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Solitary wave solutions of a generalized scale-invariant analog of the Korteweg–de Vries equation via applications of four mathematical methods

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In this paper, we have studied a generalized scale-invariant analog of the well-known Korteweg–de Vries (KdV) equation. The generalized scale-invariant analog of the Korteweg–de Vries (SIdV) plays as a bridge between the KdV equation. The generalized SIdV model was discovered recently, and shares the same one-soliton solution as the KdV equation. By employing four mathematical methods, several types of exact and solitary wave solutions are established. For the physical behavior of the model, some solutions are plotted graphically by imparting specific values to the parameters under constrain condition. Hence, reconnoitered elucidations have profitable rewards in the field of mathematical physics.

Keywords: Generalized SIdV equation; soliton solutions.

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1. Introduction

Consider the generalized scale-invariant analog of the Korteweg–de Vries (KdV) equation proposed in Ref. 1 and given by

$$U_t + \left(3(1-\delta_1)U + (\delta_1+1)\frac{U_{xx}}{U}\right)U_x = \gamma_1 U_{xxx}. \tag{1}$$

Here, δ_1 and γ_1 are nonzero constants. Equation (1) can be thought of as a KdV-like equation with an advecting velocity given by the expression $(3(1-\delta_1)U + (\delta_1+1)\frac{U_{xx}}{U})$.

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Note that when $\delta_1 = -1$ and $\gamma_1 = -1$ then Eq. (1) reduces into KdV equation and also SIdV is called a natural extension of KdV model. A recent work[2] has shown that there are resilient associations between the Sylvester equation and integrable systems such as the KdV and SIdV models. Equation (1) is extensively used in control theory, image restoration and in signal processing similar to Sylvester equation.[3] Equation (1) was lately studied in Ref. 4 employing the dynamical system theory, and traveling waves of bell type and valley type were investigated. Our main goal here is to construct the exact and solitary wave solutions of the generalized SIdV equation in closed forms even when δ_1 and γ_1 are not equal to ± 1.

There are several known authoritative schemes that can be utilized to establish exact solutions of nonlinear partial differential equations (NPDEs)[5–10] such as Hirota's bilinear technique,[11] the inverse scattering transform approach,[12] the (G/G')-expansion scheme,[13,14] the Riccati–Bernoulli sub-ODE method,[15,16] the homogeneous balance method,[17] the generalized Riccati equation mapping approach[18–29] and many more.[30–47] Here, we have applied four mathematical methods called extended simple equation method,[48] modified extended auxiliary method,[49] (G/G')-expansion method[50] and modified F-expansion method to construct exact and solitary wave solutions of SIdV model mentioned in Eq. (1). The derived solutions have fruitful applications in nonlinear science. The remaining part of this paper is arranged as follows. Three mathematical methods are temporarily designated in Sec. 2. In Sec. 3, the proposed schemes are functionalized to the generalized SIdV model. Section 4 concludes.

2. Description of the Methods

Let

$$T_1(U, U_t, U_x, U_{xx}, U_{xt}, \ldots) = 0. \tag{2}$$

Let

$$U = U(\xi), \quad \xi = kx - \omega t. \tag{3}$$

Put (3) into (2)

$$T_2(U, U', U'', \ldots) = 0. \tag{4}$$

2.1. *Extended simple equation method*

Let (4) has solution

$$U(\xi) = \sum_{i=-N}^{N} A_i \Psi^i(\xi). \tag{5}$$

Let Ψ satisfy

$$\Psi' = c_0 + c_1\Psi + c_2\Psi^2 + c_3\Psi^3. \tag{6}$$

Put (5) with (6) into (4). Solve the achieved system for the solution of (2).

2.2. *Modified extended auxiliary equation mapping method*

Let solution of (4) be

$$U = \sum_{i=0}^{N} A_i \Psi^i + \sum_{i=-1}^{-N} B_{-i} \Psi^i + \sum_{i=2}^{N} C_i \Psi^{i-2} \Psi' + \sum_{i=1}^{N} D_i \left(\frac{\Psi'}{\Psi}\right)^i. \tag{7}$$

Let Ψ satisfy

$$\Psi' = \sqrt{\beta_1 \Psi^2 + \beta_2 \Psi^3 + \beta_3 \Psi^4}. \tag{8}$$

Put (7) with (8) into (4), solve the obtained system for the require destination of (2).

2.3. *The (G'/G)-expansion method*

Let (4) have the solution

$$U = A_0 + \sum_{i=1}^{N} A_i \left(\frac{G'}{G}\right). \tag{9}$$

Let

$$G'' = -\lambda_1 G' - \mu_1 G. \tag{10}$$

Put (9) with (10) into (4), solve the obtained system for the require destination of (2).

2.4. *Modified F-expansion method*

Let solution of (2) be

$$U = a_0 + \sum_{i=1}^{N} a_i F^i(\xi) + \sum_{i=1}^{N} b_i F^{-i}(\xi). \tag{11}$$

Let F oblige

$$F' = A + BF + CF^2. \tag{12}$$

Put (11) with (12) into (4). Solve obtained system to establish the required solution of (2).

3. Applications

3.1. *Application of extended simple equation method*

Putting Eq. (3) into Eq. (1), integrating once by omitting the constant of integration, we have

$$k^3 U^2(\gamma_1 + \delta_1 + 1) - 2\gamma_1 k^3 U U'' + 2(1-\delta_1) k U^3 - \omega U^2 = 0. \tag{13}$$

Let (13) have the solution

$$U = A_2\Psi^2 + A_1\Psi + \frac{A_{-2}}{\Psi^2} + \frac{A_{-1}}{\Psi} + A_0. \tag{14}$$

Put (14) with (6) into (13).

Case 1. $c_3 = 0$

FAMILY-I

$$\begin{aligned} A_0 &= -\frac{\gamma_1(c_1^2 + 2c_0c_2)k^2}{\delta_1 - 1}, \quad A_2 = -\frac{6\gamma_1 c_2^2 k^2}{\delta_1 - 1}, \quad A_1 = -\frac{6\gamma_1 c_1 c_2 k^2}{\delta_1 - 1}, \\ \omega &= k^3(2\gamma_1 c_1^2 - 8\gamma_1 c_0 c_2 + \gamma_1 + \delta_1 + 1), \quad A_{-2} = 0, \quad A_{-1} = 0. \end{aligned} \tag{15}$$

Put (15) into (14)

$$\begin{aligned} U_1 = &\left(\frac{(3\gamma_1 c_1 k^2)\left(c_1 - \sqrt{4c_2c_0 - c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0 - c_1^2}(\xi + \xi_0)\right)\right)}{\delta_1 - 1}\right) \\ &- \left(\frac{\gamma_1(c_1^2 + 2c_0c_2)k^2}{\delta_1 - 1}\right) - \left(\frac{(3\gamma_1 k^2)\left(c_1 - \sqrt{4c_2c_0 - c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0 - c_1^2}(\xi + \xi_0)\right)\right)^2}{2(\delta_1 - 1)}\right), \quad 4c_0c_2 > c_1^2. \end{aligned} \tag{16}$$

FAMILY-II

$$\begin{aligned} A_0 &= -\frac{\gamma_1(c_1^2 + 2c_0c_2)k^2}{\delta_1 - 1}, \quad A_{-2} = -\frac{6\gamma_1 c_0^2 k^2}{\delta_1 - 1}, \quad A_{-1} = -\frac{6\gamma_1 c_0 c_1 k^2}{\delta_1 - 1}, \\ A_2 &= 0, \quad A_1 = 0, \quad \omega = k^3(2\gamma_1 c_1^2 - 8\gamma_1 c_0 c_2 + \gamma_1 + \delta_1 + 1). \end{aligned} \tag{17}$$

Substitute (17) into (14)

$$\begin{aligned} U_2 = &-\left(\frac{\gamma_1(c_1^2 + 2c_0c_2)k^2}{\delta_1 - 1}\right) - \left(\frac{6\gamma_1 c_0 c_1 k^2}{\frac{(\delta_1 - 1)\left(-\left(c_1 - \sqrt{4c_2c_0 - c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0 - c_1^2}(\xi + \xi_0)\right)\right)\right)}{2c_2}}\right) \\ &- \left(\frac{6\gamma_1 c_0^2 k^2}{(\delta_1 - 1)\left(-\frac{c_1 - \sqrt{4c_2c_0 - c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0 - c_1^2}(\xi + \xi_0)\right)}{2c_2}\right)^2}\right), \quad 4c_0c_2 > c_1^2. \end{aligned} \tag{18}$$

Case 2. $c_0 = c_3 = 0$

$$\begin{aligned} A_0 &= A_{-2} = A_{-1} = 0, \quad A_2 = -\frac{6\gamma_1 c_2^2 k^2}{\delta_1 - 1}, \quad A_1 = -\frac{6\gamma_1 c_1 c_2 k^2}{\delta_1 - 1}, \\ \omega &= -k^3(2\gamma_1 c_1^2 - \gamma_1 - \delta_1 - 1). \end{aligned} \tag{19}$$

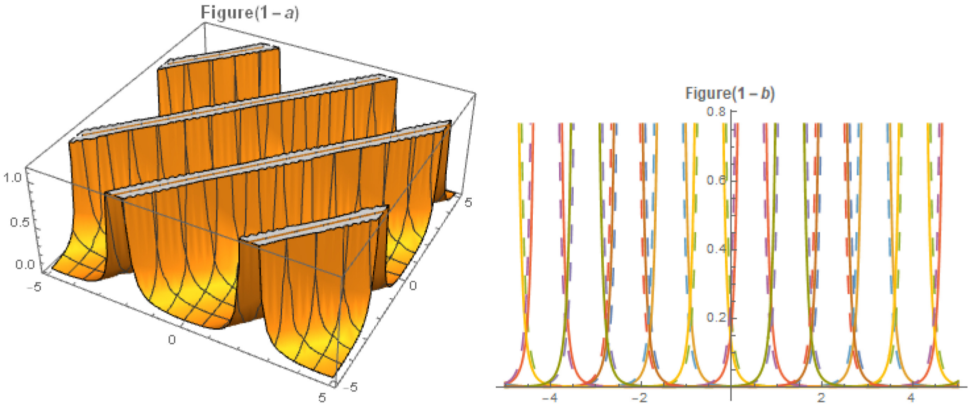

Fig. 1. (Color online) Solution U_1 with $c_0 = 1$, $c_1 = 0.5$, $c_2 = 2$, $\gamma_1 = 0.03$, $\delta_1 = 3$, $\xi_0 = 4.5$, $k = -0.5$.

Put (19) into (14)

$$U_3 = \left(\frac{(c_1\exp(c_1(\xi+\xi_0)))(-(6\gamma_1 c_1 c_2 k^2))}{(\delta_1-1)(1-c_2\exp(c_1(\xi+\xi_0)))}\right) - \left(\frac{\left(\frac{c_1\exp(c_1(\xi+\xi_0))}{1-c_2\exp(c_1(\xi+\xi_0))}\right)^2(6\gamma_1 c_2^2 k^2)}{\delta_1-1}\right), \quad c_1 > 0, \tag{20}$$

$$U_4 = -\left(\frac{\left(-\frac{c_1\exp(c_1(\xi+\xi_0))}{c_2\exp(c_1(\xi+\xi_0))+1}\right)^2(6\gamma_1 c_2^2 k^2)}{\delta_1-1}\right) - \left(\frac{(c_1\exp(c_1(\xi+\xi_0)))(-(6\gamma_1 c_1 c_2 k^2))}{(\delta_1-1)(c_2\exp(c_1(\xi+\xi_0))+1)}\right), \quad c_1 < 0. \tag{21}$$

Case 3. $c_1 = 0$, $c_3 = 0$

FAMILY-I

$$A_0 = -\frac{2\gamma_1 c_0 c_2 k^2}{\delta_1-1}, \quad A_{-2} = 0, \quad A_{-1} = 0, \quad A_2 = -\frac{6\gamma_1 c_2^2 k^2}{\delta_1-1}, \quad A_1 = 0, \quad \omega = -k^3(8c_0c_2\gamma_1 - \gamma_1 - \delta_1 - 1). \tag{22}$$

Put (22) into (14)

$$U_5 = -\frac{(6\gamma_1 c_2^2 k^2)\left(\frac{\sqrt{c_0c_2}\tan\left(\sqrt{c_0c_2}(\xi+\xi_0)\right)}{c_2}\right)^2}{\delta_1-1} - \left(\frac{2\gamma_1 c_0 c_2 k^2}{\delta_1-1}\right), \quad c_0c_2 > 0, \tag{23}$$

$$U_6 = -\frac{(6\gamma_1 c_2^2 k^2)\left(-\frac{\sqrt{-c_0c_2}\tanh\left(\sqrt{-c_0c_2}(\xi+\xi_0)\right)}{c_2}\right)^2}{\delta_1 - 1} - \left(\frac{2\gamma_1 c_0 c_2 k^2}{\delta_1 - 1}\right), \quad c_0c_2 < 0. \tag{24}$$

FAMILY-II

$$\begin{aligned} &A_0 = -\frac{2\gamma_1 c_0 c_2 k^2}{\delta_1 - 1}, \quad A_{-2} = -\frac{6\gamma_1 c_0^2 k^2}{\delta_1 - 1}, \quad A_{-1} = 0, \quad A_2 = 0, \\ &A_1 = 0, \quad \omega = -k^3(8c_0c_2\gamma_1 - \gamma_1 - \delta_1 - 1). \end{aligned} \tag{25}$$

Put (25) into (14)

$$U_7 = \left(-\frac{6\gamma_1 c_0^2 k^2}{(\delta_1 - 1)\left(\frac{\sqrt{c_0c_2}\tan\left(\sqrt{c_0c_2}(\xi+\xi_0)\right)}{c_2}\right)^2}\right) - \left(\frac{2\gamma_1 c_0 c_2 k^2}{\delta_1 - 1}\right), \quad c_0c_2 > 0, \tag{26}$$

$$U_8 = \left(-\frac{6\gamma_1 c_0^2 k^2}{(\delta_1 - 1)\left(\frac{\sqrt{-c_0c_2}\tanh\left(\sqrt{-c_0c_2}(\xi+\xi_0)\right)}{c_2}\right)^2}\right) - \left(\frac{2\gamma_1 c_0 c_2 k^2}{\delta_1 - 1}\right), \quad c_0c_2 < 0. \tag{27}$$

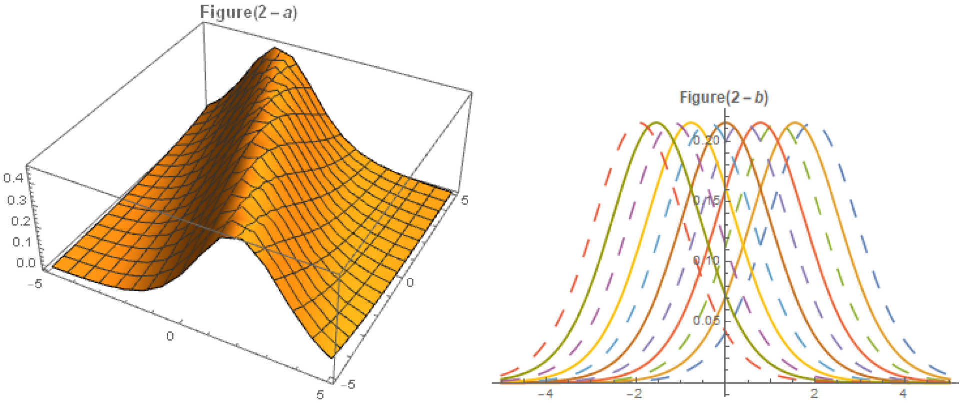

Fig. 2. (Color online) Solution U_4 with $c_1 = -2.5$, $c_2 = 3.5$, $\gamma_1 = 0.3$, $\delta_1 = 0.03$, $\xi_0 = 0.5$, $k = 0.4$.

FAMILY-III

$$\begin{aligned} &A_0 = \frac{4\gamma_1 c_0 c_2 k^2}{\delta_1 - 1}, \quad A_{-2} = -\frac{6\gamma_1 c_0^2 k^2}{\delta_1 - 1}, \quad A_{-1} = 0, \quad A_2 = -\frac{6\gamma_1 c_2^2 k^2}{\delta_1 - 1}, \\ &A_1 = 0, \quad \omega = -k^3(32c_0c_2\gamma_1 - \gamma_1 - \delta_1 - 1). \end{aligned} \tag{28}$$

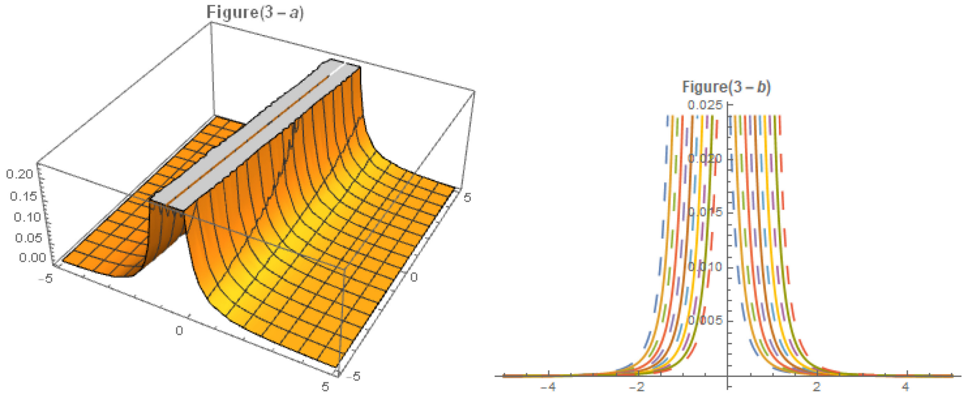

Fig. 3. (Color online) Solution U_{10} with $c_0 = 0.5$, $c_2 = -0.1$, $\gamma_1 = 0.01$, $\delta_1 = 0.2$, $\xi_0 = 0.03$, $k = 0.3$.

Put (28) into (14)

$$U_9 = -\left(\frac{(6\gamma_1 c_2^2 k^2)\left(\frac{\sqrt{c_0 c_2}\tan\left(\sqrt{c_0 c_2}(\xi+\xi_0)\right)}{c_2}\right)^2}{\delta_1 - 1}\right) - \left(\frac{6\gamma_1 c_0^2 k^2}{(\delta_1 - 1)\left(\frac{\sqrt{c_0 c_2}\tan\left(\sqrt{c_0 c_2}(\xi+\xi_0)\right)}{c_2}\right)^2}\right) + \left(\frac{4c_0 c_2 \gamma_1 k^2}{\delta_1 - 1}\right), \quad c_0 c_2 > 0, \tag{29}$$

$$U_{10} = -\left(\frac{(6\gamma_1 c_2^2 k^2)\left(\frac{\sqrt{-c_0 c_2}\tanh\left(\sqrt{-c_0 c_2}(\xi+\xi_0)\right)}{c_2}\right)^2}{\delta_1 - 1}\right) - \left(\frac{6\gamma_1 c_0^2 k^2}{(\delta_1 - 1)\left(\frac{\sqrt{-c_0 c_2}\tanh\left(\sqrt{-c_0 c_2}(\xi+\xi_0)\right)}{c_2}\right)^2}\right) + \left(\frac{4c_0 c_2 \gamma_1 k^2}{\delta_1 - 1}\right), \quad c_0 c_2 < 0. \tag{30}$$

3.2. *Application of modified extended auxiliary equation mapping method*

Let solution of (13) be

$$U = A_2\Psi^2 + A_1\Psi + A_0 + \frac{B_1}{\Psi} + \frac{B_2}{\Psi^2} + C_2\Psi' + D_2\left(\frac{\Psi'}{\Psi}\right)^2 + \frac{D_1\Psi'}{\Psi}. \tag{31}$$

Put (31) with (8) into (13)

$$\begin{aligned}&A_0=-\frac{\beta_1(\delta_1 D_2-D_2+\gamma_1 k^2)}{\delta_1-1},\quad A_1=-\frac{\beta_2(2\delta_1 D_2-2D_2+3\gamma_1 k^2)}{2(\delta_1-1)},\ B_1=0,\ B_2=0,\\&D_1=0,\quad A_2=-\frac{\beta_3(\delta_1 D_2-D_2+3\gamma_1 k^2)}{\delta_1-1},\quad C_2=\frac{3\sqrt{\frac{\beta_3\gamma_1^2k^4}{\delta_1-1}}}{\sqrt{\delta_1-1}},\\&\ \omega=k^3(2\beta_1\gamma_1+\gamma_1+\delta_1+1).\end{aligned}\tag{32}$$

Put (32) into (31)

Case I.

$$\begin{aligned}U_{11}=&-\left(\frac{\left(-\beta_1\left(\epsilon\coth\left(\frac{1}{2}\sqrt{\beta_1}(\xi+\xi_0)\right)+1\right)\right)\left(\beta_2(2\delta_1 D_2-2D_2+3\gamma_1 k^2)\right)}{\beta_2(2(\delta_1-1))}\right)\\&-\left(\frac{\beta_1(\delta_1 D_2-D_2+\gamma_1 k^2)}{\delta_1-1}\right)-\left(\frac{\left(-\frac{\beta_1\left(\epsilon\coth\left(\frac{1}{2}\sqrt{\beta_1}(\xi+\xi_0)\right)+1\right)}{\beta_2}\right)^2\left(\beta_3(\delta_1 D_2-D_2+3\gamma_1 k^2)\right)}{\delta_1-1}\right)\\&+\left(\frac{\beta_1^{3/2}\epsilon\operatorname{csch}^2\left(\frac{1}{2}\sqrt{\beta_1}(\xi+\xi_0)\right)}{2\beta_2}\right)\left(\frac{3\sqrt{\frac{\beta_3\gamma_1^2k^4}{\delta_1-1}}}{\sqrt{\delta_1-1}}\right)\\&+\left(\left(\frac{\beta_1^{3/2}\epsilon\operatorname{csch}^2\left(\frac{1}{2}\sqrt{\beta_1}(\xi+\xi_0)\right)}{\frac{(2\beta_2)\left(-\beta_1\left(\epsilon\coth\left(\frac{1}{2}\sqrt{\beta_1}(\xi+\xi_0)\right)+1\right)\right)}{\beta_2}}\right)^2\right),\quad \beta_1>0,\ \ \beta_2^2-4\beta_1\beta_3=0.\end{aligned}\tag{33}$$

Case II.

$$\begin{aligned}U_{12}=&-\left(\frac{\left(\beta_2(2\delta_1 D_2-2D_2+3\gamma_1 k^2)\right)\left(-\sqrt{\frac{\beta_1}{4\beta_3}}\left(\frac{\epsilon\sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta}+1\right)\right)}{2(\delta_1-1)}\right)\\&-\left(\frac{\beta_1(\delta_1 D_2-D_2+\gamma_1 k^2)}{\delta_1-1}\right)+\frac{1}{2}\left(\frac{3\sqrt{\frac{\beta_3\gamma_1^2k^4}{\delta_1-1}}}{\sqrt{\delta_1-1}}\right)\\&\times\left(\sqrt{\frac{\beta_1}{\beta_3}}\left(-\left(\frac{\sqrt{\beta_1}\epsilon\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta}-\frac{\sqrt{\beta_1}\epsilon\sinh^2\left(\sqrt{\beta_1}(\xi+\xi_0)\right)}{\left(\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\right)^2}\right)\right)\right)\end{aligned}$$

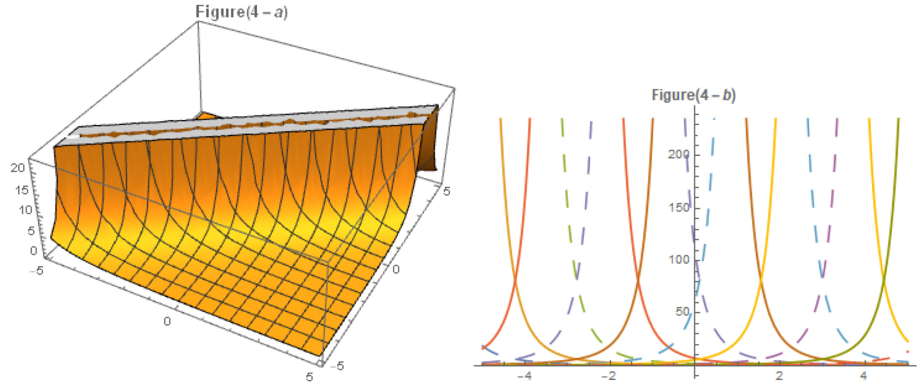

Fig. 4. (Color online) Solution U_{11} with $\beta_1 = \beta_3 = 2$, $\beta_2 = 4$, $\gamma_1 = 3.01$, $\delta_1 = 0.01$, $D_2 = 0.03$, $\xi_0 = -0.03$, $k = 0.3$, $\epsilon = 1$.

$$- \left(\frac{(\beta_3(\delta_1 D_2 - D_2 + 3\gamma_1 k^2))\left(-\sqrt{\frac{\beta_1}{4\beta_3}}\left(\frac{\epsilon \sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta}+1\right)\right)^2}{\delta_1 - 1} \right)$$

$$+ D_2 \left(- \frac{\sqrt{\frac{\beta_1}{\beta_3}}\left(\frac{\sqrt{\beta_1}\epsilon \cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta} - \frac{\sqrt{\beta_1}\epsilon \sinh^2\left(\sqrt{\beta_1}(\xi+\xi_0)\right)}{\left(\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\right)^2}\right)}{2\left(-\sqrt{\frac{\beta_1}{4\beta_3}}\left(\frac{\epsilon \sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta}+1\right)\right)} \right)^2,$$

$$\beta_1 > 0, \quad \beta_3 > 0, \quad \beta_2 = (4\beta_1\beta_3)^{1/2}. \tag{34}$$

Case III.

$$U_{13} = -\left(\frac{\beta_1(\delta_1 D_2 - D_2 + \gamma_1 k^2)}{\delta_1 - 1}\right) - \left(\frac{(\beta_2(2\delta_1 D_2 - 2D_2 + 3\gamma_1 k^2)) \left(-\beta_1\left(\frac{\epsilon\left(\sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+P\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\sqrt{P^2+1}}+1\right)\right)}{\beta_2(2(\delta_1 - 1))} \right)$$

$$+ \left(\frac{\frac{3\sqrt{\frac{\beta_3\gamma_1^2 k^4}{\delta_1-1}}}{\sqrt{\delta_1-1}} \left(-\left(\beta_1\left(\frac{\sqrt{\beta_1}\epsilon\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\sqrt{P^2+1}} - \frac{\sqrt{\beta_1}\epsilon\sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)\left(\sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+P\right)}{\left(\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\sqrt{P^2+1}\right)^2}\right)\right)\right)}{\beta_2} \right)$$

$$
- \left(\frac{(\beta_3(\delta_1 D_2 - D_2 + 3\gamma_1 k^2)) \left(- \frac{\beta_1 \left(\frac{\epsilon\left(\sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+P\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\sqrt{P^2+1}} + 1 \right)}{\beta_2} \right)^2}{\delta_1 - 1} \right) + \left(\frac{3\sqrt{\frac{\beta_3\gamma_1^2 k^4}{\delta_1-1}}}{\sqrt{\delta_1 - 1}} \right)
$$

$$
- \left(\frac{\beta_1 \left(\frac{\sqrt{\beta_1}\epsilon\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\sqrt{P^2+1}} - \frac{\sqrt{\beta_1}\epsilon\sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)\left(\sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+P\right)}{\left(\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\sqrt{P^2+1}\right)^2} \right)}{\beta_2} \right) + D_2 L, \quad \beta_1 > 0, \tag{35}
$$

where

$$
L = \left(- \frac{\beta_1 \left(\frac{\sqrt{\beta_1}\epsilon\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\sqrt{P^2+1}} - \frac{\sqrt{\beta_1}\epsilon\sinh(\sqrt{\beta_1}(\xi+\xi_0))\left(\sinh(\sqrt{\beta_1}(\xi+\xi_0))+P\right)}{\left(\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\sqrt{P^2+1}\right)^2} \right)}{\frac{\beta_2 \left(-\beta_1 \left(\frac{\epsilon\left(\sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+P\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\sqrt{P^2+1}} + 1 \right) \right)}{\beta_2}} \right)^2 .
$$

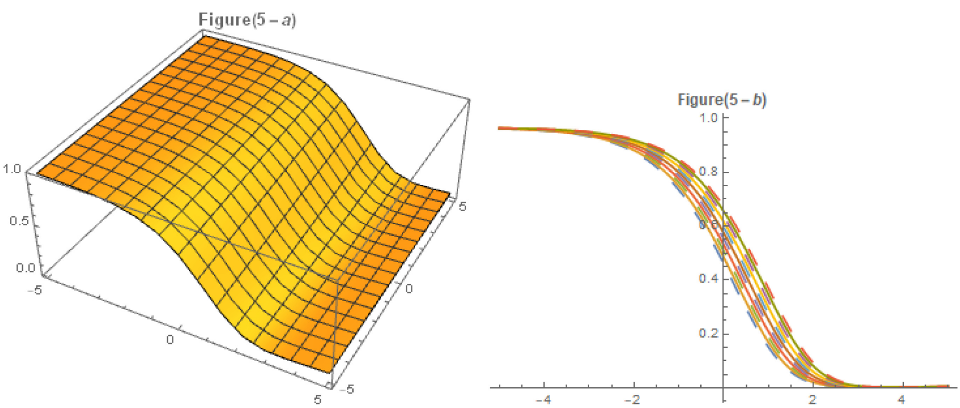

Fig. 5. (Color online) Solution U_{13} with $\beta_1 = 0.9$, $\beta_2 = 4$, $\beta_3 = 2$, $\gamma_1 = 0.2$, $\delta_1 = -1.5$, $D_2 = 2.1$, $\eta = 1$, $\xi_0 = 1.3$, $k = -1.1$, $P = 1$, $\epsilon = 1$.

3.3. *Application of* (G'/G)*-expansion method*

Let solution of (13) be

$$U = A_0 + A_1\left(\frac{G'}{G}\right) + A_2\left(\frac{G'}{G}\right)^2. \tag{36}$$

Put (36) with (10) into (13)

$$\begin{aligned} &A_2 = -\frac{6\gamma_1 k^2}{\delta_1 - 1}, \quad A_0 = -\frac{\gamma_1 k^2(\lambda_1^2 + 2\mu_1)}{\delta_1 - 1}, \quad A_1 = -\frac{6\gamma_1 k^2 \lambda_1}{\delta_1 - 1}, \\ &\omega = k^3(2\gamma_1\lambda_1^2 - 8\gamma_1\mu_1 + \gamma_1 + \delta_1 + 1). \end{aligned} \tag{37}$$

Put (37) into (36), we have following cases.

Case I. $\lambda_1^2 - 4\mu_1 > 0$

$$\begin{aligned} U_{14} = &-\left(\frac{(\gamma_1 k^2(\lambda_1^2 + 2\mu_1))\left(\frac{\xi P_1 \sinh\left(\frac{1}{2}\sqrt{\lambda_1^2 - 4\mu_1}\right) + \xi P_2 \cosh\left(\frac{1}{2}\sqrt{\lambda_1^2 - 4\mu_1}\right)}{\xi P_2 \sinh\left(\frac{1}{2}\sqrt{\lambda_1^2 - 4\mu_1}\right) + \xi P_1 \cosh\left(\frac{1}{2}\sqrt{\lambda_1^2 - 4\mu_1}\right)} - \frac{\lambda_1}{2}\right)}{\delta_1 - 1}\right) \\ &-\left(\frac{6\gamma_1 k^2}{\delta_1 - 1}\right)\left(\frac{\xi P_1 \sinh\left(\frac{1}{2}\sqrt{\lambda_1^2 - 4\mu_1}\right) + \xi P_2 \cosh\left(\frac{1}{2}\sqrt{\lambda_1^2 - 4\mu_1}\right)}{\xi P_2 \sinh\left(\frac{1}{2}\sqrt{\lambda_1^2 - 4\mu_1}\right) + \xi P_1 \cosh\left(\frac{1}{2}\sqrt{\lambda_1^2 - 4\mu_1}\right)} - \frac{\lambda_1}{2}\right)^2. \end{aligned} \tag{38}$$

Case II. $\lambda_1^2 - 4\mu_1 < 0$

$$\begin{aligned} U_{15} = &-\left(\frac{(\gamma_1 k^2(\lambda_1^2 + 2\mu_1))\left(\frac{\sqrt{4\mu_1 - \lambda_1^2}\left(\xi P_2 \cos\left(\frac{1}{2}\sqrt{4\mu_1 - \lambda_1^2}\right) - \xi P_1 \sin\left(\frac{1}{2}\sqrt{4\mu_1 - \lambda_1^2}\right)\right)}{2\left(\xi P_2 \sin\left(\frac{1}{2}\sqrt{4\mu_1 - \lambda_1^2}\right) + \xi P_1 \cos\left(\frac{1}{2}\sqrt{4\mu_1 - \lambda_1^2}\right)\right)} - \frac{\lambda_1}{2}\right)}{\delta_1 - 1}\right) \\ &\times\left(\frac{(6\gamma_1 k^2)\left(\frac{\sqrt{4\mu_1 - \lambda_1^2}\left(\xi P_2 \cos\left(\frac{1}{2}\sqrt{4\mu_1 - \lambda_1^2}\right) - \xi P_1 \sin\left(\frac{1}{2}\sqrt{4\mu_1 - \lambda_1^2}\right)\right)}{2\left(\xi P_2 \sin\left(\frac{1}{2}\sqrt{4\mu_1 - \lambda_1^2}\right) + \xi P_1 \cos\left(\frac{1}{2}\sqrt{4\mu_1 - \lambda_1^2}\right)\right)} - \frac{\lambda_1}{2}\right)^2}{\delta_1 - 1}\right). \end{aligned} \tag{39}$$

Case III. $\lambda_1^2 - 4\mu_1 = 0$

$$\begin{aligned} U_{16} = &-\left(\frac{\gamma_1 k^2(\lambda_1^2 + 2\mu_1)}{\delta_1 - 1}\right) - \left(\frac{(6\gamma_1 k^2)\left(\frac{P_2}{\xi P_2 + P_1} - \frac{\lambda_1}{2}\right)^2}{\delta_1 - 1}\right) \\ &-\left(\frac{(6\gamma_1 k^2 \lambda_1)\left(\frac{P_2}{\xi P_2 + P_1} - \frac{\lambda_1}{2}\right)}{\delta_1 - 1}\right). \end{aligned} \tag{40}$$

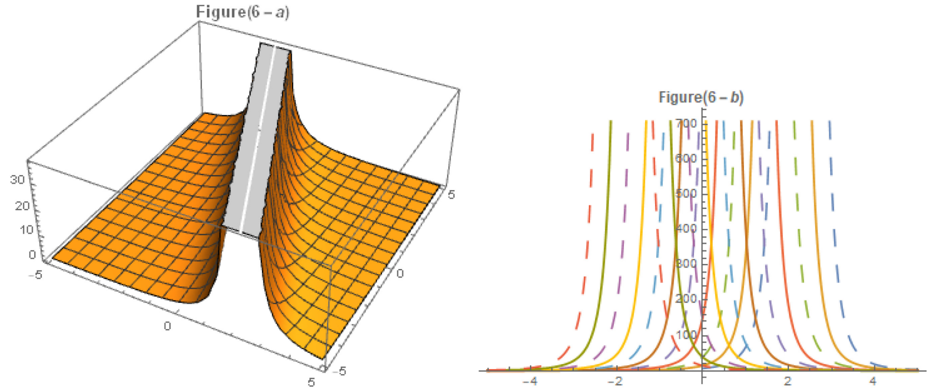

Fig. 6. (Color online) Solution U_{16} with $\gamma_1 = 3.2$, $\delta_1 = -0.5$, $k = -0.1$, $\lambda_1 = 1$, $\mu_1 = 2$, $P_2 = 0.5$, $P_1 = 0.01$.

3.4. *Application of modified F-expansion method*

Let solution of (13) be

$$U = a_2F^2 + a_1F + a_0 + \frac{b_2}{F^2} + \frac{b_1}{F}. \tag{41}$$

Put (41) with (12) into (13)

$\boldsymbol{A = 0}$, $\boldsymbol{B = 1}$, $\boldsymbol{C = -1}$

$$a_0 = -\frac{\gamma_1 k^2}{\delta_1 - 1}, \quad a_2 = -\frac{6\gamma_1 k^2}{\delta_1 - 1}, \quad a_1 = \frac{6\gamma_1 k^2}{\delta_1 - 1}, \quad b_1 = 0, \quad b_2 = 0,$$
$$\omega = k^3(3\gamma_1 + \delta_1 + 1). \tag{42}$$

Put (42) into (41)

$$U_{17} = -\left(\frac{\gamma_1 k^2}{\delta_1 - 1}\right) + \left(\frac{6\gamma_1 k^2}{\delta_1 - 1}\right)\left(\frac{1}{2}\tanh\left(\frac{\xi}{2}\right) + \frac{1}{2}\right) + \left(\frac{6\gamma_1 k^2}{\delta_1 - 1}\right)$$
$$\times \left(\frac{1}{2}\tanh\left(\frac{\xi}{2}\right) + \frac{1}{2}\right)\right)^2. \tag{43}$$

$\boldsymbol{A = 0}$, $\boldsymbol{C = 1}$, $\boldsymbol{B = -1}$

$$a_0 = -\frac{\gamma_1 k^2}{\delta_1 - 1}, \quad a_2 = -\frac{6\gamma_1 k^2}{\delta_1 - 1}, \quad a_1 = \frac{6\gamma_1 k^2}{\delta_1 - 1}, \quad b_1 = 0,$$
$$b_2 = 0, \quad \omega = k^3(3\gamma_1 + \delta_1 + 1). \tag{44}$$

Put (44) into (41)

$$U_{18} = -\left(\frac{\gamma_1 k^2}{\delta_1 - 1}\right) + \left(\frac{6\gamma_1 k^2}{\delta_1 - 1}\right)\left(\frac{1}{2} - \frac{1}{2}\coth\left(\frac{\xi}{2}\right)\right) - \left(\frac{6\gamma_1 k^2}{\delta_1 - 1}\right)$$
$$\times \left(\frac{1}{2} - \frac{1}{2}\coth\left(\frac{\xi}{2}\right)\right)^2. \tag{45}$$

$\boldsymbol{A = 1/2}$, $\boldsymbol{B = 0}$, $\boldsymbol{C = -1/2}$

FAMILY-I

$$\begin{aligned}&a_0 = \frac{3\gamma_1 k^2}{2(\delta_1 - 1)}, \quad a_2 = -\frac{3\gamma_1 k^2}{2(\delta_1 - 1)}, \quad a_1 = 0, \quad b_1 = 0, \quad b_2 = 0,\\ &\omega = -k^3(\gamma_1 - \delta_1 - 1).\end{aligned} \tag{46}$$

Substitute (46) into (41)

$$U_{19,1} = \left(\frac{3\gamma_1 k^2}{2(\delta_1 - 1)}\right) - \left(\frac{(3\gamma_1 k^2)(\coth(\xi) + \operatorname{csch}(\xi))^2}{2(\delta_1 - 1)}\right). \tag{47}$$

FAMILY-II

$$\begin{aligned}&a_0 = \frac{\gamma_1 k^2}{2(\delta_1 - 1)}, \quad a_2 = 0, \quad a_1 = 0, \quad b_1 = 0, \quad b_2 = -\frac{3\gamma_1 k^2}{2(\delta_1 - 1)},\\ &\omega = k^3(3\gamma_1 + \delta_1 + 1).\end{aligned} \tag{48}$$

Put (48) in (41)

$$U_{19,2} = \left(\frac{\gamma_1 k^2}{2(\delta_1 - 1)}\right) - \left(\frac{3\gamma_1 k^2}{(2(\delta_1 - 1))(\coth(\xi) + \operatorname{csch}(\xi))^2}\right). \tag{49}$$

FAMILY-III

$$\begin{aligned}&a_0 = \frac{3\gamma_1 k^2}{\delta_1 - 1}, \quad a_2 = -\frac{3\gamma_1 k^2}{2(\delta_1 - 1)}, \quad a_1 = 0, \quad b_1 = 0, \quad b_2 = -\frac{3\gamma_1 k^2}{2(\delta_1 - 1)},\\ &\omega = -7\gamma_1 k^3 + \delta_1 k^3 + k^3.\end{aligned} \tag{50}$$

Put (50) into (41)

$$\begin{aligned}U_{19,3} = {}& \left(\frac{3\gamma_1 k^2}{\delta_1 - 1}\right) - \left(\frac{(3\gamma_1 k^2 t)(\coth(\xi) + \operatorname{csch}(\xi))^2}{2(\delta_1 - 1)}\right)\\ &- \left(\frac{3\gamma_1 k^2}{2(\delta_1 - 1)}\,\frac{1}{(\coth(\xi) + \operatorname{csch}(\xi))^2}\right).\end{aligned} \tag{51}$$

$\boldsymbol{A = 1,\ B = 0,\ C = -1}$

FAMILY-I

$$\begin{aligned}&a_0 = \frac{2\gamma_1 k^2}{\delta_1 - 1}, \quad a_2 = -\frac{6\gamma_1 k^2}{\delta_1 - 1}, \quad a_1 = 0, \quad b_1 = 0, \quad b_2 = 0,\\ &\omega = k^3(9\gamma_1 + \delta_1 + 1).\end{aligned} \tag{52}$$

Put (52) into (41)

$$U_{20,1} = \left(\frac{2\gamma_1 k^2}{\delta_1 - 1}\right) - \left(\frac{(6\gamma_1 k^2)\tanh^2(\xi)}{\delta_1 - 1}\right). \tag{53}$$

FAMILY-II

$$
\begin{aligned}
&a_0 = \frac{6\gamma_1 k^2}{\delta_1 - 1}, \quad a_2 = 0, \quad a_1 = 0, \quad b_1 = 0, \quad b_2 = -\frac{6\gamma_1 k^2}{\delta_1 - 1}, \\
&\omega = -7\gamma_1 k^3 + \delta_1 k^3 + k^3.
\end{aligned} \tag{54}
$$

Put (54) into (41)

$$
U_{20,2} = \left(\frac{6\gamma_1 k^2}{\delta_1 - 1}\right) - \left(\frac{6\gamma_1 k^2}{\delta_1 - 1}\,\frac{1}{\tanh^2(\xi)}\right). \tag{55}
$$

FAMILY-III

$$
\begin{aligned}
&a_0 = \frac{12\gamma_1 k^2}{\delta_1 - 1}, \quad a_2 = -\frac{6\gamma_1 k^2}{\delta_1 - 1}, \quad a_1 = 0, \quad b_1 = 0, \quad b_2 = -\frac{6\gamma_1 k^2}{\delta_1 - 1}, \\
&\omega = -31\gamma_1 k^3 + \delta_1 k^3 + k^3.
\end{aligned} \tag{56}
$$

Put (56) into (41)

$$
U_{20,3} = \left(\frac{12\gamma_1 k^2}{\delta_1 - 1}\right) - \left(\frac{6\gamma_1 k^2}{\delta_1 - 1}\tanh^2(\xi)\right) - \left(\frac{6\gamma_1 k^2}{\delta_1 - 1}\,\frac{1}{\tanh^2(\xi)}\right). \tag{57}
$$

$\boldsymbol{A = C = 1/2,\ B = 0}$

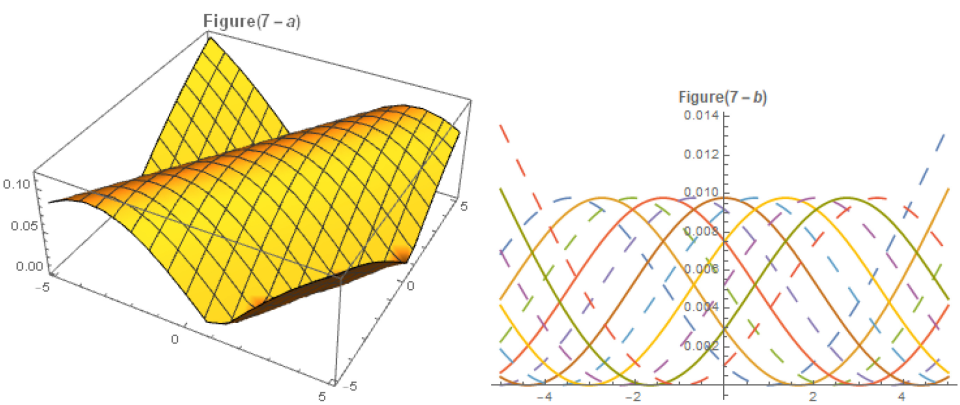

Fig. 7. (Color online) Solution U_{17} with $\gamma_1 = 2.2$, $\delta_1 = -0.001$, $k = -0.3$.

FAMILY-I

$$
\begin{aligned}
&a_0 = -\frac{3\gamma_1 k^2}{2(\delta_1 - 1)}, \quad a_2 = -\frac{3\gamma_1 k^2}{2(\delta_1 - 1)}, \quad a_1 = 0, \quad b_1 = 0, \quad b_2 = 0, \\
&\omega = k^3(3\gamma_1 + \delta_1 + 1).
\end{aligned} \tag{58}
$$

Put (58) into (41)

$$U_{21,1} = -\left(\frac{3\gamma_1 k^2}{2(\delta_1 - 1)}\right) - \frac{3\gamma_1 k^2}{2(\delta_1 - 1)}((\tan(\xi) + \sec(\xi))^2). \tag{59}$$

FAMILY-II

$$a_0 = -\frac{3\gamma_1 k^2}{2(\delta_1 - 1)}, \quad a_2 = 0, \quad a_1 = 0, \quad b_1 = 0, \quad b_2 = -\frac{3\gamma_1 k^2}{2(\delta_1 - 1)}, \quad \omega = k^3(3\gamma_1 + \delta_1 + 1). \tag{60}$$

Put (60) into (41)

$$U_{20,2} = -\frac{3\gamma_1 k^2}{2(\delta_1 - 1)} - \left(\frac{3\gamma_1 k^2}{2(\delta_1 - 1)} \frac{1}{(\tan(\xi) + \sec(\xi))^2}\right). \tag{61}$$

FAMILY-III

$$a_0 = \frac{\gamma_1 k^2}{\delta_1 - 1}, \quad a_2 = -\frac{3\gamma_1 k^2}{2(\delta_1 - 1)}, \quad a_1 = 0, \quad b_1 = 0, \quad b_2 = -\frac{3\gamma_1 k^2}{2(\delta_1 - 1)}, \quad \omega = -7\gamma_1 k^3 + \delta_1 k^3 + k^3. \tag{62}$$

Put (62) into (41)

$$U_{21,3} = \left(\frac{\gamma_1 k^2}{\delta_1 - 1} - \frac{3\gamma_1 k^2}{2(\delta_1 - 1)}(\tan(\xi) + \sec(\xi))^2\right) - \left(\frac{3\gamma_1 k^2}{2(\delta_1 - 1)} \frac{1}{(\tan(\xi) + \sec(\xi))^2}\right). \tag{63}$$

***A* = *C* = −1/2, *B* = 0**

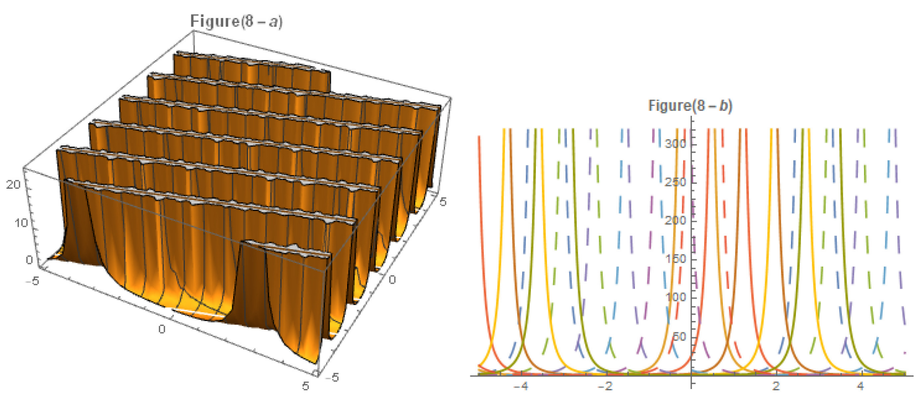

Fig. 8. (Color online) Solution $U_{21,1}$ with $\gamma_1 = 1.2$, $\delta_1 = -1.1$, $k = 1.001$.

FAMILY-I

$$a_0 = -\frac{3\gamma_1 k^2}{2(\delta_1 - 1)}, \quad a_2 = -\frac{3\gamma_1 k^2}{2(\delta_1 - 1)}, \quad a_1 = 0, \quad b_1 = 0, \quad b_2 = 0, \quad \omega = k^3(3\gamma_1 + \delta_1 + 1). \tag{64}$$

Put (64) into (41)

$$U_{22,1} = -\frac{3\gamma_1 k^2}{2(\delta_1 - 1)} - \frac{3\gamma_1 k^2}{2(\delta_1 - 1)}(\sec(\xi) - \tan(\xi))^2. \tag{65}$$

FAMILY-II

$$a_0 = -\frac{\gamma_1 k^2}{2(\delta_1 - 1)}, \quad a_2 = 0, \quad a_1 = 0, \quad b_1 = 0, \quad b_2 = -\frac{3\gamma_1 k^2}{2(\delta_1 - 1)}, \quad \omega = -k^3(\gamma_1 - \delta_1 - 1). \tag{66}$$

Put (66) into (41)

$$U_{22,2} = -\frac{\gamma_1 k^2}{2(\delta_1 - 1)} - \frac{3\gamma_1 k^2}{2(\delta_1 - 1)} \frac{1}{(\sec(\xi) - \tan(\xi))^2}. \tag{67}$$

FAMILY-III

$$a_0 = \frac{\gamma_1 k^2}{\delta_1 - 1}, \quad a_2 = -\frac{3\gamma_1 k^2}{2(\delta_1 - 1)}, \quad a_1 = 0, \quad b_1 = 0, \quad b_2 = -\frac{3\gamma_1 k^2}{2(\delta_1 - 1)}, \quad \omega = -7\gamma_1 k^3 + \delta_1 k^3 + k^3. \tag{68}$$

Put (68) into (41)

$$U_{22,3} = \frac{\gamma_1 k^2}{\delta_1 - 1} - \left(\frac{3\gamma_1 k^2}{2(\delta_1 - 1)}(\sec(\xi) - \tan(\xi))^2\right) - \left(\frac{3\gamma_1 k^2}{2(\delta_1 - 1)} \frac{1}{(\sec(\xi) - \tan(\xi))^2}\right). \tag{69}$$

$\boldsymbol{A = C = -1,\ B = 0}$

FAMILY-I

$$a_0 = -\frac{6\gamma_1 k^2}{\delta_1 - 1}, \quad a_2 = -\frac{6\gamma_1 k^2}{\delta_1 - 1}, \quad a_1 = 0, \quad b_1 = 0, \quad b_2 = 0, \quad \omega = k^3(9\gamma_1 + \delta_1 + 1). \tag{70}$$

Put (70) into (41)

$$U_{23,1} = -\frac{6\gamma_1 k^2}{\delta_1 - 1} - \frac{6\gamma_1 k^2}{\delta_1 - 1}\tan^2(\xi). \tag{71}$$

FAMILY-II

$$\begin{aligned} a_0 &= -\frac{2\gamma_1 k^2}{\delta_1 - 1}, \quad a_2 = 0, \quad a_1 = 0, \quad b_1 = 0, \quad b_2 = -\frac{6\gamma_1 k^2}{\delta_1 - 1}, \\ \omega &= -7\gamma_1 k^3 + \delta_1 k^3 + k^3. \end{aligned} \tag{72}$$

Put (72) into (41)

$$U_{23,2} = -\frac{6\gamma_1 k^2}{\delta_1 - 1} - \left(\frac{2\gamma_1 k^2}{\delta_1 - 1}\frac{1}{\tan^2(\xi)}\right). \tag{73}$$

FAMILY-III

$$\begin{aligned} a_0 &= \frac{4\gamma_1 k^2}{\delta_1 - 1}, \quad a_2 = -\frac{6\gamma_1 k^2}{\delta_1 - 1}, \quad a_1 = 0, \quad b_1 = 0, \quad b_2 = -\frac{6\gamma_1 k^2}{\delta_1 - 1}, \\ \omega &= -31\gamma_1 k^3 + \delta_1 k^3 + k^3. \end{aligned} \tag{74}$$

Put (74) into (41)

$$U_{23,3} = \frac{4\gamma_1 k^2}{\delta_1 - 1} - \left(\frac{6\gamma_1 k^2}{\delta_1 - 1}\tan^2(\xi)\right) - \left(\frac{6\gamma_1 k^2}{\delta_1 - 1}\frac{1}{\tan^2(\xi)}\right). \tag{75}$$

$\boldsymbol{A = B = 0}$

$$\begin{aligned} a_0 &= 0, \quad a_2 = -\frac{6\gamma_1 C^2 k^2}{\delta_1 - 1}, \quad a_1 = 0, \quad b_1 = 0, \quad b_2 = 0, \\ \omega &= k^3(\gamma_1 + \delta_1 + 1). \end{aligned} \tag{76}$$

Put (76) into (41)

$$U_{24} = -\frac{6\gamma_1 C^2 k^2}{\delta_1 - 1}\left(-\frac{1}{C\xi}\right)^2. \tag{77}$$

$\boldsymbol{B = C = 0}$

$$a_0 = a_2 = a_1 = b_1 = 0, \quad b_2 = \frac{6\sqrt[3]{-1}A^2\gamma_1\omega^{2/3}}{(\delta_1 - 1)(\gamma_1 + \delta_1 + 1)^{2/3}}, \quad k = \frac{(-1)^{2/3}\sqrt[3]{\omega}}{\sqrt[3]{\gamma_1 + \delta_1 + 1}}. \tag{78}$$

Put (78) into (41)

$$U_{25} = \frac{6\sqrt[3]{-1}A^2\gamma_1\omega^{2/3}}{(\delta_1 - 1)(\gamma_1 + \delta_1 + 1)^{2/3}}\left(\frac{1}{A\xi}\right)^2. \tag{79}$$

$\boldsymbol{C = 0}$

$$\begin{aligned} a_0 &= -\frac{B^2\gamma_1 k^2}{\delta_1 - 1}, \quad a_2 = 0, \quad a_1 = 0, \quad b_1 = -\frac{6AB\gamma_1 k^2}{\delta_1 - 1}, \quad b_2 = -\frac{6A^2\gamma_1 k^2}{\delta_1 - 1}, \\ \omega &= k^3(2B^2\gamma_1 + \gamma_1 + \delta_1 + 1). \end{aligned} \tag{80}$$

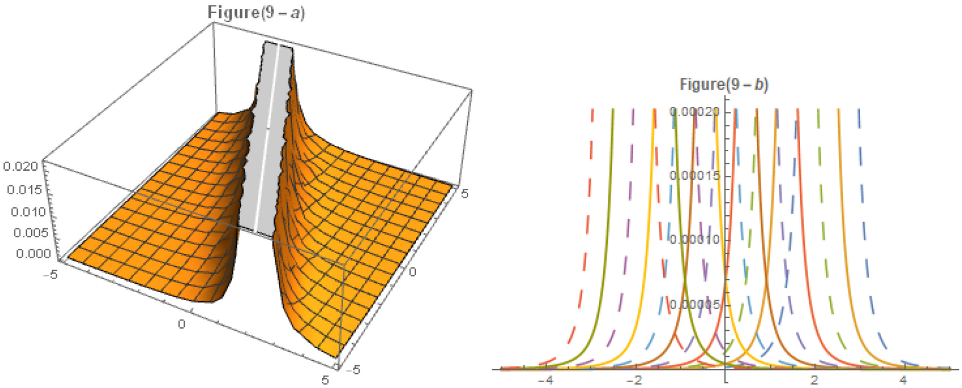

Fig. 9. (Color online) Solution U_{25} with $A = 1.5$, $\gamma_1 = -0.1$, $\delta_1 = 3.1$, $k = -5.5$, $\omega = 2.5$.

Put (80) into (41)

$$U_{26} = -\frac{6AB\gamma_1 k^2}{\delta_1 - 1} - \left(\frac{B^2\gamma_1 k^2}{\delta_1 - 1}\frac{B}{\exp(B\xi) - A}\right) - \frac{6A^2\gamma_1 k^2}{\delta_1 - 1}\left(\frac{B}{\exp(B\xi) - A}\right)^2. \quad (81)$$

4. Conclusion

We have applied three mathematical schemes to establish soliton solutions of generalized SIdV equation which is likely to be KdV equation. The derived solutions are in different types like exponential, hyperbolic, trigonometric and rational functions. Some obtained solutions are plotted graphically in two-dimensional and three-dimensional forms by imparting particular value to the parameters under the constrain condition on each disquiet solution. Hence, it shows that our proposed mathematical methods are powerful, methodical and have the capacity to be used in supplementary works to originate novel results for NPDEs.

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