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PAPERS

Robust adaptive beamforming using modified constant modulus algorithms

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This paper addresses the self-nulling phenomenon also known as the self-cancellation in adaptive beamformers. Optimum beamforming requires knowledge of the desired signal characteristics, either its statistics, its direction-of-arrival, or its response vector. Inaccuracies in the required information lead the beamformer to attenuate the desired signal as if it were interference. Self-nulling is caused by the desired signal having large power (high SNR) relative to the interference signal in case of the minimum variance distortion less response beamformer, and low power desired signal in the case of the constant modulus algorithm (CMA) beamformer, which leads the beamformer to suppress the desired signal and lock onto the interference signal. The least-square constant modulus algorithm is a prominent blind adaptive beamforming algorithm. We propose two CMA-based algorithms which exploit the constant modularity as well as power or DOA of the desired signal to avoid self-nulling in beamforming. Simulations results verify the effectiveness of the proposed algorithms.

K e y w o r d s: array signal processing, constant modulus algorithm, robust adaptive beamforming, self-nulling

1 Introduction

Adaptive beamforming is a fundamental technique in array signal processing and has been widely used in radar, sonar, seismology, and wireless communication [1-6]. Many beamforming algorithms and techniques have been proposed in the last several decades. Among them, the two basic types of beamformers are the minimum variance distortionless response (MVDR) beamformer which is based on the desired signal direction-of-arrival (DOA) [7] and constant modulus (CM) beamformers which exploits the constant modulus property of the signal of interest [8, 9]. The MVDR beamformer is well-known to be sensitive to model mismatch, especially when the desired signal is present in the training data. Since, the separation of interference from the target signal in many real-time applications is not practical, the weights computed by the MVDR beamformer using data includes the target signal. When the desired signal component is present in the beamformer training data, a small estimation error in the signal steering vector and/or array covariance matrix may severely degrade the beamformer performance. Consequently, these estimation errors result in the cancellation of the desired signal. The suppression of the desired signal due to model errors is commonly referred to as self-nulling or self-cancellation [10, 11]. In contrast, the CM beamformers are robust to array imperfections, because they exploit the constant modularity of the received signals. The main idea of constant modulus algorithm (CMA) is to adjust the weight vector of the adaptive array to minimize the variation of the envelope at the output of the array, so that the output weighted sum signal of the array must possess constant modulus property [8-11]. However, the constant modularity of the communication signal is undermined due to a series of fading effect, such as frequency selective fading due to multipath, the time selective fading caused by doppler frequency shift, and the effect of additive white noise in the channel. Consequently, the amplitude of transmitted constant modulus signals varies. A small DOA error will lead the MVDR beamformer to suppress the high-power desired signal [4], whereas if the desired signal power is small, the CM beamformer lock onto the high power interference signal possessing constant modularity [8-11]. Other array errors leading to self-nulling include steering vector errors [2, 3], mutual coupling error [12, 13], and channel response errors [14, 15], etc.

Self-nulling is a fundamental issue in many state-ofthe-art beamformers. Generally, array calibration and robust adaptive beamforming techniques are employed to overcome the array deterministic and stochastic errors [16-19]. Diagonal loading, linear and non-linear constraints are some of the popular strategies to overcome self-nulling [12-17]. Numerous related works focuses on improving the robustness of MVDR beamformer against self-nulling [20-28], the basic idea of which is to minimize the maximum output power for the set of presumed array covariance matrices subject to constraints on the beamformer response for the set of presumed signal steering vectors.

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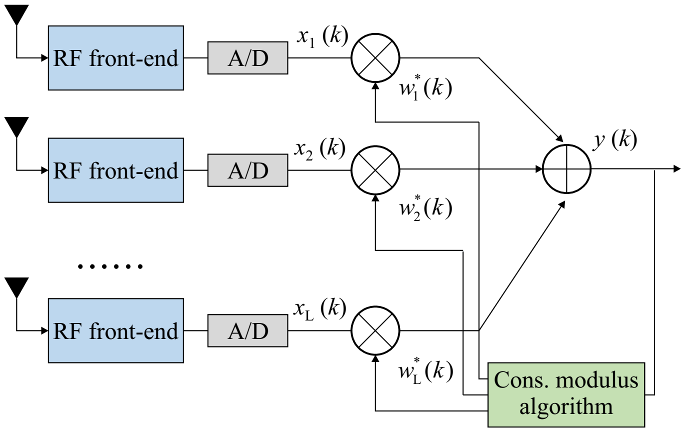

Fig. 1. The structure of CMA beamforming

In this paper, we propose two novel CMA-based algorithms for robust adaptive beamforming, which exploits the DOA or power information in addition to the constant modularity of the signals. The proposed algorithms avoid the self-cancellation of desired signals caused by the high-power desired signal in the case of the MVDR beamformer, and the high-power interference signal in the case of the CM beamformers. In addition, this paper also discusses the family of CMA-based algorithms and their deficiencies. Simulations results are presented to verify the performance of the proposed algorithms.

2 Constant modulus algorithms

Assume an L-element CM array as shown in Fig. 1. The output of this beamformer is

$$y(k) = \mathbf{w}^{\mathsf{H}}(k)\mathbf{x}(k), \tag{1}$$

where $\mathbf{w}(k) = [w_1(k)\ w_2(k)\ \dots\ w_L(k)]^{\top}$ is the complex weight vector for beamforming, and the array of the received signal is $\mathbf{x}(k) = [x_1(k)\ x_2(k)\ \dots\ x_L(k)]^T$. The constant modulus algorithm adjusts the weight vector $\mathbf{w}(k)$ such that the output signal has a constant modulus. The CMA is well-known for its faster convergence, reduced computational complexity, and strong robustness [11].

2.1 Conventional steepest descent CMA

Assume that the transmitted signal has a constant envelope, the cost function is defined as [11]

$$J\big(\mathbf{w}(k)\big)= E\Big\{\Big||y(k)|^p - |y_0|^p\Big|^q\Big\}, \tag{2}$$

where y_0 is the desired output signal amplitude of the array weighted sum, and p and q are parameters which take one of the values $\{1, 2\}$. The CMA cost function is used to adjust the beamformer complex weights, such that the output signal has a constant envelope. Different values of p and q yield different types of steepest descent CMA with varying computational complexities and convergence rates.

When $p = 1$ and $q = 2$, we get the type 1-2 steepest descent CMA. Suppose $y_0 = 1$, then

$$e(k) = 2\left[y(k) - \frac{y(k)}{|y(k)|}\right], \tag{3}$$

$$\mathbf{w}(k+1) = \mathbf{w}(k) - \mu\mathbf{x}(k)e^{*}(k). \tag{4}$$

where μ is the step-size parameter, asterisk denotes conjugation, and to guarantee convergence of the iterations, it is required that [29]

$$0 < \mu < \frac{2}{\lambda_{\max}}. \tag{5}$$

Here $\lambda_{\max}$ is the maximal eigenvalue of the covariance matrix of the received signal.

Table 1 shows various types of steepest descent CMA with their corresponding error function $e(k)$.

Table 1. Types of steepest descent CMA and their corresponding error functions

Type	p	q	Error function
1-1	1	1	$e(k) = \frac{y(k)}{\lvert y(k)\rvert}\operatorname{sgn}\big(y(k) - 1\big)$
2-1	2	1	$e(k) = 2y(k)\operatorname{sgn}\big(\lvert y(k)\rvert^2 - 1\big)$
2-2	2	2	$e(k) = 4y(k)\operatorname{sgn}\big(\lvert y(k)\rvert^2 - 1\big)$

The steepest descent CMA is very similar to the least mean square (LMS) algorithm, the only difference is that the LMS algorithm uses a training sequence for the adaptation process, whereas the receiver does not know the transmitted sequence in CMA. Any sequence with a constant phase offset may be the correct sequence at the receiver since the phase shift does not change the constant

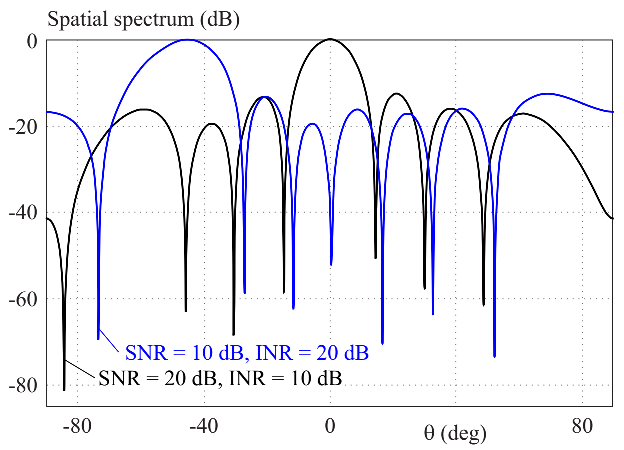

Fig. 2. The structure of CMA beamforming

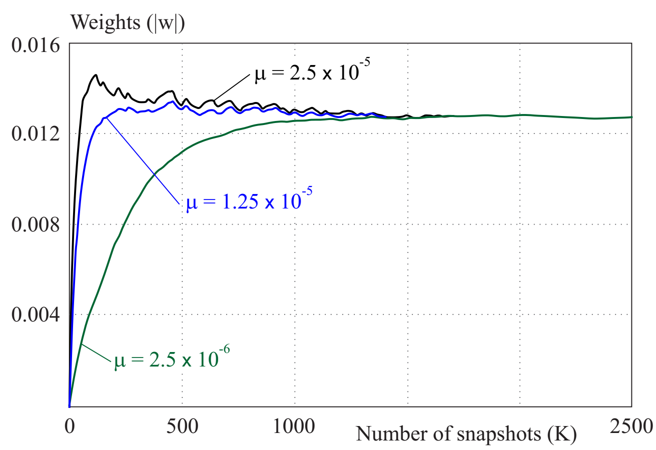

Fig. 3. Weights convergence curve

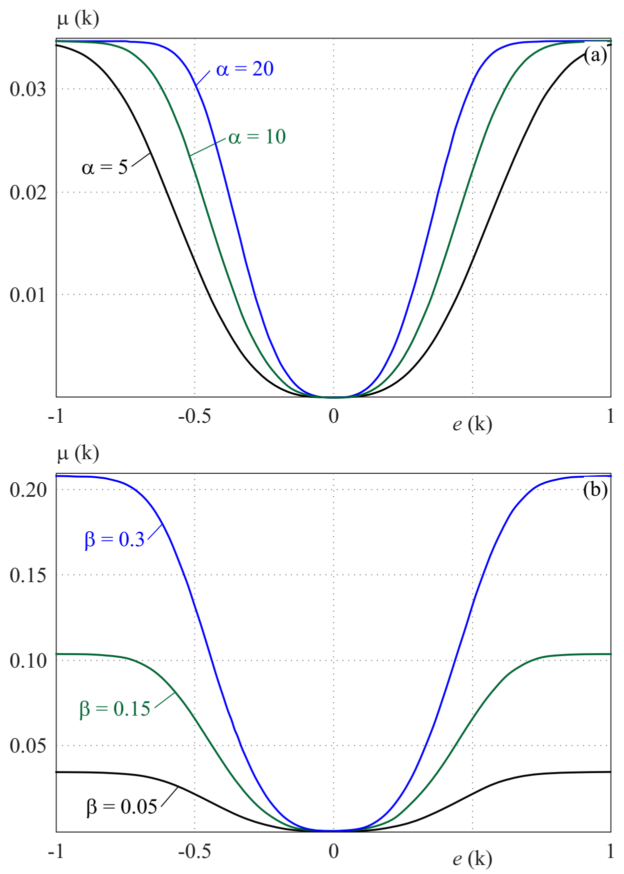

Fig. 4. The relationship between error function and step-size parameter: (a – fixed α, while β is variable, and (b) $-\alpha$ is variable, β is fixed

modularity property of a signal, and that is the desired signal is taken as $y(k)/|y(k)|$.

Assume a uniform linear array (ULA) composed of 8 omni-directional sensors, and the inter-element spacing is $d = \lambda/2$. Desired signal $s_0(t)$ and interference signal $s_1(t)$ are both constant modulus QPSK modulated signals incident on the array from direction $\theta_0 = 0$ and $\theta_1 = -45\,°$, respectively. The variance (power) of additive white gaussian noise (AWGN) is 1. Using type 1-2 steepest descent CMA, we compare the performance of beamformer with (a) SNR=20 dB and INR=10 dB (b) SNR=10 dB and INR=20 dB. The results are shown in Fig. 2, from where it can be observed that if the desired signal power is greater than the interference signal power *ie*, SNR=20 dB, and INR=10 dB, the beamformer captures the high-power desired signal, and null out the interference signal. The main lobe of the array pattern is aligned in the desired look direction *ie*, 0 °, and a deep null is generated in the direction of interference signal at-45 °. Next, when the power of the desired signal power is less than the interference signal, the beamformer generates a deep null in the direction of the desired signal *ie*, 0 °. Moreover, the beamformer locked onto the high-power interference signal and form a main lobe in the direction of interference at -45 °.

The reason for the self-nulling is that the cost function makes y_0 equal to 1, *ie* the algorithm makes the output signal power equal to 1 and not to the desired signal amplitude, so the beamformer will always capture the constant modularity of the high-power signal.

2.2 Variable step-size steepest descent CMA

In this section, we first discuss the effect of step size μ on the convergence of the beamformer weights. Then we discuss the variable step-size parameter $\mu(k)$ and variable step-size steepest descent CMA.

Assume the identical simulation parameters as in the previous simulation, the effect of different step-sizes, $\mu = 1.25 \times 10^{-5}$, $\mu = 2.5 \times 10^{-5}$, and $\mu = 2.5 \times 10^{-6}$, is analyzed. The weight convergence with different step-sizes is shown in Fig. 3. We can see that when $\mu = 2.5 \times 10^{-6}$, the convergence is slower as compared to the other two cases, but the residual error (offset) is small; when $\mu = 2.5 \times 10^{-5}$, the convergence is faster than the other two cases, but the residual error is large; and when $\mu = 1.25 \times 10^{-5}$, the convergence rate of the weight is faster than that of $\mu = 2.5 \times 10^{-6}$, and the offset is larger than $\mu = 2.5 \times 10^{-6}$, while the convergence rate of the weight is slower than that of $\mu = 2.5 \times 10^{-5}$,

Table 2. The comparison between various constant modulus algorithms

	SNR > INR	SNR < INR	Large SNR-INR Difference	Small SNR-INR Difference
Conventional steepest descent CMA [33]	Effective	Failed		
Variable step size steepest descent CMA [31]	Effective	Failed		
Steepest descent CMA based on the minimum Rayleigh entropy [34]			Effective	Failed
Conventional LSCMA [8]	Effective	Failed		
Pre-processing LSCMA [35]	Failed	Effective		
LSCMA based on minimum Rayleigh entropy [36]			Effective	Failed

but the residual error (offset) is smaller than that of $\mu = 2.5 \times 10^{-5}$.

It is concluded that large μ speed up the convergence of the weights, but large μ also lowers the precision of the steady-state solution of the algorithm, and the corresponding residual error (offset) become large. On the other hand, when the step-size parameter μ is small, the algorithm converges slowly but the residual error (offset) is small. Therefore, the step-size parameter μ, in a general view, exhibit a trade-off between convergence speed and the required offset value.

For a better convergence, some works have considered the variable step-size steepest descent CMA. The basic idea is that if the difference between the instantaneous weight and the optimal weight is large, select a larger step-size, and if the difference is relatively small, choose a smaller step-size. So, the variable step-size steepest descent CMA utilizes a large step-size during the transient state, and switchers to a smaller step-size during the steady-state [30].

In [31], the steepest descent CMA with variable step-size has been proposed, and the following relationship is established, which makes the variable step-size parameter $\mu(k)$ and the absolute value of the error function $e(k)$ to be inversely correlated

$$\mu(k) = \beta \ln \left[\frac{2}{1 + \exp(-\alpha |e(k)|^3)} \right], \tag{6}$$

where α is a positive control parameter, and β is the proportionality factor or scale factor. α is used to control the shape of the function of the tilt coefficient, and β is used to control the value scope of step-size μ or in other words, β is the bounds of the step-size. These two values are usually determined through experiments.

Next, we analyze parameters α and β as

1) Fix $\alpha = 10$, the relationship between the step-size $\mu(k)$ and error function $e(k)$ with different values of β is shown in Fig. 4(a).
2) Fix $\beta = 0.05$, the relationship between the step-size $\mu(k)$ and error function $e(k)$ with different values of α is shown in Fig. 4(b).

Figure 4(a) demonstrates that, when α is fixed, the larger the β, the larger will be the $\mu(k)$ corresponding to the same error function $e(k)$. Similarly, Fig. 4(b) demonstrates that when β is fixed, the larger the α, the larger will be the $\mu(k)$ corresponding to the same error function $e(k)$. Therefore, appropriate values of α and β should be chosen according to the actual need.

Moreover, $\mu(k)$ is large when the absolute value of the error function $|e(k)|$ is large, and $\mu(k)$ is small when the absolute value of the error function $|e(k)|$ approaches zero. Therefore, the convergence speed of the variable step-size algorithm is faster than that with the fixed step-size.

Although the variable step-size steepest descent CMA converges faster than fixed-step steepest descent CMA, the desired output signal amplitude is still considered to be 1, therefore, the self-nulling issue still prevails.

2.3 Least square CMA

Least square CMA (LSCMA) is a well-known algorithm due to its global convergence and stability [32]. The LSCMA is derived using the nonlinear Gauss method with cost function given as Euclidean norm [32]

$$J(\mathbf{w}) = \sum_{k=1}^{K} \left| g_k(\mathbf{w}) \right|^2 = \left\| g(\mathbf{w}) \right\|_2^2, \tag{7}$$

$$g_k(\mathbf{w}) = \left| y(k) \right| - 1, \tag{8}$$

$$\mathbf{g}(\mathbf{w}) = \left[g_1(\mathbf{w}) \; g_2(\mathbf{w}) \; \dots \; g_K(\mathbf{w}) \right]^\top. \tag{9}$$

The cost function has a partial Taylor-series expansion [11]

$$J(\mathbf{w}+\mathbf{\Delta}) = \left\| g(\mathbf{w}) + \mathbf{D}^{\mathrm{H}}(\mathbf{w})\mathbf{\Delta} \right\|_2^2, \tag{10}$$

where

$$\mathbf{D}(\mathbf{w}) = \Big[\nabla g_1(\mathbf{w})\ \nabla g_2(\mathbf{w})\ \ldots\ \nabla g_K(\mathbf{w})\Big], \tag{11}$$

is the Jacobian of $\mathbf{g}(\mathbf{w})$ and $\mathbf{\Delta}$ is an offset vector.

Taking the gradient of the cost function $J(\mathbf{w}+\mathbf{\Delta})$ with respect to $\mathbf{\Delta}$ yields

$$\begin{aligned}\nabla J(\mathbf{w}+\mathbf{\Delta}) &= 2\frac{\partial J(\mathbf{w}+\mathbf{\Delta})}{\partial \mathbf{\Delta}^*} = \\ &= 2\Big[\mathbf{D}(\mathbf{w})\mathbf{g}(\mathbf{w}) + \mathbf{D}(\mathbf{w})\mathbf{D}^{\mathrm{H}}(\mathbf{w})\mathbf{\Delta}\Big].\end{aligned} \tag{12}$$

Setting the result in (12) equal to zero yields an offset $\mathbf{\Delta}$ which minimizes the cost function,

$$\mathbf{\Delta} = -[\mathbf{D}(\mathbf{w})\mathbf{D}^{\mathrm{H}}(\mathbf{w})]^{-1}\mathbf{D}(\mathbf{w})\mathbf{g}(\mathbf{w}). \tag{13}$$

The weight vector can then be updated by adding this offset to the current weight vector as

$$\mathbf{w}(k+1) = \mathbf{w}(k) - \mathbf{\Delta}(k). \tag{14}$$

The LSCMA is obtained by applying (14) to the type 1-2 CMA cost function,

$$J(\mathbf{w}) = \sum_{k=1}^{K} \big||y(k)| - 1\big|^2 = \sum_{k=1}^{K} \big||\mathbf{w}^{\mathrm{H}}(k)\mathbf{x}(k)| - 1\big|. \tag{15}$$

To compute $\mathbf{\Delta}$ given in (13), substitute (8) into (9)

$$g_k(\mathbf{w}) = \Big[|y(1) - 1\ |y(2) - 1 \ldots |y(K) - 1\Big]^{\top}. \tag{16}$$

The gradient with respect to $\mathbf{w}$, is

$$\nabla\Big(g_k(\mathbf{w})\Big) = 2\frac{\partial g_k(\mathbf{w})}{\partial \mathbf{w}^*} = \mathbf{x}(k)\frac{y^*(k)}{|y(k)|}. \tag{17}$$

Substituting (17) into (11), we have

$$\begin{aligned}\mathbf{D}(\mathbf{w}) &= \left[x(1)\frac{y^*(1)}{|y(1)|}\ x(2)\frac{y^*(2)}{|y(2)|}\ \ldots\ x(K)\frac{y^*(K)}{|y(K)|}\right] = \\ &= \mathbf{XY}_{\mathrm{CM}},\end{aligned} \tag{18}$$

where

$$\mathbf{X} = \Big[x(1)\ x(2)\ \ldots\ x(K)\Big], \tag{19}$$

$$\mathbf{Y}_{\mathrm{CM}} = \begin{bmatrix} \frac{y^*(1)}{|y(1)|} & 0 & \ldots & 0 \\ 0 & \frac{y^*(2)}{|y(2)|} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \frac{y^*(K)}{|y(K)|} \end{bmatrix}. \tag{20}$$

From (16) and (18)

$$\mathbf{D}(\mathbf{w})\mathbf{D}^{\mathrm{H}}(\mathbf{w}) = \mathbf{XY}_{\mathrm{CM}}\mathbf{Y}_{\mathrm{CM}}^{\mathrm{H}}\mathbf{X}^{\mathrm{H}} = \mathbf{XX}^{\mathrm{H}}, \tag{21}$$

and

$$\begin{aligned}\mathbf{D}(\mathbf{w})\mathbf{g}(\mathbf{w}) &= \mathbf{XY}_{\mathrm{CM}} \begin{bmatrix} |y(1)| - 1 \\ |y(2)| - 1 \\ \vdots \\ |y(K)| - 1) \end{bmatrix} = \\ &= \mathbf{X} \begin{bmatrix} y^*(1) - \frac{y^*(1)}{|y(1)|} \\ y^*(2) - \frac{y^*(2)}{|y(2)|} \\ \vdots \\ y^*(K) - \frac{y^*(K)}{|y(K)|} \end{bmatrix}.\end{aligned} \tag{22}$$

Let

$$\mathbf{y} = \Big[y(1)\ y(2)\ \ldots\ y(K)\Big]^{\top}, \tag{23}$$

$$\mathbf{r} = \left[\frac{y(1)}{|y(1)|}\ \frac{y(2)}{|y(2)|} \ldots \frac{y(K)}{|y(K)|}\right]^{\top} = \mathbf{L}(\mathbf{y}), \tag{24}$$

where $\mathbf{L}(\mathbf{y})$ represents the hard-clipping operation on $\mathbf{y}$, then (22) can be written as

$$\mathbf{D}(\mathbf{w})\mathbf{g}(\mathbf{w}) = \mathbf{X}(\mathbf{y} - \mathbf{r})^*. \tag{25}$$

Here, $\mathbf{X}$ is the input data matrix, and $\mathbf{y}$ and $\mathbf{r}$ are the output-data and complex-limited output-data vectors. Substituting (21) and (25) into (13), we have

$$\mathbf{\Delta} = -\big(\mathbf{XX}^{\mathrm{H}}\big)^{-1}\mathbf{X}\big(\mathbf{y} - \mathbf{r}\big)^*. \tag{26}$$

Finally, the LSCMA weight vector update equation (14) can be rewritten as

$$\begin{aligned}\mathbf{w}(k+1) &= \mathbf{w}(k) - \big[\mathbf{XX}^{\mathrm{H}}\big]^{-1}\mathbf{X}\big[\mathbf{y}(k) - \mathbf{r}(k)\big]^* = \\ &= \big[\mathbf{XX}^{\mathrm{H}}\big]^{-1}\mathbf{X}\mathbf{r}^*(k),\end{aligned} \tag{27}$$

where

$$\mathbf{y}(k) = \big[\mathbf{w}^{\mathrm{H}}(k)\mathbf{X}\big]^{\top}, \tag{28}$$

$$\mathbf{r}(k) = \mathbf{L}(\mathbf{y}(k)). \tag{29}$$

3 Comparison of existing CM beamforming algorithm

We have discussed conventional constant modulus algorithms for beamforming, namely the steepest descent CMA and LSCMA. Many improved algorithms have been reported in recent years. The comparison of various CM beamforming algorithms under different scenarios is shown in Tab. 2.

Among them, the improved algorithms based on the steepest descent CMA are the variable step-size steepest descent CMA [31] and the steepest descent CMA based on the minimum Rayleigh entropy [34]. The variable step-size steepest descent CMA can accelerate the convergence speed of the algorithm, but it does not overcome the problem of the self-nulling in the conventional CMA. The steepest descent CMA based on the minimum Rayleigh entropy overcomes the self-nulling to a certain extent. That is, when the difference between the power of the desired signal and the power of the interference signal is large enough, the algorithm can accurately capture the desired signal and suppress the interference signal, but when the difference is small between the power of the desired signal and the interference signal, the algorithm is no more effective to avoid self-nulling.

The improved algorithms based on the LSCMA are the Pre-processing LSCMA [35] and the LSCMA based on minimum Rayleigh entropy [36]. The pre-processing LSCMA can capture the desired signal and suppress the interference signal in the case when the desired signal power is less than the power of the interference signal, but when the desired signal power is greater than the interference signal power, the algorithm lock onto the interference signal and suppress the desired signal. The LSCMA based on minimum Rayleigh entropy can accurately capture the desired signals and suppress the interference signal when the difference between desired signal power and the interference signal power is large, but when the desired signal power and the interference signal power is closer, the algorithm fails.

4 Proposed algorithms

In beamforming, finding a weight vector is essential step for directing the beam in the desired direction, and to suppress the interferes. The inaccurate initial value of the weight vector leads the CMA beamformer to suffer severe performance degradation because of convergence to the local minima instead of the actual minima. Thus, the beamformer is more prone to self-nulling. Therefore, the proposed algorithms in this paper calculate suitable initial values for the beamformer weights. The steepest descent CMA or LSCMA with known desired signal power and DOA is utilized with these initial values of weight vector to avoid self-nulling.

4.1 Covariance matrix decomposition

The power of the beamformer output signal is

$$\overline{|y|^2} = E\left[yy^*\right] = \mathbf{w}^{\mathsf{H}}\mathbf{R}_{\mathrm{xx}}\mathbf{w}, \tag{30}$$

where

$$\mathbf{R}_{\mathrm{xx}} = E\left[\mathbf{x}^*\mathbf{x}^{\top}\right], \tag{31}$$

is the covariance matrix of the array received signal $\mathbf{x}$. Since $\mathbf{R}_{xx}$ is a positive definite Hermitian matrix, there exists an orthogonal matrix $\mathbf{Q}$ such that

$$\mathbf{Q}^*\mathbf{R}_{\mathrm{xx}}\mathbf{Q}^{\top} = \Lambda = diag(\lambda_1, \lambda_2, \dots, \lambda_L), \tag{32}$$

where $\lambda_i > 0 \ \ (i = 1, 2, \dots, L)$ are the eigenvalues of the covariance matrix $\mathbf{D}_{\mathrm{xx}}$, and $\mathbf{Q}$ is an orthogonal matrix which satisfies

$$\mathbf{Q}^*\mathbf{Q}^{\top} = \mathbf{Q}^{\top}\mathbf{Q}^* = \mathbf{I}_L. \tag{33}$$

Here, $\mathbf{I}_L$ is an $L-th$ order identity matrix, and $\mathbf{Q}$ is given as

$$\mathbf{Q} = \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1L} \\ q_{21} & q_{22} & \cdots & q_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ q_{L1} & q_{L2} & \cdots & q_{LL} \end{pmatrix} = \begin{pmatrix} \mathbf{q}_1^{\top} \\ \mathbf{q}_2^{\top} \\ \vdots \\ \mathbf{q}_L^{\top} \end{pmatrix}, \tag{34}$$

where $\mathbf{q}_i = \begin{bmatrix} q_{i1} & q_{i2} & \dots & q_{iL} \end{bmatrix}^{\top}$ is the eigenvector of the covariance matrix $\mathbf{R}_{\mathrm{xx}}$. The eigenvalues and eigenvectors of $\mathbf{R}_{\mathrm{xx}}0$. should satisfy [11]

$$|\mathbf{R}_{\mathrm{xx}} - \lambda_i\mathbf{I}_L| = 0. \tag{35}$$

After the eigenvalues λ_i are obtained, the eigenvector $\mathbf{q}_i$ can be computed. Since the received signal $\mathbf{x}$ is sum of the desired signal, interference and the noise, therefore the covariance matrix can be written as

$$\mathbf{R}_{\mathrm{xx}} = \mathbf{R}_{\mathrm{ss}} + \mathbf{R}_{\mathrm{nn}}, \tag{36}$$

where $\mathbf{R}_{\mathrm{ss}}$ is the covariance matrix of the desired signal and the interference, and $\mathbf{R}_{\mathrm{nn}}$ is the covariance matrix of the noise. $\mathbf{R}_{\mathrm{ss}}$ has $M+1$ nonzero eigenvalues, λ_i' $i = 0, 1, \dots, M)$, and $\mathbf{R}_{\mathrm{nn}}$ has L root σ^2. Eigenvalues of $\mathbf{R}_{\mathrm{xx}}$

$$\begin{cases} \lambda_i = \lambda_i' + \sigma^2 & \text{for } i = 0, 1, \dots, M, \\ \lambda_i = \sigma^2 & \text{for } i = M+1, \dots, L, \end{cases} \tag{37}$$

are the sum of the $\mathbf{R}_{\mathrm{ss}}$ eigenvalues and the $\mathbf{R}_{\mathrm{nn}}$ eigenvalues. Obviously, the eigenvalues of $\mathbf{R}_{\mathrm{xx}}$ depends on the noise power as well as the signal power.

Assume two time-independent sources with powers P_1 and P_2 in the space, we have

$$|\mathbf{R}_{\mathrm{xx}} - \lambda_i'\mathbf{I}_2| = 0. \tag{38}$$

Solving for the eigenvalues of $\mathbf{R}_{xx}$, we have [11]

$$\lambda' = \frac{L}{2}(P_1+P_2)\left[1 \pm \sqrt{1-\frac{4P_1P_2(1-|\rho_{12}|^2)}{P_1+P_2)^2}}\right]+\sigma^2, \quad (39)$$

where ρ_{12} is the spatial correlation coefficient. If there is no spatial correlation, then $\rho_{12}=0$ and

$$\begin{cases} \lambda_1' = LP_1 + \sigma^2 \\ \lambda_2' = LP_2 + \sigma^2 \end{cases}. \quad (40)$$

The eigenvalues obtained from the covariance matrix eigen decomposition are the sum of the number of array elements multiplied by the signal power and the noise power.

4.2 Steepest descent CMA with known signal power

We propose a new algorithm to realize beamforming under the condition that the power of the desired signal is known. Since the covariance matrix of the array received signal contains some information about the desired signal, such as power and DOA and so on, and from the theoretical knowledge of the previous section, the eigenvalues of the array covariance matrix are related to the signal power and the number of the array elements.

The proposed algorithm first estimates appropriate initial values for the weight vector by constructing a new covariance matrix as

$$\overline{\mathbf{R}}_{xx} = \frac{\mathbf{R}_{xx}}{L} - P_0\mathbf{I}, \quad (41)$$

where P_0 is the power of the desired signal, and $\mathbf{I}$ denotes a unit matrix.

Next, we perform the eigen-decomposition of the covariance matrix constructed in (41) to get eigen vectors. The weight vector is initialized with the value of eigen-vector a_0 corresponding to minimum eigen value as $\mathbf{w}(0)=\mathbf{a}_0$. The initial value of the weight vector is substituted in the steepest descent CMA to iterate as follows

$$y(k) = \mathbf{w}^{\mathsf{H}}(k)\mathbf{x}(k), \quad (42)$$

$$e(k) = 2\left[y(k) - \frac{y(k)}{|y(k)|}\right], \quad (43)$$

$$\mathbf{w}(k+1) = \mathbf{w}(k) - \mu(k)\mathbf{x}(k)e^*(k). \quad (44)$$

After the weight vector $\mathbf{w}(k)$ converges, the iteration is stopped and $\mathbf{w}(K)$ is the final weight vector for beamforming.

4.2.1 Simulation analysis

In this experiment, we assume a ULA with $L=8$ half-a wavelength wavelength spaced sensors. There are three narrowband constant-modulus QPSK modulated signals in the space. The desired signal is assumed to be from the direction 0°, while two interferers are assumed to have the directions -45°and 30°, respectively.

We consider three different scenarios having large SNR-INR difference, small SNR-INR difference, and equal SNR-INR. In scenario I, the power of the desired signal and each interferer are 10, 20, and 2 dB, respectively. In scenario II, they are 10, 8, and 12 dB, respectively, and scenario III assumes equal SNR-INR of 10 dB. The performance of the proposed steepest descent CMA with known signal power is analyzed. The beampattern is shown in Fig. 5.

In scenario I and II, the main lobe of the array pattern is aligned to the correct look direction of the desired signal *ie*, 0°, and deep nulls are generated in the direction of interferers *ie*, -45°, and 30°. Thus, under the condition of large SNR-INR difference and small SNR-INR difference, the proposed steepest descent CMA with known signal power is effective regardless of the power of the desired signal or interferers. However, in scenario III, when the power of desired signal and interferers are equal, the proposed algorithm is ineffective and the beamformer lock onto the interference signal.

The proposed steepest descent CMA with known signal power is more accurate than the conventional CMA. The new covariance matrix of the beamforming algorithm makes it possible to accurately capture the desired signal by minimizing the diagonalization of the desired signal when constructing a new covariance matrix. The proposed CMA is better able to overcome the self-nulling, but when the SNR-INR difference continues to be reduced and any of the interferer power is equal to the desired signal power, the beamformer captures the interference signal. Although the proposed CMA with known signal power is ineffective when the SNR-INR difference is small, it is still more accurate than the conventional CMA.

4.3 LSCMA with known desired signal DOA

The direction vector $\mathbf{a}_0$ corresponding to the desired signal is taken as the initial value of the weight vector such that $\mathbf{w}(0)=\mathbf{a}_0$, increasing the gain of the antenna in the desired signal direction. Then, the LSCMA is iterated to converge, and accurately capture the desired signal. The iterative equations for the weight vector of LSCMA are given below

$$y(k) = \left[\mathbf{w}^{\mathsf{H}}(k)\mathbf{X}\right]^{\top}, \quad (45)$$

$$\mathbf{r} = \left[\frac{y(1)}{|y(1)|}\frac{y(2)}{|y(2)|}\cdots\frac{y(K)}{|y(K)|}\right]^{\top}, \quad (46)$$

$$\mathbf{w}(k+1) = (\mathbf{X}\mathbf{X}^{\mathsf{H}})^{-1}\mathbf{X}\mathbf{r}^*(k). \quad (47)$$

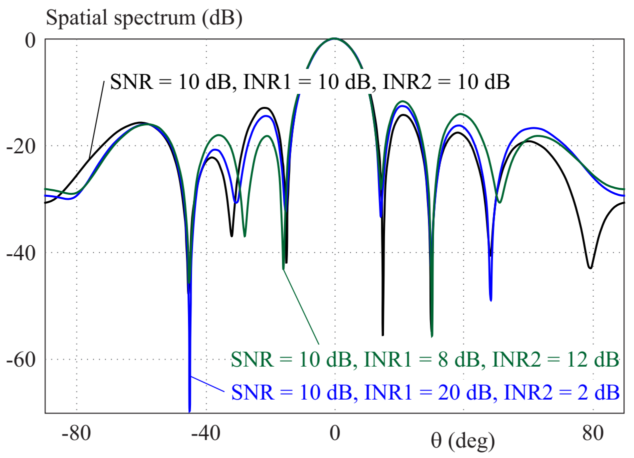

Fig. 5. Resultant beam pattern for CMA beamforming with known signal power

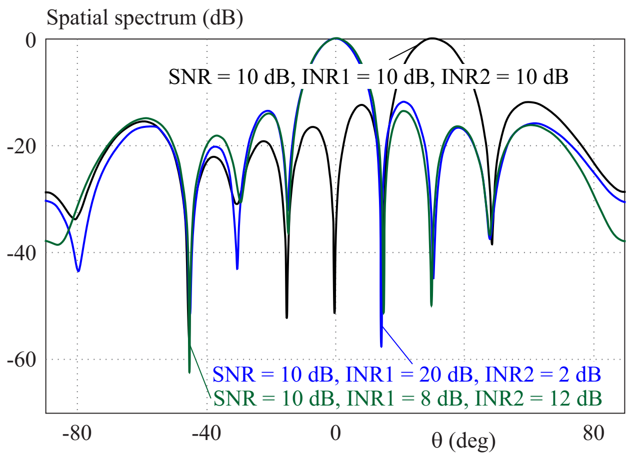

Fig. 6. Resultant beam pattern for CMA beamforming based on desired signal DOA

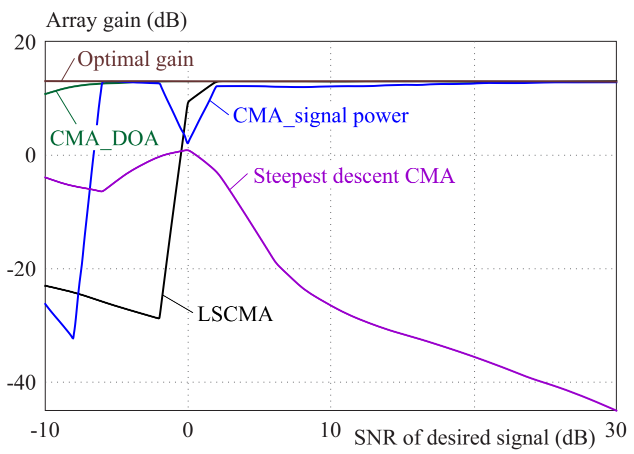

Fig. 7. Array gain versus SNR of the desired signal

4.3.1 Simulation analysis

Assume the identical simulation conditions as in the previous section, the proposed LSCMA with known DOA of the desired signal is analyzed. The resultant beam pattern is shown in Fig. 6.

In all the three scenarios, the main lobe of the array pattern is aligned to the correct look direction of the desired signal *ie*, 0°, and deep nulls are placed in the direction of interferes. Thus, the proposed LSCMA with known DOA is effective in all scenarios regardless of the power of the desired signal or interfering signal.

4.4 Array gain

To verify and validate the performance of a beamformer, array gain is the most commonly used parameter. Array gain is defined as

$$G_{\mathrm{A}} = \frac{S_{\mathrm{OUT}}}{S_{\mathrm{IN}}}, \quad (48)$$

where S stands for signal to interference-plus-noise ratio (SINR).

Figure 7 shows the array gain as a function of the SNR of the desired signal. In this simulation scenario, we assume one interference signal with INR equal to 0 dB and one desired signal whose SNR varies from -10 dB to 30 dB. The results depicted in Fig. 7 show that both the proposed CMA-based algorithms provide a significant gain over the conventional algorithms. It is also clear that the proposed DOA-based CMA reaches the level of the idealistic algorithm more quickly than the proposed CMA based on desired signal power. The reason for performance deterioration in low SNR region (SNR¡0) in the case of proposed CMA based on desired signal power is that when the power of the desired signal is less than the interference, there is a possibility of self-nulling. As the power of desired signal increases, the performance of the proposed desired signal power-based CMA improves.

4.5 Discussion

The original idea of CMA is that it does not require any training sequence such as signal DOA or power information. CMA employs the constant modularity of signals to capture the desired signal [37]. However, the proposed algorithms with known signal power and DOA of the desired signal seem to have lost the significance of the blind beamforming algorithms. Since, the proposed algorithms not only utilize the DOA information or power of the desired signal but also the constant modularity property of the signal. Integrating the advantages of the blind and non-blind beamforming algorithms, the proposed algorithms are more robust compared to existing beamforming algorithms. The commonly used non-blind adaptive beamforming algorithms are ineffective, because the beamformer is based on the estimates of the DOA, and the estimation error is inevitable. The direct matrix inversion (DMI) algorithm and Frost LMS algorithm are very sensitive to the DOA errors and robustness is poor especially when the power of the desired signal power is high. So, the use of DMI or LMS algorithm is often prone to self-nulling.

Diagonal loading is an approach used to overcome the DOA errors by adding white noise in the received signal [26, 38], but the load value is difficult to determine [26]. Similarly, [39] proposed a model-switched beamformer with a large dynamic range that can switch the models between phased array and adaptive array to overcome the self-nulling phenomenon caused by high power signal. But this model ignores the self-nulling that occurs due to low power desired signal in the case of CM algorithms. Similarly, convex optimization methods are extensively deployed in robust beamforming, but their implementation process is very complex. The proposed algorithms with known desired signal power or DOA are robust, simple, and effective to achieve the desired results for the signals with a large dynamic range.

5 Conclusion

In this paper, we briefly discuss the statistical signal modeling and fundamentals of the CMA and its variants. The self-nulling caused by low-power desired signal in CMA beamformers and high-power desired signal in MVDR beamformers is found to be the main limiting factor of these algorithms. Furthermore, we present two improved CMA based beamforming algorithms to avoid self-nulling in beamformers. The proposed algorithms exploit the power and DOA information of the desired signal to avoid self-nulling, especially for desired signals having a large dynamic range. Simulation results demonstrate the effectiveness of the proposed algorithms.

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