, \overline{\varphi}_{ji}^{ku}]$. The RF direct relation $RF^k = \left[RF(\varphi_{ji}^k)\right]_{r\times m}$ for kth DM is written in Eq. (19) (see Box I). Similarly, the RF number evaluations of risk factors are given in Eq. (20).

$$RF(\varphi_i) = \left(RF(\varphi_i^1), RF(\varphi_i^2), \ldots, RF(\varphi_i^k), \ldots, RF(\varphi_i^s)\right) \quad (20)$$

where the RF number $RF(\varphi_i^k) = [\underline{\varphi}_i^k, \overline{\varphi}_i^k] = \left[(\underline{\varphi}_i^{kl}, \underline{\varphi}_i^{km}, \underline{\varphi}_i^{ku}), (\overline{\varphi}_i^{kl}, \overline{\varphi}_i^{km}, \overline{\varphi}_i^{ku})\right]$ is an RF interval of ith risk factor given by kth expert.

4.1.3. Computing cloud evaluations of RF numbers

The RF numbers are converted into cloud evaluations to study randomness and vagueness simultaneously. The cloud evaluation of RF number φ_{ji}^k is expressed as $C_{ji}^k = \left(Ex_{ji}^k, En_{ji}^k, HEn_{ji}^k\right)$, where the expectation, entropy and hyper entropy terms are illustrated in Eqs. (21)–(23).

$$Ex_{ji}^k = \frac{1}{6}\left(\underline{\varphi}_{ji}^{kl} + \overline{\varphi}_{ji}^{kl} + \underline{\varphi}_{ji}^{km} + \overline{\varphi}_{ji}^{km} + \underline{\varphi}_{ji}^{ku} + \overline{\varphi}_{ji}^{ku}\right), \quad (21)$$

$$En_{ji}^k = \frac{1}{18}\left(\overline{\varphi}_{ji}^{kl} - \underline{\varphi}_{ji}^{kl} + \overline{\varphi}_{ji}^{km} - \underline{\varphi}_{ji}^{km} + \overline{\varphi}_{ji}^{ku} - \underline{\varphi}_{ji}^{ku}\right), \quad (22)$$

$$HEn_{ji}^k = \mu. \quad (23)$$

C_{ji}^k is the cloud evaluation of jth failure mode corresponding to ith risk factor given by kth expert. μ is a constant to be determined according to degree of vagueness in the given evaluations. The RFIC direct relation matrix is given in Eq. (24).

$$RFIC^k = [C_{ji}^k]_{r\times m} = \begin{bmatrix} (Ex_{11}^k, En_{11}^k, HEn_{11}^k) & (Ex_{12}^k, En_{12}^k, HEn_{12}^k) & \cdots & (Ex_{1m}^k, En_{1m}^k, HEn_{1m}^k) \\ (Ex_{21}^k, En_{21}^k, HEn_{21}^k) & (Ex_{22}^k, En_{22}^k, HEn_{22}^k) & \cdots & (Ex_{2m}^k, En_{2m}^k, HEn_{2m}^k) \\ \vdots & \vdots & \ddots & \vdots \\ (Ex_{r1}^k, En_{r1}^k, HEn_{r1}^k) & (Ex_{r2}^k, En_{r2}^k, HEn_{r2}^k) & \cdots & (Ex_{rm}^k, En_{rm}^k, HEn_{rm}^k) \end{bmatrix} \quad (24)$$

The cloud evaluations of importance of risk factors given by the experts are $\left(C_i^1, C_i^2, \ldots, C_i^k, \ldots, C_i^s\right)$, where $C_i^k = \left(Ex_i^k, En_i^k, HEn_i^k\right)$ is the cloud importance of ith risk factor given by kth expert. The expectation, entropy are TF numbers and hyper entropy is the uncertainty degree as illustrated in Eqs. (25), (26) and (27), respectively.

$$Ex_i^k = \frac{1}{6}\left(\underline{\varphi}_i^{kl} + \overline{\varphi}_i^{kl} + \underline{\varphi}_i^{km} + \overline{\varphi}_i^{km} + \underline{\varphi}_i^{ku} + \overline{\varphi}_i^{ku}\right), \quad (25)$$

$$En_i^k = (En_i^{kl}, En_i^{km}, En_i^{ku}) = \frac{1}{18}\left(\overline{\varphi}_i^{kl} - \underline{\varphi}_i^{kl} + \overline{\varphi}_i^{km} - \underline{\varphi}_i^{km} + \overline{\varphi}_i^{ku} - \underline{\varphi}_i^{ku}\right), \quad (26)$$

$$HEn_i^k = \nu. \quad (27)$$

ν is a constant to be computed for the degree of uncertainty.

4.1.4. Aggregation of cloud evaluations of different experts

The group cloud evaluation $CRF((\varphi_{ji})) = \left(Ex_{ji}, En_{ji}, HEn_{ji}\right)$ of the judgement set $J = \left\{ \varphi_{ji}^k = (\varphi_{ji}^{kl}, \varphi_{ji}^{km}, \varphi_{ji}^{ku}) \mid k = 1, 2, \ldots, r \right\}$ can be acquired by taking average of s cloud drops $C_{ji}^1, C_{ji}^2, \ldots, C_{ji}^s$ as explained in Eq. (28).

$$C_{ji} = \left(\frac{1}{s}\sum_{k=1}^{s} Ex_{ji}^k, \sqrt{\frac{1}{s}\sum_{k=1}^{s}(En_{ji}^k)^2}, \sqrt{\frac{1}{s}\sum_{k=1}^{s}(HEn_{ji}^k)^2}\right). \quad (28)$$

The average cloud decision matrix is written in Eq. (29).

$$RFIC = [C_{ji}]_{r\times m} \times \begin{bmatrix} (Ex_{11}, En_{11}, HEn_{11}) & (Ex_{12}, En_{12}, HEn_{12}) & \cdots & (Ex_{1m}, En_{1m}, HEn_{1m}) \\ (Ex_{21}, En_{21}, HEn_{21}) & (Ex_{22}, En_{22}, HEn_{22}) & \cdots & (Ex_{2m}, En_{2m}, HEn_{2m}) \\ \vdots & \vdots & \ddots & \vdots \\ (Ex_{r1}, En_{r1}, HEn_{r1}) & (Ex_{r2}, En_{r2}, HEn_{r2}) & \cdots & (Ex_{rm}, En_{rm}, HEn_{rm}) \end{bmatrix} \quad (29)$$

The group cloud importance of all the risk factors is given in the form of row vector as:

$$[C_1 \quad C_2 \quad \ldots \quad C_m]$$

where $C_i = (Ex_i, En_i, HEn_i)$ is the group cloud evaluation of ith risk factor as explained in Eq. (30).

$$C_i = \left(\frac{1}{s}\sum_{k=1}^{s} Ex_i^k, \sqrt{\frac{1}{s}\sum_{k=1}^{s}(En_i^k)^2}, \sqrt{\frac{1}{s}\sum_{k=1}^{s}(HEn_i^k)^2}\right). \quad (30)$$

4.2. Computing weighted cloud decision matrix

In this subsection, the subjective, objective and combined weights of risk factors are computed to determine the weighted cloud decision matrix.

4.2.1. Weight determination of risk factors

The subjective weights of risk factors can be computed using Formula (31).

$$w_i^s = \frac{Ex_i}{\sum_{i=1}^{m} Ex_i} \quad (31)$$

w_i^s is the subjective weight of the ith risk factor and Ex_i is the expectation value of the group cloud evaluation $C(RF(\varphi_i))$.

Inspired by Rao et al. [56] and Liu et al. [57], the objective weights of risk factors can be computed using the notion of statistical variance. It is more simpler than Shannon [58] entropy weights. The objective weights of risk factors can be computed from group cloud evaluations. The mean value of ith risk factor for r failure modes is described in Eqs. (32).

$$\begin{aligned}\widehat{C}_i &= (C_{ji})_{mean} = (Ex_i, En_i, HEn_i)\\ &= \left(\frac{1}{r}\sum_{j=1}^{r} Ex_{ji}, \sqrt{\frac{1}{r}\sum_{j=1}^{r}(En_{ji})^2}, \sqrt{\frac{1}{r}\sum_{j=1}^{r}(HEn_{ji})^2}\right).\end{aligned} \quad (32)$$

The statistical variance Var_i of ith risk factor is computed in Eqs. (33) and (34).

$$Var_i = \frac{1}{r}\sum_{j=1}^{r}\left(d(C_{ji}, \widehat{C}_i)\right)^2, \quad (33)$$

$$\Rightarrow \quad Var_i = \frac{1}{r}\sum_{j=1}^{r}\left|\left(1 - \frac{En_{ji} + HEn_{ji}}{Ex_{ji}}\right)Ex_{ji} - \left(1 - \frac{En_i + HEn_i}{Ex_i}\right)Ex_i\right|^2. \quad (34)$$

The objective weight w_i^o of ith risk factor is computed in Eq. (35).

$$w_i^o = \frac{Var_i}{\sum_{i=1}^{m} Var_i}. \quad (35)$$

Here $\widehat{C}_i$ is the mean value and Var_i is the variance of group cloud evaluations of failure modes $C(RF(\varphi_{ji}))$, for $1 \le j \le r$ under ith risk factor.

The weight of ith risk factor computed using subjective and objective weights is given in Formula (36).

$$w_i = \frac{w_i^o \times w_i^s}{\sum_{i=1}^{m} w_i^o \times w_i^s}. \quad (36)$$

4.2.2. Determine normalized cloud decision matrix

Due to different magnitudes and dimensions of failure modes and risk factors, the cloud decision matrix can be transformed into comparable one by normalizing values of the matrix. The normalized value of group cloud evaluation $C(RF(\varphi_{ji}))$ of jth failure mode for ith risk factor is computed in Eqs. (37) and (38).

$$C'_{ji} = (Ex'_{ji}, En'_{ji}, HEn'_{ji}) = \frac{(Ex_{ji}, En_{ji}, HEn_{ji})}{(Ex_i^{max}, En_i^{max}, HEn_i^{max})}, \quad (37)$$

$$= \left(\frac{Ex_{ji}}{Ex_i^{max}}, \left|\frac{Ex_{ji}}{Ex_i^{max}}\right| \times \sqrt{\frac{En_{ji}^2}{Ex_{ji}^2} + \frac{(En_i^{max})^2}{(Ex_i^{max})^2}}, \left|\frac{Ex_{ji}}{Ex_i^{max}}\right| \times \sqrt{\frac{HEn_{ji}^2}{Ex_{ji}^2} + \frac{(HEn_i^{max})^2}{(Ex_i^{max})^2}}\right) \quad (38)$$

where $Ex_i^{max} = \max_j Ex_{ji}$, $En_i^{max} = \max_j En_{ji}$ and $HEn_i^{max} = \max_j HEn_{ji}$. Clearly, Eq. (38) shows the division of two clouds. The normalized cloud decision matrix is written in Eq. (39).

$$NC = [C'_{ji}]_{r\times m} \times \begin{bmatrix} (Ex'_{11}, En'_{11}, HEn'_{11}) & (Ex'_{12}, En'_{12}, HEn'_{12}) & \dots & (Ex'_{1m}, En'_{1m}, HEn'_{1m})\\ (Ex'_{21}, En'_{21}, HEn'_{21}) & (Ex'_{22}, En'_{22}, HEn'_{22}) & \dots & (Ex'_{2m}, En'_{2m}, HEn'_{2m})\\ \vdots & \vdots & \ddots & \vdots\\ (Ex'_{r1}, En'_{r1}, HEn'_{r1}) & (Ex'_{r2}, En'_{r2}, HEn'_{r2}) & \dots & (Ex'_{rm}, En'_{rm}, HEn'_{rm}) \end{bmatrix} \quad (39)$$

4.2.3. Determine weighted cloud decision matrix

The importance of failure modes against different risk factors can be observed by multiplying cloud evaluations by the weights of risk factors. The weighted value of cloud evaluation $C(\vartheta_{ji})$ of jth failure mode for ith risk factor is computed in Eq. (40).

$$C^*_{ji} = (Ex^*_{ji}, En^*_{ji}, HEn^*_{ji}) = w_i C'_{ji} = \left(w_i Ex'_{ji}, \sqrt{w_i} En'_{ji}, \sqrt{w_i} HEn'_{ji}\right) \quad (40)$$

where $w_i C(\vartheta_{ji})$ is the weighted value of $C(\vartheta_{ji})$ for ith risk factor. The weighted cloud decision matrix is written in Eq. (41).

$$WC = [C^*_{ji}]_{r\times m} = \begin{bmatrix} w_1 C'_{11} & w_2 C'_{12} & \dots & w_m C'_{1m}\\ w_1 C'_{21} & w_2 C'_{22} & \dots & w_m C'_{2m}\\ \vdots & \vdots & \ddots & \vdots\\ w_1 C'_{r1} & w_2 C'_{r2} & \dots & w_m C'_{rm} \end{bmatrix}. \quad (41)$$

4.3. Ranking of failure modes

In this subsection, the failure modes are ranked from most risky to less risky. For this purpose, the PI and NI solutions, distances and closeness degrees are computed from weighted cloud matrix given in Eq. (41).

4.3.1. Identifying PI and NI solutions

Based on the weighted cloud decision matrix, the PI and NI solutions of risk factors are computed as given in Eqs. (42) and (43).

$$S^+(z_i) = \begin{cases} \left(\max_j Ex^*_{ji}, \min_j En^*_{ji}, \min_j HEn^*_{ji}\right), & i \in B\\ \left(\min_j Ex^*_{ji}, \max_j En^*_{ji}, \max_j HEn^*_{ji}\right), & i \in C \end{cases} \quad (42)$$

$$S^-(z_i) = \begin{cases} \left(\min_j Ex^*_{ji}, \max_j En^*_{ji}, \max_j HEn^*_{ji}\right), & i \in B\\ \left(\max_j Ex^*_{ji}, \min_j En^*_{ji}, \min_j HEn^*_{ji}\right), & i \in C \end{cases} \quad (43)$$

where z_i is the ith risk factor, B and C are the benefit and cost criteria. The benefit criteria means the greater the value, the higher risk of the failure mode; the cost criteria means that the greater the value, the lower the risk of the failure mode. In PI solution $S^+(z_i)$ of the risk factor z_i, the cloud corresponding to benefit criteria are maximized and to cost criteria are minimized. In NI solution $S^-(z_i)$ of the risk factor z_i, the cloud corresponding to benefit criteria are minimized and to cost criteria are maximized.

4.3.2. Computing the distance of failure modes from PI and NI solutions

The distance of each failure mode from PI and NI solutions are computed in Eqs. (44) and (45).

$$D_j^+ = \sqrt{\sum_{i=1}^{m}\left[d(C_{ji}^*, S^+(z_i))\right]^2} \tag{44}$$

$$D_j^- = \sqrt{\sum_{i=1}^{m}\left[d(C_{ji}^*, S^-(z_i))\right]^2} \tag{45}$$

where D_j^+ is the distance of jth failure mode from PI solution and D_j^- is the distance of jth failure mode from NI solution.

4.3.3. Obtaining the closeness degree of failure modes

The closeness degree of jth failure mode is computed in Eq. (46).

$$cd_j = \frac{D_j^+}{D_j^+ + D_j^-} \tag{46}$$

where cd_j is the closeness degree which is the strength of risk priority. The greater the value of cd_j, the lower the risk of jth failure mode.

Based on all the discussion, the procedure of proposed RFIC modified TOPSIS method is given in Algorithm 4.1.

Algorithm 4.1. *The RFIC TOPSIS method*

1. Identify the r failure modes $F_{m1}, F_{m2}, \dots, F_{mr}$ and their m risk factors provided by s experts $DM_1, DM_2, \dots, DM_s$.
2. Construct the initial decision matrix $L^k = [\lambda_{ji}^k]_{r\times m}$ of failure modes based on the scales given in Table 2, evaluated by kth expert, $1 \le k \le s$.
3. Construct the matrix $[\lambda_i^k]_{m\times s}$ of importance of risk factors given by the decision-makers.
4. For each $1 \le k \le r$, convert each initial decision matrix L^k into TF relation matrix
   $$TF^k = \left[(\varphi_{ji}^{kl}, \varphi_{ji}^{km}, \varphi_{ji}^{ku})\right]_{r\times m},$$
   where $(\varphi_{ji}^{kl}, \varphi_{ji}^{km}, \varphi_{ji}^{ku}) = TF(\lambda_{ji}^k)$ is a TF number corresponding to the scale λ_{ji}^k.
5. Convert the weights of risk factors into TF numbers and obtain a TF relation matrix $\left[(\varphi_i^{kl}, \varphi_i^{km}, \varphi_i^{ku})\right]_{m\times s}$, where $(\varphi_i^{kl}, \varphi_i^{km}, \varphi_i^{ku}) = TF(\lambda_i^k)$.
6. Using Eqs. (14)–(18), convert the TF numbers into RF numbers $RF(\varphi_{ji}^k)$ to obtain the RF direct relation matrix
   $$RF^k = \left[RF(\varphi_{ji}^k)\right]_{r\times m} = \left[\left[(\underline{\varphi}_{ji}^{kl}, \underline{\varphi}_{ji}^{km}, \underline{\varphi}_{ji}^{ku}), (\overline{\varphi}_{ji}^{kl}, \overline{\varphi}_{ji}^{km}, \overline{\varphi}_{ji}^{ku})\right]\right]_{r\times m}.$$
7. Convert the TF number weights of risk factors into RF numbers to obtain RF relation matrix
   $$\left[RF(\varphi_i^k)\right]_{m\times s} = \left[\left[(\underline{\varphi}_{i}^{kl}, \underline{\varphi}_{i}^{km}, \underline{\varphi}_{i}^{ku}), (\overline{\varphi}_{i}^{kl}, \overline{\varphi}_{i}^{km}, \overline{\varphi}_{i}^{ku})\right]\right]_{m\times s}.$$
8. Compute the cloud evaluations $C_{ji}^k = \left(Ex_{ji}^k, En_{ji}^k, HEn_{ji}^k\right)$ of RF numbers $RF(\varphi_{ji}^k)$, for each $1 \le j \le r$, $1 \le i \le m$, $1 \le k \le s$ using Eqs. (21)–(23).
9. Compute the cloud evaluations $C_i^k = (Ex_i, En_i, HEn_i)$ of RF number weights $RF(\varphi_i^k)$, for each $1 \le i \le m$, $1 \le k \le s$ using Eqs. (25)–(27).
10. Using Eq. (28), obtain the aggregated cloud matrix $AC = [C_{ji}]_{r\times m} = \left[\left(Ex_{ji}, En_{ji}, HEn_{ji}\right)\right]_{r\times m}$ by taking average of cloud evaluations.
11. Aggregate the weights of risk factors using Eq. (30).
12. Compute the subjective weights of risk factors using Eq. (31), objective weights using Eqs. (32)–(35) and then combined weights w_i, $1 \le i \le m$, using Eq. (36).
13. Obtain the normalized cloud decision matrix
    $$NC = [C'_{ji}]_{r\times m} = \left[\left(Ex'_{ji}, En'_{ji}, HEn'_{ji}\right)\right]_{r\times m},$$
    where C_{ji} is obtained using Formulae (37)–(39).
14. Determine the weighted cloud decision matrix
    $$WC = [w_i C'_{ji}]_{r\times m} = \left[\left(w_i Ex'_{ji}, \sqrt{w_i} En'_{ji}, \sqrt{w_i} HEn'_{ji}\right)\right]_{r\times m},$$
    where C'_{ji} is computed using Formulae (40) and (41).
15. Compute PI and NI solution of risk factors using Eqs. (42) and (43).
16. Compute the distances D_j^+ and D_j^-, for each $1 \le j \le r$, of failure modes from PI and NI solutions using Eqs. (44) and (45).
17. Determine the closeness cd_j, $1 \le j \le r$, of failure modes using Eq. (46).
18. Rank the failure modes according to the descending order of closeness degrees.

Computational Complexity of Algorithm Algorithm 4.1: In Algorithm 4.1, there are r failure modes, m risk factors and s decision makers. On step 2, the matrix representing the judgements of DMs contains $r \times m$ entries and there are s total matrices. The inner loop runs rm times and the outer loop runs s times to input the data. The computational complexity of step 2 is $O(rms)$. On step 3, the matrix representing importance of risk factors contains $m \times s$centries, so the complexity at level is $O(ms)$. On lines 4,5,6,8 and 9, the initial data is converted into TF numbers, RF numbers and then RFI clouds, so the complexity for this data transformation in $O(rms)$ as s matrices of order $r \times m$ are converted. The computational complexity of line 7 is $O(ms)$. On line 10, the s matrices are aggregated into a single matrix. The loop runs s times so the complexity of data aggregation is $O(s)$. On lines 13 and 14, the cloud evaluations are converted into weighted cloud evaluations. The inner loop runs m times and outer loop runs r times, therefore the complexity at this step is $O(rm)$. On lines 16, 17 and 18, the r failures modes are ranked using positive and negative distances and, closeness coefficient. Thus, the computational complexity at this level is $O(r)$. Hence, the net computational complexity of the algorithm is $O(rms)$.

5. Case study: FMEA of steam valve shaft working in turbine system

The proposed RFIC modified TOPSIS is applied to steam valve (SV) system of a power generation plant to illustrate its significance and advantages in engineering applications. An SV system is important for the reliability of a steam turbine operation in a power generation plant, as it is considered an essential part to control transporting medium with high pressure and temperature in complicated working conditions. SV system is located between l-pressure (low pressure) cylinder and MSR (Moisture Separator Re-heater) to control the amount of steam in the cylinder. The MSR, in the turbine system, has a pipeline with large capacity and the excessive water and steam can cause the failures in turbine rotor speed. The reliability and safety of the turbine system can be assured if the SV opens and closes automatically to stop the heat or steam entering into l-pressure cylinder. The major failures in a turbine system identified by a team of DMs are F_{m1} (extensive closing time of SV), F_{m2} (not properly closed), F_{m3} (leakage of steam from valve shaft), F_{m4} (fluctuations in SV), F_{m5} (jam in SV), F_{m6} (fractures valve shaft), F_{m7} (breakdown of SV support

Table 3
Evaluation of DMs for failure modes in SV system.

Risk factors	Decision makers	Fm_1	Fm_2	Fm_3	Fm_4	Fm_5	Fm_6	Fm_7	Fm_8
D	DM_1	9	3	3	4	3	2	2	3
	DM_2	9	5	3	2	3	2	2	5
	DM_3	7	5	2	3	2	3	4	3
	DM_4	5	3	4	4	3	5	3	3
O	DM_1	3	3	8	2	3	1	5	5
	DM_2	3	3	7	2	3	2	5	8
	DM_3	5	5	5	4	5	2	7	6
	DM_4	2	4	5	3	5	3	6	7
S	DM_1	10	2	3	7	10	10	7	5
	DM_2	10	2	4	8	10	10	7	5
	DM_3	10	1	5	7	7	10	9	8
	DM_4	9	3	3	5	9	10	9	7

Table 4
Importance of risk factors.

Risk	Decision makers				Corresponding TF numbers			
factor	DM_1	DM_2	DM_3	DM_4	DM_1	DM_2	DM_3	DM_4
D	5	7	8	9	(3, 4, 5)	(5, 6, 7)	(6, 7, 8)	(7, 8, 9)
O	4	5	5	7	(2, 3, 4)	(3, 4, 5)	(3, 4, 5)	(5, 6, 7)
S	5	6	6	7	(3, 4, 5)	(4, 5, 6)	(4, 5, 6)	(5, 6, 7)

bearing) and F_{m8} (extra noise of the system). The causes, effects and detection measures of failure modes are given in Table I of [7] and Table 3 of [59]. Song et al. [7] discussed this problem of failures in a turbine using rough TOPSIS approach. We apply our proposed RFIC modified TOPSIS method on Song's problem and study its advantages, differences and similarities. The evaluations of failure modes for three risk factors D (detection measures), O (occurrence of failure modes) and S (severity of effects of failure modes) in the form of quantitative measures given by four DMs are given in Table 3.

The importance of risk factor given by DMs are used to determine their weights for all the failure modes. Using the scales of Table 2, the importance of risk factors is given in the form of TF numbers, which is provided in Table 4.

Using Table 2, the TF number evaluations of failure modes, the TF relation matrices of DMs are computed in Table 5.

To discuss the uncertainty in the data, the TF numbers are then converted into RF numbers. Using Eqs. (14)–(18), the RF numbers of TF number evaluations are computed in Table 6.

In Table 6, the symbols $\hat{\varphi} = (\varphi - 2, \varphi - 1, \varphi)$ is a TF number. If $\varphi - 2 < 0$, then $\varphi - 2 = 0$. Moreover, $\hat{\alpha} = (0.25, 1, 2)$. The RF number evaluations of importance of risk factors (given in Table 4) are computed in Table 7.

Using Eqs. (21)–(23), the cloud evaluations of RF numbers of failure modes (given in Table 6) are computed in Tables 8–10.

Using Eqs. (25)–(27), the cloud evaluations of RF weights of risk factors (given in Table 7) are computed in Table 11.

Using Eq. (28), the aggregated cloud decision matrix of failure modes containing group cloud evaluations computed from Tables 8–10 is given in Table 12.

Using Eqs. (30) and (31), the aggregated cloud weights and subjective weights of risk factors (computed using Table 11) are given in Table 13.

Using Formulae (37)–(39), the normalized cloud decision matrix is computed in Table 14.

Using Eq. (32), the mean values of risk factors are $\hat{C}_D = (2.7487, 0.2568, 0.01)$, $\hat{C}_O = (3.2873, 0.2718, 0.01)$ and $\hat{C}_S = (6.1103, 0.1950, 0.01)$. Using Eqs. (32)–(36), the distances, variance of risk factors, objective weights (w^O) and combined weights (w) are computed in Table 15.

Using Eqs. (40)–(43), the weighted cloud matrix, PI solutions and NI solutions are computed in Table 16.

The distances of weighted cloud evaluations of failure modes for each risk factor from PI and NI solutions for each risk factor are computed in Table 17.

Using Eqs. (44)–(46), the distances of failure modes from PI and NI solutions and closeness degrees are computed in Table 18.

The failure modes are ranked as $Fm_1 \succ Fm_6 \succ Fm_5 \succ Fm_7 \succ Fm_8 \succ Fm_4 \succ Fm_3 \succ Fm_2$ which shows that Fm_1 has the highest risk and Fm_2 has the lowest among all the failure modes.

5.1. Sensitivity analysis

The sensitivity analysis of the proposed scheme is subdivided into two parts (1) changing or disturbance of weights of risk factors (2) control parameter in closeness degree of failure modes. The selection of weight information and parameters by the DMs is important for the accuracy of results. To analyse the robustness of the proposed method and effect of parameters and weight on the results, a sensitivity analysis in performed in this section.

5.1.1. Control parameter α

The relative importance of the PI and NI solutions of the evaluation results can be controlled by a parameter α as given in Eq. (47).

Table 5
TF number evaluations for failure modes.

Risk factors	Decision makers	Fm_1	Fm_2	Fm_3	Fm_4	Fm_5	Fm_6	Fm_7	Fm_8
D	DM_1	(7, 8, 9)	(1, 2, 3)	(1, 2, 3)	(2, 3, 4)	(1, 2, 3)	(0, 1, 2)	(0, 1, 2)	(1, 2, 3)
	DM_2	(7, 8, 9)	(3, 4, 5)	(1, 2, 3)	(0, 1, 2)	(1, 2, 3)	(0, 1, 2)	(0, 1, 2)	(3, 4, 5)
	DM_3	(5, 6, 7)	(3, 4, 5)	(0, 1, 2)	(1, 2, 3)	(0, 1, 2)	(1, 2, 3)	(2, 3, 4)	(1, 2, 3)
	DM_4	(3, 4, 5)	(1, 2, 3)	(2, 3, 4)	(2, 3, 4)	(1, 2, 3)	(3, 4, 5)	(1, 2, 3)	(1, 2, 3)
O	DM_1	(1, 2, 3)	(1, 2, 3)	(6, 7, 8)	(0, 1, 2)	(1, 2, 3)	(0, 0, 1)	(3, 4, 5)	(3, 4, 5)
	DM_2	(1, 2, 3)	(1, 2, 3)	(5, 6, 7)	(0, 1, 2)	(1, 2, 3)	(0, 1, 2)	(3, 4, 5)	(6, 7, 8)
	DM_3	(3, 4, 5)	(3, 4, 5)	(3, 4, 5)	(2, 3, 4)	(3, 4, 5)	(0, 1, 2)	(5, 6, 7)	(4, 5, 6)
	DM_4	(0, 1, 2)	(2, 3, 4)	(3, 4, 5)	(1, 2, 3)	(3, 4, 5)	(1, 2, 3)	(4, 5, 6)	(5, 6, 7)
S	DM_1	(8, 9, 10)	(0, 1, 2)	(1, 2, 3)	(5, 6, 7)	(8, 9, 10)	(8, 9, 10)	(5, 6, 7)	(3, 4, 5)
	DM_2	(8, 9, 10)	(0, 1, 2)	(2, 3, 4)	(6, 7, 8)	(8, 9, 10)	(8, 9, 10)	(5, 6, 7)	(3, 4, 5)
	DM_3	(8, 9, 10)	(0, 0, 1)	(3, 4, 5)	(5, 6, 7)	(5, 6, 7)	(8, 9, 10)	(7, 8, 9)	(6, 7, 8)
	DM_4	(7, 8, 9)	(1, 2, 3)	(1, 2, 3)	(3, 4, 5)	(7, 8, 9)	(8, 9, 10)	(7, 8, 9)	(5, 6, 7)

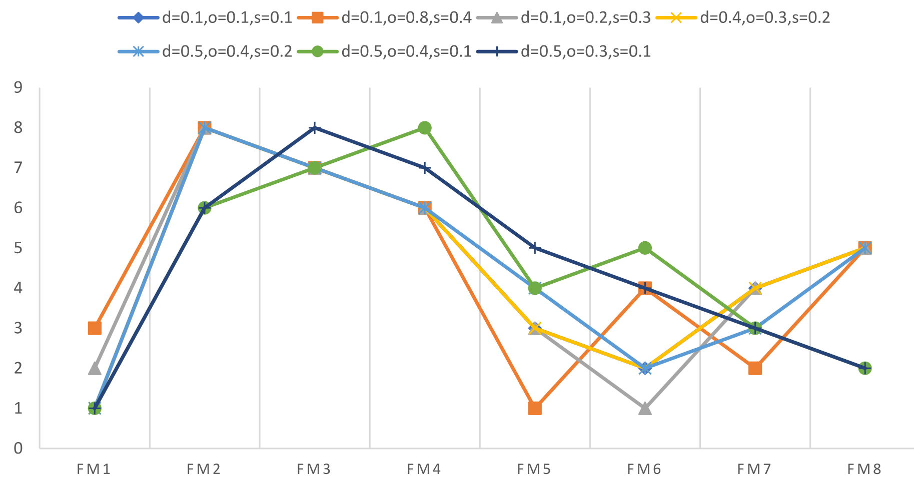

Fig. 2. Comparison of ranking results for different variations of parameters.

Table 6
RF number evaluations for failure modes.

Risk factors	Decision makers	Fm_1	Fm_2	Fm_3	Fm_4	Fm_5	Fm_6	Fm_7	Fm_8
D	DM_1	$[\hat{7.5}, \hat{9}]$	$[\hat{3}, \hat{4}]$	$[\hat{2.6667}, \hat{3.3333}]$	$[\hat{3.25}, \hat{4}]$	$[\hat{2.75}, \hat{3}]$	$[\hat{2}, \hat{3}]$	$[\hat{2}, \hat{2.75}]$	$[\hat{3}, \hat{3.5}]$
	DM_2	$[\hat{7.5}, \hat{9}]$	$[\hat{4}, \hat{5}]$	$[\hat{2.6667}, \hat{3.3333}]$	$[\hat{2}, \hat{3.25}]$	$[\hat{2.75}, \hat{3}]$	$[\hat{2}, \hat{3}]$	$[\hat{2}, \hat{2.75}]$	$[\hat{3.5}, \hat{5}]$
	DM_3	$[\hat{6}, \hat{8.3333}]$	$[\hat{4}, \hat{5}]$	$[\hat{2}, \hat{3}]$	$[\hat{2.5}, \hat{3.6667}]$	$[\hat{2}, \hat{2.75}]$	$[\hat{2.3333}, \hat{4}]$	$[\hat{2.75}, \hat{4}]$	$[\hat{3}, \hat{3.5}]$
	DM_4	$[\hat{5}, \hat{7.5}]$	$[\hat{3}, \hat{4}]$	$[\hat{3}, \hat{4}]$	$[\hat{3.25}, \hat{4}]$	$[\hat{2.75}, \hat{3}]$	$[\hat{3}, \hat{5}]$	$[\hat{2.333}, \hat{3.5}]$	$[\hat{3}, \hat{3.5}]$
O	DM_1	$[\hat{2.6667}, \hat{4}]$	$[\hat{3}, \hat{3.75}]$	$[\hat{6.25}, \hat{8}]$	$[\hat{2}, \hat{2.75}]$	$[\hat{3}, \hat{4}]$	$[\hat{1}, \hat{\alpha}]$	$[\hat{5}, \hat{5.75}]$	$[\hat{5}, \hat{6.5}]$
	DM_2	$[\hat{2.6667}, \hat{4}]$	$[\hat{3}, \hat{3.75}]$	$[\hat{5.6667}, \hat{7.5}]$	$[\hat{2}, \hat{2.75}]$	$[\hat{3}, \hat{4}]$	$[\hat{1.5}, \hat{1.6667}]$	$[\hat{5}, \hat{5.75}]$	$[\hat{6.5}, \hat{8}]$
	DM_3	$[\hat{3.25}, \hat{5}]$	$[\hat{3.75}, \hat{5}]$	$[\hat{5}, \hat{6.25}]$	$[\hat{2.75}, \hat{4}]$	$[\hat{4}, \hat{5}]$	$[\hat{1.5}, \hat{1.6667}]$	$[\hat{5.75}, \hat{7}]$	$[\hat{5.5}, \hat{7}]$
	DM_4	$[\hat{2}, \hat{3.25}]$	$[\hat{3.3333}, \hat{4}]$	$[\hat{5}, \hat{6.25}]$	$[\hat{2.3333}, \hat{3.5}]$	$[\hat{4}, \hat{5}]$	$[\hat{\alpha}, \hat{3}]$	$[\hat{5.3333}, \hat{6.5}]$	$[\hat{6}, \hat{7.5}]$
S	DM_1	$[\hat{9.75}, \hat{10}]$	$[\hat{1.5}, \hat{1.6667}]$	$[\hat{3}, \hat{3.75}]$	$[\hat{6.3333}, \hat{7.5}]$	$[\hat{9}, \hat{10}]$	$\hat{10}$	$[\hat{7}, \hat{8}]$	$[\hat{5}, \hat{6.75}]$
	DM_2	$[\hat{9.75}, \hat{10}]$	$[\hat{1.5}, \hat{1.6667}]$	$[\hat{3.3333}, \hat{4.5}]$	$[\hat{6.75}, \hat{8}]$	$[\hat{9}, \hat{10}]$	$\hat{10}$	$[\hat{7}, \hat{8}]$	$[\hat{5}, \hat{6.75}]$
	DM_3	$[\hat{9.75}, \hat{10}]$	$[\hat{1}, \hat{\alpha}]$	$[\hat{3.75}, \hat{5}]$	$[\hat{6.3333}, \hat{7.5}]$	$[\hat{7}, \hat{9}]$	$\hat{10}$	$[\hat{8}, \hat{9}]$	$[\hat{6}, \hat{8.5}]$
	DM_4	$[\hat{9}, \hat{9.75}]$	$[\hat{\alpha}, \hat{3}]$	$[\hat{3}, \hat{3.75}]$	$[\hat{5}, \hat{6.75}]$	$[\hat{8}, \hat{9.6667}]$	$\hat{10}$	$[\hat{8}, \hat{9}]$	$[\hat{6.75}, \hat{9}]$

Table 7
RF number evaluations of importance of risk factors.

Risk	Decision makers			
factor	DM_1	DM_2	DM_3	DM_4
D	$[\hat{5}, \hat{7.25}]$	$[\hat{6}, \hat{8}]$	$[\hat{6.6667}, \hat{8.5}]$	$[\hat{7.25}, \hat{9}]$
O	$[\hat{4}, \hat{5.25}]$	$[\hat{4.6667}, \hat{5.6667}]$	$[\hat{4.6667}, \hat{5.6667}]$	$[\hat{5.25}, \hat{7}]$
S	$[\hat{5}, \hat{6}]$	$[\hat{5.6667}, \hat{6.3333}]$	$[\hat{5.6667}, \hat{6.3333}]$	$[\hat{6}, \hat{7}]$

$$cd_j = \frac{\alpha D_j^+}{\alpha D_j^+ + (1-\alpha)D_j^-} \tag{47}$$

where cd_j is the closeness degree which depends on PI solution, NI solution and a control parameter α belonging to [0, 1]. Taking $\alpha = 0.7, 0.8, 0.9$, the closeness degree and ranking results are given in Table 19.

It can be seen that with the variation of control parameter, the closeness degrees fluctuate in some range. However, the ranking results of failure modes remain unaffected: $Fm_1 \succ Fm_6 \succ Fm_5 \succ Fm_7 \succ Fm_8 \succ Fm_4 \succ Fm_3 \succ Fm_2$. The stability of the results is very high. As the ranking results do not change with control parameter α for the given assessment data, therefore the sensitivity of the parameter is low. Further, it can be checked from Table 19 that the closeness coefficient is somehow consistent and do not much effected by α, that is, α no longer effects the risk priority of failure modes. But, with the change of initial input data by the expert, the inconsistencies can occur. In this case, α can play a significant role in balancing the closeness coefficient for balancing the results.

5.1.2. Change of weight by parameters $\gamma_D, \gamma_O, \gamma_S$

The accuracy of the results also depend on weight information. Suppose a disturbance $\gamma \in [0, 1]$ is imposed on the weight w_i, $i \in \{D, O, S\}$ and w_k changes into $w_k^* = \gamma_k w_k$. The weights are changed for different variations of the parameters. The new weights are given in Table 20.

The closeness coefficients and ranking results of failure modes for different variations of γ_k are given in Table 21.

The ranking results of failure modes for different parameters are also demonstrated in Fig. 2.

In Fig. 2, the parameters $\gamma_D, \gamma_O, \gamma_S$ are labeled as d, o, s. It is easy to check that when the parameters vary in similar pattern, the ranking results are same, For example, the ranking of failure modes is same for $\gamma_k = 0.1, 0.2, 0.3$, for all $k \in \{D, O, S\}$. The ranking results of Fm_5 and Fm_6 fluctuate between 1 and 5, and the ranking of other failure modes do not fluctuate. If the parameter vary in a different pattern, then ranking results change little. So, the ranking results for different range of parameters do not much

Table 8
Cloud evaluations of failure modes for risk factor D.

Failure	D			
modes	DM_1	DM_2	DM_3	DM_4
Fm_1	(7.25, 0.25, 0.01)	(7.25, 0.25, 0.01)	(6.1665, 0.3889, 0.01)	(5.25, 0.4167, 0.01)
Fm_2	(2.5, 0.1667, 0.01)	(3.5, 0.1667, 0.01)	(3.5, 0.1667, 0.01)	(2.5, 0.1667, 0.01)
Fm_3	(2, 0.6666, 0.01)	(2, 0.6666, 0.01)	(1.5, 0.1667, 0.01)	(2.5, 0.1667, 0.01)
Fm_4	(2.625, 0.125, 0.01)	(1.625, 0.2083, 0.01)	(2.0834, 0.1945, 0.01)	(2.625, 0.125, 0.01)
Fm_5	(1.875, 0.0417, 0.01)	(1.875, 0.0417, 0.01)	(1.375, 0.125, 0.01)	(1.875, 0.0417, 0.01)
Fm_6	(1.5, 0.1667, 0.01)	(1.5, 0.1667, 0.01)	(2.1667, 0.2778, 0.01)	(3, 0.3333, 0.01)
Fm_7	(1.375, 0.125, 0.01)	(1.375, 0.125, 0.01)	(3.25, 0.2083, 0.01)	(1.9167, 0.1945, 0.01)
Fm_8	(2.25, 0.0833, 0.01)	(3.25, 0.25, 0.01)	(2.25, 0.0833, 0.01)	(2.25, 0.0833, 0.01)

Table 9
Cloud evaluations of failure modes for risk factor O.

Failure	O			
modes	DM_1	DM_2	DM_3	DM_4
Fm_1	(2.3334, 0.2222, 0.01)	(2.3334, 0.2222, 0.01)	(3.125, 0.2917, 0.01)	(1.625, 0.2083, 0.01)
Fm_2	(2.375, 0.125, 0.01)	(2.375, 0.125, 0.01)	(3.375, 0.2083, 0.01)	(2.9167, 0.1945, 0.01)
Fm_3	(6.125, 0.2917, 0.01)	(5.5834, 0.3056, 0.01)	(4.625, 0.2083, 0.01)	(4.625, 0.2083, 0.01)
Fm_4	(5.9167, 0.1945, 0.01)	(6.375, 0.2083, 0.01)	(5.9167, 0.1945, 0.01)	(4.875, 0.2917, 0.01)
Fm_5	(8.5, 0.1667, 0.01)	(8.5, 0.1667, 0.01)	(7, 0.3333, 0.01)	(7.8334, 0.2778, 0.01)
Fm_6	(10, 0, 0.01)	(10, 0, 0.01)	(10, 0, 0.01)	(10, 0, 0.01)
Fm_7	(6.5, 0.1667, 0.01)	(6.5, 0.1667, 0.01)	(7.5, 0.1667, 0.01)	(7.5, 0.1667, 0.01)
Fm_8	(4.875, 0.2917, 0.01)	(4.875, 0.2917, 0.01)	(6.25, 0.4167, 0.01)	(6.875, 0.375, 0.01)

Table 10
Cloud evaluations of failure modes for risk factor S.

Failure	S			
modes	DM_1	DM_2	DM_3	DM_4
Fm_1	(8, 875, 0.0417, 0.01)	(8.875, 0.0417, 0.01)	(8.875, 0.0417, 0.01)	(8.375, 0.125, 0.01)
Fm_2	(0.7223, 0.0185, 0.01)	(0.7223, 0.0185, 0.01)	(0.7083, 0.125, 0.01)	(1.5417, 0.1528, 0.01)
Fm_3	(2.375, 0.125, 0.01)	(2.9167, 0.1945, 0.01)	(3.375, 0.2083, 0.01)	(2.375, 0.125, 0.01)
Fm_4	(1.375, 0.125, 0.01)	(1.375, 0.125, 0.01)	(2.375, 0.2083, 0.01)	(0.9167, 0.1945, 0.01)
Fm_5	(2.5, 0.1667, 0.01)	(2.5, 0.1667, 0.01)	(3.5, 0.1667, 0.01)	(3.5, 0.1667, 0.01)
Fm_6	(0.7083, 0.125, 0.01)	(0.7223, 0.0185, 0.01)	(0.7223, 0.0185, 0.01)	(1.5417, 0.1528, 0.01)
Fm_7	(4.375, 0.125, 0.01)	(4.375, 0.125, 0.01)	(5.375, 0.2083, 0.01)	(4.9167, 0.1945, 0.01)
Fm_8	(4.75, 0.25, 0.01)	(6.25, 0.25, 0.01)	(5.25, 0.25, 0.01)	(5.75, 0.25, 0.01)

Table 11
Cloud evaluations of importance of risk factors.

Risk	Decision makers			
factor	DM_1	DM_2	DM_3	DM_4
D	(5.125, 0.375, 0.01)	(6, 0.3333, 0.01)	(6.5834, 0.3056, 0.01)	(7.125, 0.2917, 0.01)
O	(3.625, 0.2083, 0.01)	(4.1667, 0.1667, 0.01)	(4.1667, 0.1667, 0.01)	(5.125, 0.2917, 0.01)
S	(4.5, 0.1667, 0.01)	(5, 0.1111, 0.01)	(5, 0.1111, 0.01)	(5.5, 0.1667, 0.01)

Table 12
Aggregated cloud decision matrix.

AC			
Failure	Risk factors		
modes	D	O	S
Fm_1	(6.4791, 0.3354, 0.01)	(2.3542, 0.2383, 0.01)	(8.75, 0.0722, 0.01)
Fm_2	(3, 0.1667, 0.01)	(2.7604, 0.1677, 0.01)	(0.9237, 0.0996, 0.01)
Fm_3	(2, 0.4859, 0.01)	(5.2396, 0.515, 0.01)	(2.7604, 0.1677, 0.01)
Fm_4	(2.2396, 0.1677, 0.01)	(1.7604, 0.3354, 0.01)	(5.7709, 0.2259, 0.01)
Fm_5	(1.75, 0.1141, 0.01)	(3, 0.1667, 0.01)	(7.9584, 0.2469, 0.01)
Fm_6	(2.0417, 0.2469, 0.01)	(0.9237, 0.0996, 0.01)	(10, 0, 0.01)
Fm_7	(1.9792, 0.1677, 0.01)	(4.7604, 0.1677, 0.01)	(7, 0.1667, 0.01)
Fm_8	(2.5, 0.1443, 0.01)	(5.5, 0.25, 0.01)	(5.7188, 0.348, 0.01)

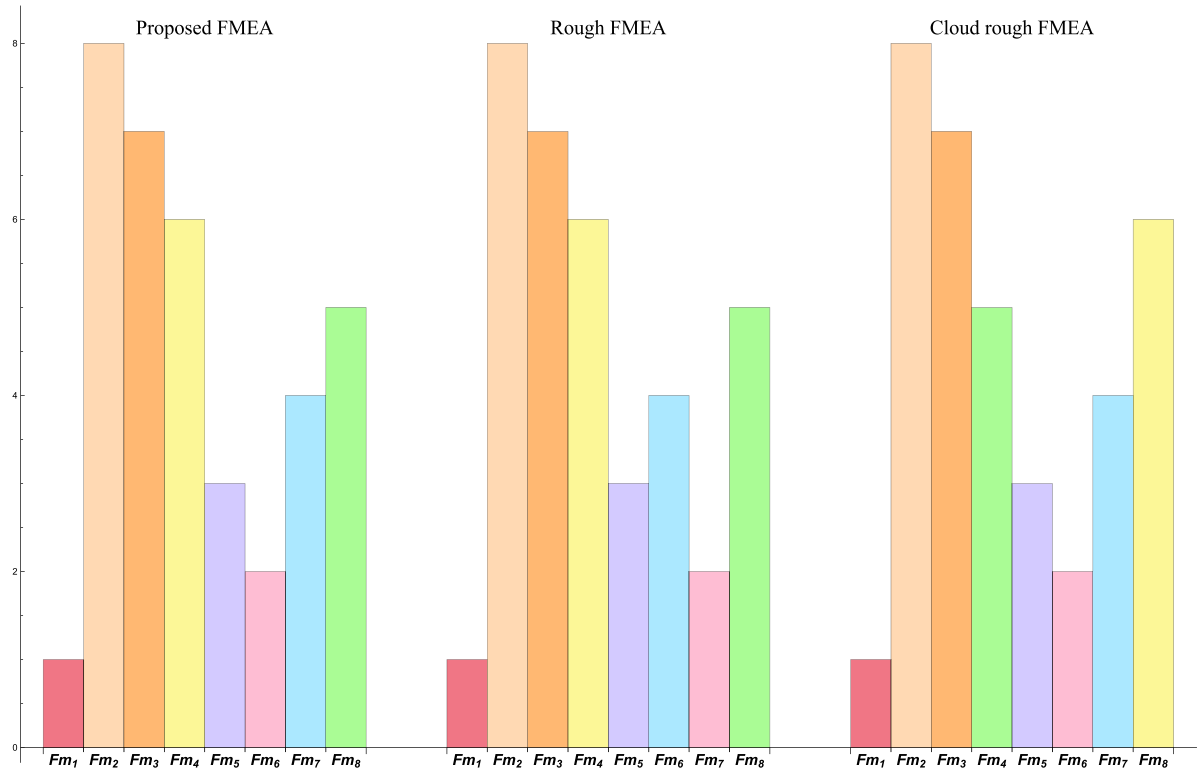

Fig. 3. Comparison of proposed FMEA with Rough FMEA and cloud rough FMEA.

Table 13
Aggregated cloud evaluations and subjective weights.

Risk factor	Aggregated cloud evaluations	Subjective weights (w^s)
D	(6.2084, 0.3279, 0.01)	0.4011
O	(4.2709, 0.2145, 0.01)	0.2759
S	(5, 0.1417, 0.01)	0.3230

fluctuate and the sensitivity of the parameters is not so high. The risk priority of failure modes do not much affect, therefore the proposed method presents good robustness.

6. Comparison analysis

In this section, the results of proposed RFIC modified TOPSIS method are compared with existing approaches, including conventional FMEA, rough TOPSIS and cloud rough TOPSIS. The advantages of the proposed model are illustrated as compared to existing FMEA approaches based on fuzzy numbers, rough sets and CM theory. In Table 22, the results failures of SV shaft working by four different methods are given.

In Table 22, the ranking of failure modes by proposed FMEA are mostly similar to rough FMEA except Fm_6 and Fm_7 which have same rankings as cloud rough FMEA which shows the authenticity of the proposed method. However, the closeness degrees are different from existing approaches as we have used the concepts of TF numbers, deviations, subjective and objective weights. The comparison of proposed FMEA with conventional FMEA and fuzzy FMEA is also given in Table 23.

Table 23 shows that there are differences between the rankings of failure modes by proposed method and fuzzy FMEA. In fuzzy FMEA, a single fixed value is considered to discuss uncertainty in DMs evaluations ignoring randomness which is not justified. In proposed method, we have discussed uncertainty using TF numbers, rough numbers as intervals and CM theory. Moreover, fuzzy FMEA does not differentiate the failure modes properly as Fm_5 and Fm_8 have the same rankings. In the proposed method, the ranking of all the failure modes is different. The comparison of proposed method with existing approaches is also shown with barcharts in Figs. 3 and 4.

To positively illustrate the out-performance and effectiveness of the proposed RFIC FMEA approach, several comparisons are made in this section. The proposed approach efficiently deals with different types of uncertainties in experts' decision analysis. It utilizes rough numbers to objectively and flexibly deal with imprecision and vagueness and, CM theory to reflect fuzziness and randomness in experts' evaluations. Both objective and subjective aspects of weight information is applied in the proposed methodology to obtain extensive importance of risk factors. The comparison results produced by various approaches are given in Table 22, 23 and Fig. 3, 4. The ranking results of all the methods show that Fm_1 and Fm_8 have, respectively the highest and lowest risk. It proves the accuracy of the proposed scheme but there are still differences among the methods.

The first comparison is performed with conventional RPN method. In RPN method, the failure modes are ranked by combining (by taking product) the three risk factors, including detection, occurrence and severity. The greater value of RPN implies the higher risk of failure mode to effect the system reliability. From Table 23 and Fig. 4, it can be seen that all the ranking results of two approaches are different except Fm_1, Fm_4 and Fm_5. The reason of these difference of ranking results between two approaches is that the RPN ignores uncertainty and gives equal importance to all risk factors. In RPN method, the different evaluations of risk factors for two different failure modes can give the same RPN, as all the risk factors are given equal relative importance in this method. The exact evaluations are used by the experts to assess failure modes, ignoring the vagueness that exists due to different approaches and experiences of experts. A slight difference of risk factor evaluations may lead to a larger difference

Table 14
Normalized cloud decision matrix.

Failure modes	Distances		
	D	O	S
Fm_1	(1.0000, 0.0579, 0.0022)	(0.4280, 0.0453, 0.0020)	(0.8750, 0.0164, 0.0013)
Fm_2	(0.4630, 0.0284, 0.0017)	(0.5019, 0.0341, 0.0020)	(0.0924, 0.0101, 0.0010)
Fm_3	(0.3087, 0.0754, 0.0016)	(0.9527, 0.0980, 0.0025)	(0.2760, 0.0174, 0.0010)
Fm_4	(0.3457, 0.0274, 0.0016)	(0.3201, 0.0618, 0.0019)	(0.5771, 0.0246, 0.0012)
Fm_5	(0.2701, 0.0189, 0.0016)	(0.5455, 0.0346, 0.0021)	(0.7958, 0.0281, 0.0013)
Fm_6	(0.3151, 0.0390, 0.0016)	(0.1679, 0.0188, 0.0018)	(1.0000, 0.0168, 0.0014)
Fm_7	(0.3055, 0.0271, 0.0016)	(0.8655, 0.0403, 0.0024)	(0.7000, 0.0204, 0.0012)
Fm_8	(0.3859, 0.0244, 0.0017)	(1.0000, 0.0547, 0.0026)	(0.5719, 0.0361, 0.0012)

Table 15
Combined weights of risk factors.

Failure modes	Distances		
	D	O	S
Fm_1	3.6518	0.8996	2.7625
Fm_2	03414	0.4228	5.0912
Fm_3	0.9778	1.7091	3.3226
Fm_4	0.4200	1.5905	0.3703
Fm_5	0.8560	0.1822	1.7962
Fm_6	0.6917	2.1914	4.0847
Fm_7	0.6804	1.5772	0.9180
Fm_8	0.1362	2.2345	0.5445
Variance	$Var_D = 2.0347$	$Var_O = 2.3443$	$Var_S = 8.2224$
w^O	0.1615	0.1860	0.6525
$w^O \times w^s$	0.0648	0.0513	0.2108
w	0.1982	0.1569	0.6448

in RPN values depending on other risk factor's assessment. Moreover, all the risk factors are given equal importance which is not effective in general practice. Some factors need to be over rated and others may be under rated. In the proposed approach, both objective and subjective uncertainties are studied by integrating rough sets with TF numbers. The importance of all risk factors is discussed according to their effects.

The second comparison is conducted with fuzzy based FMEA approaches [10,11]. Except Fm_1, Fm_4 and Fm_5, all the ranking results vary in both approaches. Fm_5 is ranked third in the proposed method, however, Fm_5 and Fm_5 are both ranked third in fuzzy FMEA, that is, repetition of ranking. The difference of ranking results is due to the techniques used to cope with uncertainty. In the proposed method, three different models are integrated to deal with interpersonal and intrapersonal uncertainty and randomness in experts' evaluations. But, in the fuzzy FMEA, the TF numbers are used to deal with uncertainty which can change the nature of the original data if different TF numbers are assumed. In the last few years, the FMEA methods based on fuzzy numbers and fuzzy sets have been effectively used to discuss uncertainty

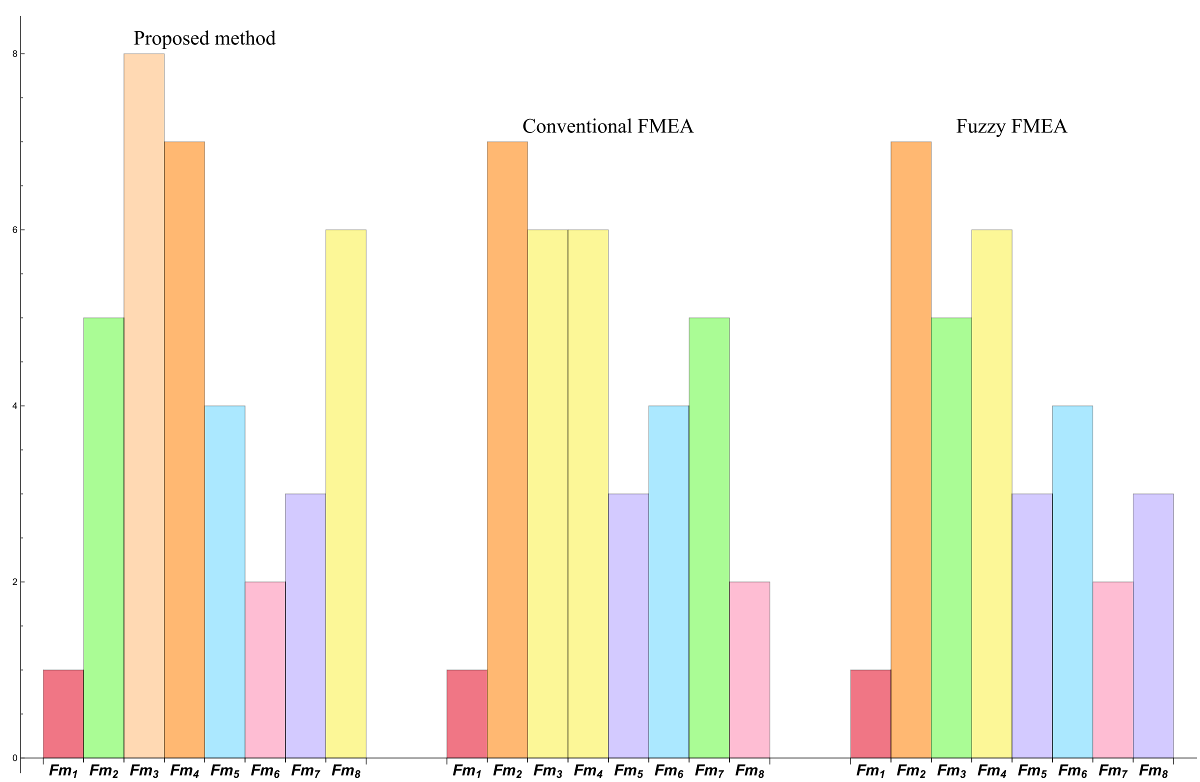

Fig. 4. Comparison of proposed FMEA with conventional FMEA and fuzzy FMEA.

Table 16
Weighted cloud decision matrix, PI and NI solutions.

Failure	Distances		
modes	*D*	*O*	*S*
Fm_1	(0.1982, 0.0258, 0.0010)	(0.0672, 0.0179, 0.0008)	(0.5642, 0.0132, 0.0010)
Fm_2	(0.0918, 0.0126, 0.0008)	(0.0787, 0.0135, 0.0008)	(0.0596, 0.0081, 0.0008)
Fm_3	(0.0612, 0.0336, 0.0007)	(0.1495, 0.0388, 0.0010)	(0.1780, 0.0140, 0.0008)
Fm_4	(0.0685, 0.0122, 0.0007)	(0.0502, 0.0245, 0.0008)	(0.3721, 0.0198, 0.0010)
Fm_5	(0.0535, 0.0084, 0.0007)	(0.0856, 0.0137, 0.0008)	(0.5131, 0.0226, 0.0010)
Fm_6	(0.0625, 0.0174, 0.0007)	(0.0263, 0.0074, 0.0007)	(0.6448, 0.0135, 0.0011)
Fm_7	(0.0606, 0.0121, 0.0007)	(0.1358, 0.0160, 0.0010)	(0.4514, 0.0164, 0.0010)
Fm_8	(0.0765, 0.0109, 0.0008)	(0.1569, 0.0217, 0.0010)	(0.3688, 0.0290, 0.0010)
S^+	(0.1982, 0.0084, 0.0007)	(0.1569, 0.0074, 0.0007)	(0.6448, 0.0081, 0.0008)
S^-	(0.0535, 0.0336, 0.0010)	(0.0263, 0.0388, 0.0010)	(0.0596, 0.0290, 0.0011)

Table 17
Distances of weighted cloud evaluations for each risk factor from PI and NI solutions.

Failure	*D*		*O*		*S*	
modes	Positive	Negative	Positive	Negative	Positive	Negative
Fm_1	0.0176	0.1525	0.1003	0.0619	0.0859	0.5205
Fm_2	0.1107	0.0594	0.0843	0.0780	0.5853	0.0212
Fm_3	0.1622	0.0079	0.0391	0.1232	0.4613	0.1337
Fm_4	0.1335	0.0366	0.1237	0.0385	0.2611	0.3219
Fm_5	0.1447	0.0254	0.0777	0.0846	0.1173	0.4601
Fm_6	0.1447	0.0254	0.1306	0.0317	0.0054	0.6007
Fm_7	0.1413	0.0288	0.0299	0.1324	0.1854	0.4045
Fm_8	0.1242	0.0459	0.0145	0.1477	0.2555	0.3093

Table 18
Ranking of failure modes.

Failure modes	D^+	D^-	closeness degree	Ranking
Fm_1	0.1332	0.5459	0.1962	1
Fm_2	0.6016	0.1003	0.8572	8
Fm_3	0.4905	0.1820	0.7294	7
Fm_4	0.3183	0.3263	0.4938	6
Fm_5	0.2018	0.4685	0.3011	3
Fm_6	0.1950	0.6021	0.2446	2
Fm_7	0.2350	0.4266	0.3552	4
Fm_8	0.2844	0.3459	0.4513	5

Table 19
Ranking results under different control parameters.

Failure	$\alpha = 0.7$		$\alpha = 0.8$		$\alpha = 0.9$	
mode	*cd*	Ranking	*cd*	Ranking	*cd*	Ranking
Fm_1	0.3629	1	0.4940	1	0.6872	1
Fm_2	0.9333	8	0.9600	8	0.9818	8
Fm_3	0.8628	7	0.9151	7	0.9604	7
Fm_4	0.6947	6	0.7960	6	0.8977	6
Fm_5	0.5013	3	0.6328	3	0.7950	3
Fm_6	0.4304	2	0.5643	2	0.7445	2
Fm_7	0.5624	4	0.6878	4	0.8322	4
Fm_8	0.6574	5	0.7669	5	0.8810	5

in experts' evaluations. However, all such approaches meet some limitations of pre-defined membership functions, large number of fuzzy rules, data distribution and considering no randomness during risk assessment. A rough number is an objective mathematical model to handle uncertainty using original data-set. In the proposed method, the CM theory is integrated with RF numbers to overcome the shortcomings of existing fuzzy based approaches and deal with fuzziness and randomness in data using upper and lower approximations, TF numbers and stochastic disturbance.

The third difference comparison is conducted with rough TOPSIS [7]. Song et al. [7] introduced the concept of rough numbers and studied rough TOPSIS method by integrating rough numbers with TOPSIS approach. There is a slight difference between the rankings of both methods as given in Table 22. All failure modes have same rank except Fm_6 and Fm_7. Fm_6 is ranked four in rough FMEA while it is ranked two in the proposed method; Fm_7 has rank two in rough FMEA and four in the other method. The difference is due to the different weighting schemes of both approach. In rough FMEA, only subjective weights are computed, however, in the proposed method, hybrid weights are computed using both subjective and objective aspects. In rough TOPSIS method, the uncertainty can be studied from original data-set without predefined parameters and additional suppositions. However, rough FMEA cannot deal with randomness and intrapersonal linguistic uncertainty in the given evaluations, while the evaluations are usually given in linguistic terms. To manipulate the randomness in experts' evaluations and intrapersonal linguistic uncertainty, the RF numbers are used and then converted into clouds in the proposed method.

The fourth comparison is performed with cloud rough TOPSIS. Li et al. [26,27] proposed CM theory based rough TOPSIS approaches to study failure modes in engineering problems and to handle randomness and uncertainty in experts' evaluations. The proposed method is an extension of [27] to enhance the uncertainty characterization and information aggregation in FMEA methods. It is based on TF number evaluations from the original data-set, without any pre-defined parameters or auxiliary suppositions in the subsequent processing. It provides a way to measure the uncertainty in fuzzy importance and linguistic variables.

Further, Chen et al. [36] introduced RF numbers to overcome the limitations of rough numbers based FMEA techniques. The MCDM methods based on RF numbers and FR numbers [8] are a hot topic of research nowadays to handle different types of uncertainties in one mathematical model. However, the randomness in evaluations or judgements of experts is not considered in these approaches. Cloud theory based on probability theory and fuzzy sets handles the issue that degrees of membership in fuzzy sets are accurate by allowing a stochastic disturbance of membership values. In the proposed RFIC modified TOPSIS method, CM theory is integrated with RF numbers to describe the randomness and uncertainty in the judgements of experts.

Table 20
New weights of risk factors for different variations of parameters $\gamma_D, \gamma_O, \gamma_S$.

Weights	Parameters						
	$\gamma_D = 1$ $\gamma_O = 1$ $\gamma_S = 1$	$\gamma_D = 0.1$ $\gamma_O = 0.1$ $\gamma_S = 0.1$	$\gamma_D = 0.1$ $\gamma_O = 0.2$ $\gamma_S = 0.3$	$\gamma_D = 0.4$ $\gamma_O = 0.3$ $\gamma_S = 0.2$	$\gamma_D = 0.5$ $\gamma_O = 0.4$ $\gamma_S = 0.2$	$\gamma_D = 0.5$ $\gamma_O = 0.4$ $\gamma_S = 0.1$	$\gamma_D = 0.1$ $\gamma_O = 0.1$ $\gamma_S = 0.1$
w_D	0.1982	0.0198	0.0198	0.0793	0.0991	0.0991	0.0991
w_O	0.1569	0.0157	0.0314	0.0471	0.0628	0.0628	0.0471
w_S	0.6448	0.0648	0.1934	0.1290	0.1290	0.0645	0.0645

Table 21
Ranking results under weight disturbance control parameters.

Failure modes	$\gamma_D = 0.1$ $\gamma_O = 0.1$ $\gamma_S = 0.1$		$\gamma_D = 0.1$ $\gamma_O = 0.8$ $\gamma_S = 0.4$		$\gamma_D = 0.1$ $\gamma_O = 0.2$ $\gamma_S = 0.3$		$\gamma_D = 0.4$ $\gamma_O = 0.3$ $\gamma_S = 0.2$		$\gamma_D = 0.5$ $\gamma_O = 0.4$ $\gamma_S = 0.2$		$\gamma_D = 0.5$ $\gamma_O = 0.4$ $\gamma_S = 0.1$		$\gamma_D = 0.5$ $\gamma_O = 0.3$ $\gamma_S = 0.1$	
	cd	Rank	*cd*	Rank	*cd*	Rank	*cd*	Rank	*cd*	Rank	*cd*	Rank	*cd*	Rank
Fm_1	0.2202	1	0.2889	3	0.1809	2	0.2356	1	0.2606	1	0.3122	1	0.2695	1
Fm_2	0.7685	8	0.7847	8	0.8679	8	0.7519	8	0.7241	8	0.6351	6	0.6517	6
Fm_3	0.7107	7	0.6224	7	0.7348	7	0.7085	7	0.6868	7	0.6571	7	0.7089	8
Fm_4	0.4701	6	0.5156	6	0.4509	6	0.5419	6	0.5764	6	0.6639	8	0.6534	7
Fm_5	0.2769	3	0.2816	1	0.2147	3	0.3976	3	0.4420	4	0.5534	4	0.5630	5
Fm_6	0.2459	2	0.3010	4	0.1436	1	0.3626	2	0.4176	2	0.5701	5	0.5535	4
Fm_7	0.3262	4	0.2832	2	0.3039	4	0.4128	4	0.4372	3	0.4990	3	0.5313	3
Fm_8	0.4030	5	0.3651	5	0.4237	5	0.4551	5	0.4581	5	0.4692	2	0.5103	2

Table 22
Comparison of proposed method with existing approaches.

Failure mode	Proposed FMEA		Rough FMEA [10]		Cloud rough FMEA [27]	
	cd	Ranking	*cd*	Ranking	*cd*	Ranking
Fm_1	0.1962	1	0.367	1	0.129	1
Fm_2	0.8572	8	0.737	8	0.895	8
Fm_3	0.7294	7	0.600	7	0.730	7
Fm_4	0.4938	6	0.587	6	0.453	5
Fm_5	0.3011	3	0.474	3	0.234	3
Fm_6	0.2446	2	0.493	4	0.200	2
Fm_7	0.3552	4	0.473	2	0.299	4
Fm_8	0.4513	5	0.495	5	0.487	6

Table 23
Comparison of proposed method with existing approaches.

Failure mode	Proposed FMEA		Conventional FMEA		Fuzzy FMEA	
	cd	Ranking	RPN	Ranking	*cd*	Ranking
Fm_1	0.1959	1	252	1	0.402	1
Fm_2	0.4796	5	48	7	0.681	7
Fm_3	0.7273	8	96	6	0.587	5
Fm_4	0.5055	7	96	6	0.592	6
Fm_5	0.3213	4	144	3	0.509	3
Fm_6	0.2491	2	120	4	0.514	4
Fm_7	0.2591	3	112	5	0.497	2
Fm_8	0.4804	6	180	2	0.509	3

6.1. Comparison of weight information

To illustrate the effect of weight values and weighting scheme on the assessment results, the group assessment values of failure modes are considered to conduct this comparison. Based on the criteria importance of DMs given in Table 4, the interval assessment information of DMs can be easily obtained. The linguistic assessment values of DM_1 for the risk factor D are $\{SM, SH, H, VH\}$. The corresponding crisp values and TF number values are $\{5, 7, 8, 9\}$ and $\{(3, 4, 5), (5, 6, 7), (6, 7, 8), (7, 8, 9)\}$, respectively. The fuzzy number intervals are $\{[4, 6], [6, 8], [7, 9], [8, 10]\}$ which are fixed at two. The rough numbers are clouds values, which are computed in Table 24. Based on the work presented by Liu et al. [60], the cloud value (Ex, En, HEn) can be converted into cloud interval $[\underline{\alpha}, \overline{\alpha}]$ as $\underline{\alpha} = Ex - 3En$ and $\overline{\alpha} = Ex + 3En$. The normal cloud intervals for the risk factor D are [5,7.25], [6.0001,7.9999], [6.6666,8.5002] and [7.2499,9.0001]. The RF numbers for the risk factor D are given in Table 7. Based on the scales in Table 2, the RF number intervals can be considered as [5,7.25], [6,8], [6.6667,8.5] and [7.25,9]. The comparison of weight information for the risk factor D is demonstrated in Fig. 5.

The crisp values are used in TOPSIS method to reflect DM's judgements which causes information loss [41,42]. Second, the fuzzy intervals are fixed at two which is not enough to cope with uncertainty. Third, the ratio of length of adjacent normal cloud intervals is fixed at one, which cannot efficiently deal with uncertainty. For example, for the normal cloud intervals [5,7.25], [6.0001,7.9999], [6.6666,8.5002] and [7.2499,9.0001]; the ratio of adjacent distances is $1.7502/1.8333 \approx 1.8333/2 \approx 2/2.25 \approx 1$. Fourth, the RF number intervals are convenient to deal with both interpersonal and intrapersonal uncertainties [36] but the randomness of DMs judgements is ignored in these intervals. The RF intervals are somehow similar to proposed RFIC intervals but still there is a major difference of integration of CM theory. All in all, the proposed RFIC model manipulates individual uncertainty along with group uncertainty and provides more realistic solutions to risk assessment problems.

6.2. Simulation analysis

In this subsection, a detailed simulation analysis is conducted to check the efficiency and merits of the proposed risk assessment model. The deviation is taken as an indicator to measure the accuracy of the results generated by different DM approaches. The deviation can be computed using the formula $|cd_j - cd_{max}|/cd_{max}$, where cd_j is the closeness degree of the jth failure mode and cd_{max} is the maximum of all closeness degrees. The deviations computed by various risk assessment models are given in Table 25 and Fig. 6.

Table 24
Rough intervals and cloud values for risk factor *D*.

Values	Decision makers			
	DM_1	DM_2	DM_3	DM_4
Crisp values	5	7	8	9
Rough intervals	[5, 7.25]	[6, 8]	[6.6667, 8.5]	[7.25, 9]
Cloud values	(6.125, 0.375, 0.01)	(7, 0.3333, 0.01)	(7.5834, 0.3056, 0.01)	(8.125, 0.2917, 0.01)
Interval clouds	[5, 7.25]	[6.0001, 7.9999]	[6.6666, 8.5002]	[7.2499, 9.0001]
RFN intervals	[5, 7.25]	[6, 8]	[6.6667, 8.5]	[7.25, 9]
RFICs	(5.125, 0.375, 0.01)	(6, 0.3333, 0.01)	(6.5834, 0.3056, 0.01)	(7.125, 0.2917, 0.01)
RFIC intervals	[4, 6.25]	[5.0001, 6.9999]	[5.6666, 7.5005]	[6.2499, 8.0001]

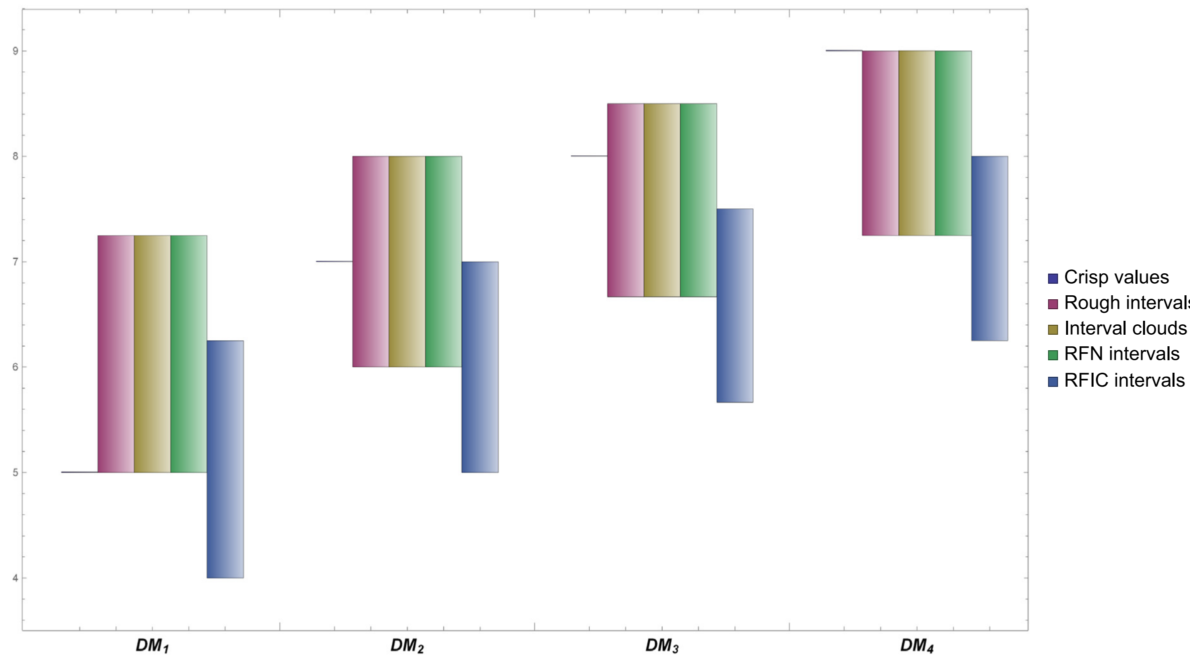

Fig. 5. Comparison of proposed FMEA with Rough FMEA and cloud rough FMEA.

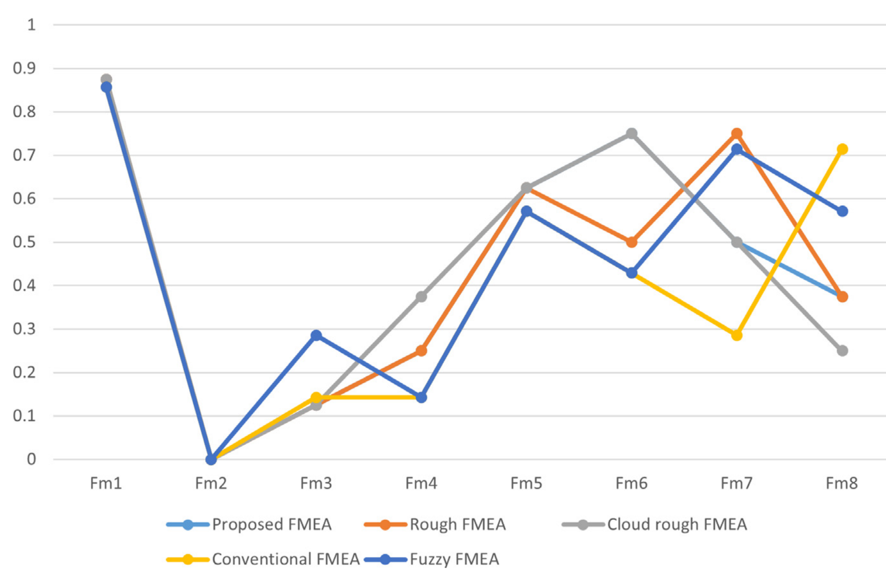

Fig. 6. Deviations of failure modes generated by different FMEA approaches.

Table 25
Result deviations of failure modes.

Failure	Proposed FMEA	Rough FMEA [10]	Cloud rough FMEA [27]	Conventional FMEA	Fuzzy FMEA
Fm_1	0.875	0.875	0.875	0.857	0.857
Fm_2	0	0	0	0	0
Fm_3	0.125	0.125	0.125	0.143	0.286
Fm_4	0.25	0.25	0.375	0.143	0.143
Fm_5	0.625	0.625	0.625	0.571	0.571
Fm_6	0.75	0.5	0.75	0.429	0.429
Fm_7	0.5	0.75	0.5	0.286	0.714
Fm_8	0.375	0.375	0.25	0.714	0.571

Table 26
Dispersion degree of failure modes.

Failure	Risk factors		
modes	*D*	*O*	*S*
Fm_1	0.3354	0.2383	0.0722
Fm_2	0.1667	0.1677	0.0996
Fm_3	0.4859	0.515	0.1677
Fm_4	0.1677	0.3354	0.2259
Fm_5	0.1141	0.1667	0.2469
Fm_6	0.2469	0.0996	0
Fm_7	0.1677	0.1677	0.1667
Fm_8	0.1443	0.25	0.348

Apparently, the developed model provides more smoothness and fluctuations among failure modes than the other approaches. It shows that the proposed RFIC TOPSIS method can generate higher discrimination among the alternatives or failure modes which can give more convenience to the experts. The deviation analysis also illustrates the strength and effectiveness of the proposed model.

6.3. Evaluation and result analysis

Figs. 3 and 4 illustrate that all the FMEA approaches recommend Fm_1 as the most risky failure mode. The risks of Fm_1 over the other failure modes are given as follows;

For risk factors S and D, Fm_1 has the highest risk as assessed by the experts. Besides its detection is low. However, S, O and D are important factors to evaluate the risk percentage of any failure mode. For risk factor O, the experts have rare leaning than other risk factors, so the risk weights are less important to influence the final results. For risk factors S and D, the impact of Fm_1 in power generation plant is widely important to consider and is mostly rated by experts. Conclusively, Fm_1 is the most significant failure mode to be analyzed and detected at earlier phase for factors S and D whose weight are important. However, by most of the experts, its occurrence is rare. Thus, it is reasonable enough to choose Fm_1 as the most important failure mode. From Table 23, the rankings of Fm_2 and Fm_3 have some differences in their rankings for different methods. From Table 3, Fm_2 and Fm_3 has no apparent and severe impact on the system but risk factor S and D. However, its occurrence is high. Hence, it is not easy to judge that which of failure mode Fm_2 or Fm_3 has less effect on the system. The relative dispersion degree which is basically the entropy value En (Li and Du [61])of each cloud evaluation is used to evaluate the vagueness in the assessment information of different failure modes. Table 26 presents the dispersion degree of the failure modes as computed from Table 12.

Table 26 shows that Fm_3 has the biggest dispersion degree, which indicates the significant uncertainty when experts evaluate risk impact of failure mode Fm_3. The dispersion degree can also be evaluated by hyper entropy HEn. In this paper, the HEn is not dependent on En, therefore, the entropy value is considered as dispersion degree. Thus, according to this view point, the proposed approach and RIC TOPSIS [27] give more flexible and reliable results in high uncertain environment than other rough theory and fuzzy based approaches. But, the fuzzy and rough number based approaches do not depict the randomness of interval boundaries which effect the efficacy of final results. However, the proposed model rationally and effectively presents the randomness in experts' judgements and discussed both intrapersonal and interpersonal uncertainties. Table 26 shows that Fm_1 has low dispersion degree as compared to Fm_1, it is reasonable to say that Fm_1 has the higher ranking and so the assessment achieved by developed method are effective and rational. Furthermore, Table 26 shows that Fm_5 and Fm_6 can also be given high ranking due to not very low dispersion degree.

In addition, the developed approach not only uses fuzzy numbers to assess failure modes towards different risk factors but also considers both objective and subjective aspect of weighting scheme. However, other weighting schemes use crisp scales to express decision makers' preferences or compute weights ignoring one of the objective or subjective aspects. Thus, it can be concluded that the developed approach is more reliable and significant for failure modes assessment and selection.

6.4. Mathematical and applied implications

This study develops a novel RFIC TOPSIS technique to discuss failure modes in complex systems regarding different risk factors under interpersonal, intrapersonal uncertain environments and unknown weights.

A study of analysis of failure modes and risk assessment in a power generation plant is put forward. This paper first contributes to the development of new linguistic assessment model, namely, RFICs for risk assessment and prioritization. The RFIC model overcomes the limitations of CM theory in dealing with subjective uncertainty in experts' evaluations and the calculation method of classical clouds. It also upgrades the performance of rough numbers and fuzzy sets for handling randomness in individual uncertainty. It has basic properties of normal cloud model and can deal with vagueness and randomness simultaneously by utilizing plenty of cloud drops instead of crisp values. In RFIC model, the intrapersonal uncertainty (personal judgment vagueness) and interpersonal uncertainty (group assessment inconsistency) can be manipulated under complicated risk assessment problems. So, the RFIC assessment model rationally and reliably study experts' evaluations to make the final results close to practical implications.

Merits of the proposed method

This study presents a hybrid DM model for risk assessment problems by integrating RFIC model, hybrid weighting scheme and TOPSIS approach to fully capture different uncertainties involved in assessment information. According to various comparisons discussed in this section, the merits of the proposed RFIC TOPSIS approach are as follows:

1. The proposed model uses rough approximations of TF numbers to study objective and subjective uncertainty without using pre-defined parameters and additional suppositions. All the information is extracted from given data which reduces the dependency on DMs and it decreases the possibility of uncertainty.

2. Another way to reduce the dependency on DMs are the entropy weights computed from original data-set. But, the computation of RF entropy weights is itself a difficult task. Therefore, in the proposed approach, combined weights are computed from given importance of risk factor which is much simpler than entropy weights.

Table 27
Merits of the proposed method.

Methods to deal with uncertainty	Proposed FMEA	Rough FMEA [10]	Cloud rough FMEA [27]	Fuzzy FMEA	Conventional FMEA
Manipulation of uncertainty	✓	✓	✓	✓	×
Hybrid weights of risk factors	✓	×	✓	×	×
Considering randomness	✓	×	✓	×	×
Considering interpersonal uncertainty	✓	✓	✓	×	×
Considering intrapersonal uncertainty	✓	×	×	✓	×
Interval flexibility	✓	✓	✓	×	×

3. The randomness in the experts' evaluations is also studied along with other uncertainties by computing cloud evaluations of RF numbers. The concepts like mathematical expectation and statistical variance are used to aggregate the cloud evaluations of different experts.

The superiority of the proposed model as compared to other existing methods in manipulating effective decision results of failure modes in illustrated in Table 27.

Limitations of the proposed method

Besides all the advantages, the proposed RFIC TOPSIS method meet some limitations at the same time. The limitations of the proposed approach are as follows:

1. The proposed model uses the concept of RF numbers which are complex mathematical structures in nature. The computations of RF numbers are complicated and time consuming, therefore increases the calculation as well as time complexity of the method.

2. The TF number evaluations given in Table 2 are arbitrary and not unique. The supposition of different TF numbers may give different results of the same problem. This phenomenon is itself an uncertainty which is not considered in this method.

Practical significance

Besides for the mathematical significance, the proposed study provides practical significance for risk assessment and prioritization. The defined failure modes and risk factors can help the engineers and DMs to evaluate a suitable solution of risk assessment from comprehensive aspect. These factors can also direct the engineers to repair the failures in power generation plant at the early stage. The proposed DM model can be implemented by engineers for utilizing different types of uncertainties in the ranking process of failure modes. The developed scheme can be studied as an application program for the large data set which can reduce the manual workload of engineers and increases accuracy of the ranking process. Based on the proposed DM model, dealing with complex risk assessment problems is quite easy, the implementation cost and effort can be reduced.

7. Conclusion and future directions

Risk assessment has obtained much attention from modern business which can reduce the early errors occurring in automatic machines and improves the accuracy of production process. In this research study, a novel FMEA approach is constructed based on RFICs and extended decision support mathematical model to address the shortcomings of existing approaches. It enhances the decision makers' risk assessment information to generate ranking results close to existent situation. Moreover, the efficacy of the proposed scheme is validated with a case study of risk priority assessment of failure modes of steam valve shaft working in turbine system. Then, a sensitivity analysis, detailed comparative study and simulation analysis with other existing approaches is given to demonstrate the performance of proposed method. The comparison illustrated using numerical tables and graphs show that the given decision model is more rational and effective to handle intrapersonal and interpersonal uncertainties in supplying risk assessment assistance. The prime contributions of the proposed decision model are summarized as follows:

1. The potentially significant failure modes and risk factors are identified to enhance the reliability, efficacy, sustainable working of automatic machines and environmental benefits.

2. RF numbers are utilized to manipulate fuzziness of experts' assessment information in risk decision analysis. It provides more flexible intervals of TF numbers to discuss subjectivity using approximations without additional data distribution.

3. The novel technique of RFIC is developed to handle linguistic information which overcomes the deficiencies of RF numbers and CM theory. It upgrades the performance of RF numbers and CM theory to solve risk assessment decision problems.

4. The hybrid weighting scheme is utilized to study the importance of experts in both objective and subjective aspects. The relationship among risk factors are established by aggregation method.

Although the proposed decision model contributes in both practical and theoretical implications, it still has some deficiencies that enhance the need of further studies in this domain. First, the team of only four experts is formed which may lead to the manipulation of final results. So, more experts need to be invited in the future to conduct a large scale risk assessment decision analysis. Secondly, the transformation from linguistic data to RFICs is a complicated process and not fully consider the possible kinds of analytical processes in evaluation of linguistic data. Third, this study considerably cope with certain kinds of uncertainties but it fails to illustrate the probability of linguistic information for each term. In the future, this factor must be considered in assessment term set. Fourth, the arbitrary selection of TF number scales may give different results for the same risk assessment problem. A mechanism needs to proposed for the computation of fuzziness from initial linguistic data. The proposed study can be further extended to (1) Rough m-polar fuzzy FMEA approaches; (2) Pythagorean fuzzy FMEA approaches; (3) FMEA based on cloud rough fuzzy numbers.

Ethical approval

This article does not contain any studies with human participants or animals performed by the author.

CRediT authorship contribution statement

Musavarah Sarwar: Worked on the concept, design, anlysis and modeling of proposed RFIC TOPSIS method, Applied it on a real-world problem and proved its effectiveness with various tests. **Ghous Ali:** Proof read the article, Checked the validity of the method. **Nauman Riaz Chaudhry:** Checked the validity of the algorithm and the applicability of cloud model theory.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

This research has no additional data.

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