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Temperature-Dependent Density and Magnetohydrodynamic Effects on Mixed Convective Heat Transfer along Magnetized Heated Plate in Thermally Stratified Medium Using Keller Box Simulation

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Abstract: The heat transmission properties along the non-magnetized geometries have been numerically obtained by various researchers. These mechanisms are less interesting in engineering and industrial processes because of excessive heating. According to current studies, the surface is magnetized and the fluid is electrically conductive, which helps to lessen excessive surface heating. The main objective of the current analysis is to numerically compute the temperature-dependent density effect on magnetohydrodynamic convective heat-transfer phenomena of electrical-conductive fluid flow along the vertical magnetized and heated plate placed in a thermally stratified medium. For the purpose of numerical analysis, the theoretical process governing heat and magnetic intensity along a vertical magnetic plate is examined. By using suitable and well-known similarity transformations for integration, the non-linear coupled PDEs for the aforementioned electrical-conductive fluid flow mechanism are changed and subsequently converted into non-similar formulation. The Keller Box method is used to numerically integrate the final non-similar equations. The MATLAB software program plots the transformed algebraic equations graphically and quantitatively. The behavior of the physical quantities such as velocity graph, magnetic field graph, and temperature plot along with their slopes that are rate of skin friction, the rate of heat transfer, and the rate of magnetic intensity for different parameters included in the flow model. The novelty of the current work is to compute the magneto-thermo analysis of electrically conducting flow along the vertical symmetric heated plate. First, we secure the numerical solution for steady part and then these results are used to find skin friction, heat transfer, and magnetic intensity. In the current work, the fluid becomes electrically conducting due to a magnetized surface which insulates heat during the mechanism and reduces the excessive heating. The results are excellent and accurate because they are satisfied by its given boundary conditions. Additionally, the current problems have a big impact on the production of polymer materials, glass fiber, petroleum, plastic films, polymer sheets, heat exchangers, catalytic reactors, and electronic devices.

Keywords: mixed convection; Keller Box method; thermally stratified medium; magnetized plate; magnetohydrodynamics; temperature dependent density

1. Introduction and Literature Review

In metallurgy and polymer technology, the heat, magnetic intensity, and momentum transfer in the laminar boundary layer flow on a magnetized heated plate immersed in thermally stratified medium are significant from both a theoretical and practical standpoint. Engineers can benefit greatly from the laws governing heat transfer and magnetohydrodynamics flows in a variety of fields, including heat exchangers, sunspot theory, intercontinental ballistic missiles, interstellar gas motion, liquid metals, cooling of nuclear reactors, plasma confinement, and geophysics and astrophysics. The dynamic study of electrically conducting fluid is known as magnetohydrodynamics, established by Maxwell for electromagnetism. The fundamental idea behind an electrically conducting fluid is that a magnetic field has the potential to cause currents to flow through conductive fluid that is flowing, which in turn exerts forces on the fluid and affects the magnetic field. In literature, most researchers have analyzed the heat transfer characteristics of mixed convection flow along different geometries. However, in current analysis, the aligned magnetic field is used to make a surface magnetized and to reduce the excessive heating along the proposed surface. Some related literature of the current problem is mentioned here to understand the theoretical and numerical points. Furthermore, the current issues have significant implications for the polymer industries, including paper production, glass-fiber production, liquid crystal solidification, petroleum production, production of unusual lubricants, suspension solutions, wire drawing, continuous cooling, fiber spinning, plastic film production, polymer sheet extraction, heat exchangers, petroleum resource recovery, fault zones, catalytic reactors, and the production of electronic devices.

Nima and Ferdows [1] performed multiple solutions of microorganisms across a non-isothermal inclined disk with gyro-tactic particles. Mahanthesh et al. [2] developed an analysis on squeezing flow along a magnetized channel with hall current, heat source, and thermal radiation effects numerically. They found the increasing behavior in concentration and temperature by reducing value of squeezing parameter. Bansood and Ambedkar [3] studied free forced convective boundary layer flow over a semi-infinite plate in a porous medium. They analyzed heat and mass transfer in boundary layer flow region for opposing flow. Hazarika and Ahmed [4] investigated the numerical development of the MHD natural convective nanofluid flow through a vertical cone with heat source effects. They discovered a decreasing behavior of concentration profile when the Brownian motion parameter is raised. The laminar mixed convection flow from a vertical surface was studied by Chaudhry et al. [5]. The melting effects on mixed convection heat and mass transfer in a non-Newtonian nanofluid submerged in a porous medium were carried out by Ram Reddy and Kairi in [6]. They discovered that when the buoyancy ratio grows, the heat transfer coefficient decreases in the entire range of porous geometry. Hussain et al. [7] developed the unsteady convection flow of two phase nanofluid around a cone embedded in porous material numerically. They found that under viscous dissipation impact, the heat transfer rate decreased with maximum viscous heat generation.

The consequences of temperature-dependent density on a steady mixed Newtonian fluid convective flow near to a vertically permeable heated plate in a stratified material has great importance in industries and engineering. In the manufacturing sector, double diffuse free forced convection over vertical bodies is crucial for the development of dependable machinery, nuclear reactors, satellites, and spacecraft. Using convective boundary conditions and induced magnetic field, Mahdy [8] studied the nanofluid including gyro-tactic microorganisms with specified density motile microorganisms for natural convection flow across a vertical flat sheet. They found that as time dilation, heat conduction, and mixed convection parameter increases, the Nusselt number decreases. In order to simulate mixed convection flow of viscous fluid from a vertical cone with the effect of chemical reaction, Pullepu et al. [9] used numerical methods. They discovered that as skin friction increases, the velocity of steady flow is raised. Srinivasacharya and Surender [10] investigated impact of dual stratification on free forced convective boundary layer flow of nanofluid over a vertical shape embedded in a porous media. They obtained a considerable decrease in

heat transmission and an increase in mass transfer rates with thermophoresis effect. Ullah et al. [11–14] elaborated impacts of MHD, velocity slip, reduced gravity, and stratification on oscillatory fluid along a non-conducting circular heated shape numerically. The baffle effects on enclosure filled with nanofluid for convective heat mechanism were analyzed in [15]. The entropy analysis on pseudo plastic flow along circular shape with MHD by using Keller Box scheme was investigated numerically in [16]. Sofiadis [17] numerically analyzed the Reynolds effects on micro polar flow in a channel.

Mehmood et al. [18] performed free forced convection flow in thermally and solutal stratified magneto nanofluid with non uniform heat source. They examined the volume fraction profile of lowered Brownian motion parameter and solute parameter increases. In the presence of internal heat generation, Elbashbeshy et al. [19] investigated the effects of thermal radiation and magnetic field on unsteady mixed convection flow and heat transfer across an exponentially extending surface with suction. They discovered that the radiative heat transfer increased as the thermal radiation parameter rose. Daniel et al. [20] developed a heat energy problem from a nanofluid flow across a stretched sheet with MHD and a chemical reaction. They observed that velocity and temperature profile rise by increments in the electrical parameter.

The impact of a chemical reaction on the flow of MHD viscoelastic fluid over vertical stretching sheet with heat source effects was described by Jena et al. in [21]. They found that the chemical reaction and Prandtl number increases the rate of heat transfer along the plate. Yasin et al. [22] performed boundary layer flow with free forced convection embedded in thermally stratified porous medium saturated by nanofluid numerically. Rawi et al. [23] introduced the influence of copper nanoparticles on Jeffrey fluid of free forced convection flow generated by g-jitter. Alzahrani et al. [24] developed a third grade hydro magnetic free/forced convection problem for nonmaterial including gyro tactic microorganisms across a stretched surface. The effects of magnetohydrodynamics and velocity-slip on convective flow along the non-conducting geometry were studied by Ullah et al. in [25]. The opposing flow through a vertical cylinder filled with a Darcy porous medium containing gyro tactic microorganisms with double diffusive mixed convection was examined by Alsenafi and Ferdows in [26]. Babu et al. [27] examined the impact of thermal conductivity and variable viscosity on heat and mass transfer of nanofluid along a vertical cone in the presence of a chemical reaction. Variable density effects on hydro magnetic mixed convective non-Newtonian fluid flow over a moving vertical plate were presented by Salem et al. in [28]. Iqbal et al. [29] investigated the forced convective flow along a cone with heat and mass flux model by using implicit finite difference method graphically. Abulnaja et al. [30] performed unsteady free/forced convection flow filled with nanofluid across the straight disc placed in a porous material in the presence of variable surface temperature. Maranna et al. [31] analyzed the mass transpiration phenomena with MHD and radiation effects on viscous fluid past a stretched shape. Mabood et al. [32] performed Jeffrey nanofluid phenomena with entropy and MHD effects along the stretched geometry for various engineering application.

It can be seen that all of the aforementioned investigations deal with a minimum temperature difference between the fluid at the surface and the ambient fluid. However, there are instances where there is a significant temperature difference. The Boussinesq approximation is no longer appropriate in this case (see Kakac and Yenar [33]). To notice these circumstances, the temperature-dependent density function is suggested in [33] for the specific cases where large temperature difference occurs. The theoretical mechanism on heat and magnetic intensity along a vertical magnetic plate has been investigated for numerical analysis. Using the idea from previous research [33], the temperature-dependent density effect on magnetohydrodynamic convective heat-transfer phenomena of electrical-conductive fluid flow along the vertical magnetized and heated plate placed in thermally stratified medium was explored numerically. The non-similar equations were numerically integrated by employing the Keller Box method. The transformed algebraic equations were plotted graphically and numerically on the MATLAB software package. The heat transfer

characteristics along the non-magnetized shapes were performed in various previous studies numerically. Due to excessive heating, these mechanisms are less interesting in engineering and industrial processes. In the current analysis, the surface was magnetized and the fluid was electrically conducting which is responsible for reducing excessive heating along the surface. The novelty of the present work is in the analysis of convective heat transfer analysis of viscous fluid flow with temperature dependent density and thermal stratification effects along the vertical symmetric heated plate numerically. First, the velocity, magnetic field, and temperature profiles were secured and then those profiles used to compute skin friction, heat transfer, and magnetic intensity along the heated plate. The obtained results are in excellent agreement by satisfying the given boundary conditions.

2. Flow Problem and Mathematical Description

The problem is devoted to numerical solutions of magneto-thermo analysis of convective heat transfer along thermally and magnetized geometry immersed in thermally stratified medium. The extended issue will change into arrangement of partial differential conditions and then transform into ordinary ones using the stream-function forms. The molded problem will be resolved by using FDM method conjunct with the Keller Box method. The numerical result for considered material properties will be depicted in graphical and tabular form.

Consider two dimensional steady and incompressible electrical conducting fluid. In Figure 1, where x and y are axes along and perpendicular to the vertical magnetized plate immersed in thermally stratified medium, respectively, u , H_x and v , H_y are the velocity-fields in x and y orders, respectively. The H_0 is the magnetic field strength, $H_x = H_0$ indicates that the magnetic field is exact at the surface, T is the temperature, T_∞ is the free stream temperature, κ is the thermal conductivity of the fluid, and C_p is the specific heat. The ρ is the fluid density, μ_0 is the magnetic permeability, $\nu = \mu/\rho$ is the kinematic fluid viscosity, β^* is the volumetric coefficient of thermal expansion, and gravity acceleration is g . The following is how the equations for continuity, momentum, magnetic force, and energy are expressed where high temperature difference arises with temperature dependent density function:

$$\rho = \rho_\infty \exp[\beta_T(T - T_\infty)], \quad \beta_T = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p \quad (1a)$$

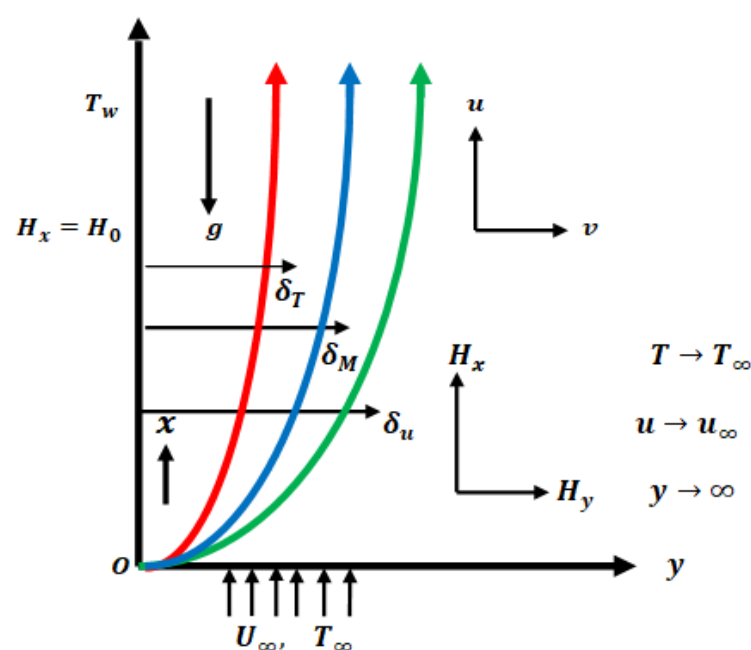


Figure 1. Coordinate system and flow geometry.

It should be emphasized that the Boussinesq approximation holds true for fluids with β_T of the magnitude of 10^{-3} when the temperature differential between the fluid and the surface is of the order of $O(10^{-2})$. Consequently, leaving out higher-order terms and expanding Equation (1a) into a Taylor series, the above equation becomes $\rho = \rho_\infty[1 - \beta_T(T_w - T_\infty)]$.

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \quad (1b)$$

$$\frac{\partial \rho H_1}{\partial x} + \frac{\partial \rho H_2}{\partial y} = 0 \quad (2)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \mu \frac{\partial}{\partial y} \left(\frac{1}{\rho} \frac{\partial u}{\partial y} \right) + \frac{g}{\rho} (\rho_\infty - \rho) + \frac{\mu_0}{4\pi\rho} \left(H_1 \frac{\partial H_1}{\partial x} + H_2 \frac{\partial H_1}{\partial y} \right) \quad (3)$$

$$u \frac{\partial H_1}{\partial x} + v \frac{\partial H_1}{\partial y} = \nu_m \frac{\partial^2 H_1}{\partial y^2} + \left(H_1 \frac{\partial u}{\partial x} + H_2 \frac{\partial u}{\partial y} \right) \quad (4)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{C_p} \frac{\partial}{\partial y} \left(\frac{1}{\rho} \frac{\partial T}{\partial y} \right) \quad (5)$$

The boundary conditions of the present model are,

$$u = 0; v = 0; H_1 = H_0, H_2 = 0; T = T_w \text{ at } y = 0 \quad (6)$$

$$U \rightarrow U_\infty, T \rightarrow T_\infty, H_1 \rightarrow 0 \text{ as } y \rightarrow \infty.$$

The proper boundary conditions for the temperature with temperature-dependent density and the thermal stratification components are described above. The Reynolds number is $Re_x = \frac{U_\infty x}{\nu}$, U_∞ being the free stream velocity, $T_w = T_o + b \frac{x}{L}$ is the variable temperature, and $T_\infty = T_o + c \frac{x}{L}$ is ambient variable temperature of the thermally and magnetized heated plate, b, c are positive constants and T_o is the temperature constant along the thermally and magnetized heated plate.

3. The Similarity Transformation and Stream Function Formulation

The proper dimensionless similarity and stream function to convert PDEs into ODEs are given as:

$$u = \frac{1}{\bar{\rho}} \frac{\partial \psi}{\partial y}, v = -\frac{1}{\bar{\rho}} \frac{\partial \psi}{\partial x}, H_1 = \frac{1}{\bar{\rho}} \frac{\delta \varphi}{\delta y}, H_2 = -\frac{1}{\bar{\rho}} \frac{\delta \varphi}{\delta x}, \theta = \frac{T - T_\infty}{T_w - T_\infty}. \quad (7)$$

The stream function ϕ and ψ are for magnetic and velocity fields, θ is dimensionless temperature and similarity parameter is η .

$$u = u_\infty f', \quad v = \frac{-1}{2\bar{\rho}} \sqrt{\frac{\nu u_\infty}{x}}, \quad (8)$$

$$H_1 = g' H_0, \quad H_2 = \frac{-1}{2\bar{\rho}} \frac{H_0}{U_\infty} \sqrt{\frac{\nu U_\infty}{x}} (g - \eta g'),$$

$$\eta = \int_0^y \bar{\rho} dy \sqrt{\frac{u_\infty}{\nu x}}, \quad \psi = \sqrt{\nu x u_\infty} f(\eta), \quad \varphi = \frac{H_0}{U_\infty} \sqrt{\nu x u_\infty} g$$

The converted form of the ODEs in Equations (1)–(6) by applying the Equations (7) and (8), we have non linear ODEs given below;

$$\frac{1}{2} f f'' + e^{-n\theta} f''' - n\theta' e^{-n\theta} f'' + \lambda \left(\frac{1 - e^{n\theta}}{1 - e^{-n}} \right) + e^{n\theta} M g g'' = 0 \quad (9)$$

$$P_m \left(e^{-n\theta} g''' - n\theta' e^{-n\theta} g'' \right) + \frac{f g''}{2} - \frac{g f''}{2} = 0 \quad (10)$$

$$e^{-n\theta}\theta' - ne^{-n\theta}\theta'^2 + Pr\left(\frac{1}{2}f\theta' - f'\theta - Stf'\right) = 0 \quad (11)$$

where $Pr = \frac{\nu}{\alpha}$ is Prandtl parameter, $P_m = \frac{\nu_m}{\nu}$ is magnetic Prandtl parameter, $M = \frac{H_0^2 L^2 \mu_0}{\nu^2 4\pi\rho}$ is a magnetic parameter, $\lambda = \frac{g\beta^* T_0}{U_\infty^2}$ is the mixed convection parameter, η is similarity variable, θ is dimensionless temperature, $\nu = \frac{\mu}{\rho}$ is kinematic fluid viscosity, and $St = \frac{c}{b}$ is stratification number. The transformed boundary conditions in (6) then becomes,

$$f' = 0, f = 0, g = 0, \theta = 1 - St, g' = 1 \quad \text{at} \quad \eta = 0 \quad (12)$$

$$f' \rightarrow 1, \theta \rightarrow 0, g' \rightarrow 0, \quad \text{at} \quad \eta \rightarrow \infty.$$

The formula for skin friction is $Cf_x = \frac{\tau_w}{\rho U_\infty^2}$, the formula for the Nusselt number is $Nu_x = \frac{xq_w}{\kappa(T_w - T_\infty)}$, and the formula for the magnetic intensity coefficient is $Mg_x = -\frac{j_w}{U_\infty^2}$. The values of τ_w , q_w , and j_w are listed below:

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -\kappa \left(\frac{\partial T}{\partial y} \right)_{y=0}, \quad j_w = \nu_m \left(\frac{\partial H}{\partial y} \right)_{y=0}$$

Consequently, the values of magnetic intensity, Nusselt number, and skin friction are provided by,

$$Re_x^{1/2} Cf_x = f''(0), \quad Re_x^{-1/2} Nu_x = -\theta'(0), \quad Re_x^{-1/2} Mg_x = -g''(0).$$

4. The Solution and Computing Techniques

A suitable stream function formulation is used to convert the linked partial differential equations for the aforementioned model into a set of ordinary differential equations. The iterative Keller Box scheme is used to solve the simplified nonlinear ordinary differential Equations (9)–(11) and boundary conditions in (12). We begin by introducing additional independent variables $p(\eta)$; $q(\eta)$, $u(\eta)$, and $v(\eta)$ by using Equation (13),

$$f' = p, \quad f'' = p' = q, \quad f''' = q', \quad g' = u,$$

$$g'' = u' = v, \quad g''' = v', \quad \theta' = l, \quad \theta'' = l' = m. \quad (13)$$

Equations (9)–(12) get simpler to solve in order to get around the problems that they present. Thus, the more simple forms are,

$$f' = p \Rightarrow f' - p = 0 \quad (14)$$

$$p' = q \Rightarrow p' - q = 0 \quad (15)$$

$$g' = u \Rightarrow g' - u = 0 \quad (16)$$

$$u' = v \Rightarrow u' - v = 0 \quad (17)$$

$$\theta' = l \Rightarrow \theta' - l = 0 \quad (18)$$

$$\frac{fq}{2} + (1 - n\theta)q' - nl(1 - n\theta)q + \lambda\theta + (1 + n\theta)Mgv = 0 \quad (19)$$

$$(1 - n\theta)[P_mv' - nlv] + \frac{fv}{2} - \frac{gq}{2} = 0 \quad (20)$$

$$(1 - n\theta)m - n(1 - n\theta)l^2 + Pr\left(-\frac{fl}{2} - p\theta - Stp\right) = 0 \quad (21)$$

The reduced boundary conditions are

$$f = 0, g = 0, p = 0, u = 1, \theta = 1 - St \quad \text{at} \quad \eta = 0, \quad (22)$$

$$p \rightarrow 1, u \rightarrow 0, \theta \rightarrow 0, \text{ as } \eta \rightarrow \infty.$$

We now consider the segment η_{n-1}, η_n with $\eta_{n-\frac{1}{2}}$ as midpoint given in Equation (23) below;

$$\eta_0 = 0, \quad \eta_n = \eta_{n-1} + h_n, \quad \eta_n = \eta_\infty. \quad (23)$$

This is how the central difference form and average form are presented:

$$f' = \frac{f_n - f_{n-1}}{h_n}, \quad f = \frac{f_n + f_{n-1}}{2} = f_{n-\frac{1}{2}}, \quad (24)$$

and

$$f_n - f_{n-1} - \frac{1}{2}h_n(p_n + p_{n-1}) = 0, \quad (25)$$

$$p_n - p_{n-1} - \frac{1}{2}h_n(q_n + q_{n-1}) = 0, \quad (26)$$

$$g_n - g_{n-1} - \frac{1}{2}h_n(u_n + u_{n-1}) = 0, \quad (27)$$

$$u_n - u_{n-1} - \frac{1}{2}h_n(v_n + v_{n-1}) = 0, \quad (28)$$

$$\theta_n - \theta_{n-1} - \frac{1}{2}h_n(l_n + l_{n-1}) = 0, \quad (29)$$

By applying the above Equations (24)–(29), the governing Equations (14)–(21) become

$$\begin{aligned} & \left(\frac{1}{8}(f_n + f_{n-1}) \right) (q_n + q_{n-1}) + \left(\frac{q_n - q_{n-1}}{h_n} \right) \\ & + n \left[\frac{3}{4}(q_n + q_{n-1})(l_n + l_{n-1}) + \frac{1}{2}(p_n + p_{n-1}) + \frac{1}{16}(f_n + f_{n-1})(p_n + p_{n-1})(l_n + l_{n-1}) \right] \\ & + n^2 \left[\frac{1}{4}(p_n + p_{n-1})(l_n + l_{n-1}) + \frac{1}{8}(f_n + f_{n-1})(l_n + l_{n-1})^2 \right] + (1 \\ & - n)(\theta_n + \theta_{n-1}) \left[\lambda(\theta_n + \theta_{n-1}) - \frac{M}{4}(g_n + g_{n-1})(v_n + v_{n-1}) \right] = 0 \end{aligned} \quad (30)$$

$$\begin{aligned} & 1 - \frac{n}{2}(\theta_n + \theta_{n-1}) \left[\frac{P_m}{h_n}(v_n - v_{n-1}) - \frac{n}{4}(l_n + l_{n-1})(v_n + v_{n-1}) \right] + \frac{1}{8}(f_n + f_{n-1})(v_n + v_{n-1}) \\ & - \frac{1}{8}(g_n + g_{n-1})(q_n + q_{n-1}) = 0 \end{aligned} \quad (31)$$

$$\begin{aligned} & 1 - \frac{n}{2}(\theta_n + \theta_{n-1}) \left[\frac{m_n + m_{n-1}}{2} - n \left(\frac{l_n + l_{n-1}}{2} \right)^2 \right] \\ & + P_r \left[\frac{1}{8}(f_n + f_{n-1})(l_n + l_{n-1}) - \frac{1}{4}(p_n + p_{n-1})(\theta_n + \theta_{n-1}) - \frac{St}{2}(p_n + p_{n-1}) \right] = 0 \end{aligned} \quad (32)$$

along with boundary conditions

$$f_0 = 0, g_0 = 0, P_0 = 0, u_0 = 1, \theta_0 = 1 - St, \text{ at } \eta = 0 \quad (33)$$

$$p_n \rightarrow 1, \theta_n \rightarrow 0, u_n \rightarrow 0, \text{ as } \eta \rightarrow \infty$$

Applying the below-mentioned iterative Newton–Raphson approach for smooth algorithm;

$$\begin{aligned} f_n^{k+1} &= f_n^k + \delta f_n^k, p_n^{k+1} = p_n^k + \delta p_n^k \\ q_n^{k+1} &= q_n^k + \delta q_n^k, \theta_n^{k+1} = \theta_n^k + \delta \theta_n^k \\ u_n^{k+1} &= u_n^k + \delta u_n^k, g_n^{k+1} = g_n^k + \delta g_n^k \\ v_n^{k+1} &= v_n^k + \delta v_n^k, l_n^{k+1} = l_n^k + \delta l_n^k \end{aligned} \quad (34)$$

By ignoring all appearances of powers of greater than the first power, just like in the standard Newton–Raphson method, equations become;

$$\delta f_n - \delta f_{n-1} - \frac{1}{2}h_n(\delta p_n + \delta p_{n-1}) = (r_1)_n \quad (35)$$

$$\delta p_n - \delta p_{n-1} - \frac{1}{2}h_n(\delta q_n + \delta q_{n-1}) = (r_2)_n \quad (36)$$

$$\delta g_n - \delta g_{n-1} - \frac{1}{2}h_n(\delta v_n + \delta v_{n-1}) = (r_3)_n \quad (37)$$

$$\delta u_n - \delta u_{n-1} - \frac{1}{2}h_n(\delta v_n + \delta v_{n-1}) = (r_4)_n \quad (38)$$

$$\delta \theta_n - \delta \theta_{n-1} - \frac{1}{2}h_n(\delta l_n + \delta l_{n-1}) = (r_5)_n \quad (39)$$

The simplified form of the equations is provided below, again utilizing Equations (35)–(39) in Equations (30)–(33);

$$(a_1)_n \delta f_n + (a_2)_n \delta f_{n-1} + (a_3)_n \delta q_n + (a_4)_n \delta q_{n-1} + (a_5)_n \delta l_n + (a_6)_n \delta l_{n-1} + (a_7)_n \delta \theta_n + (a_8)_n \delta \theta_{n-1} + (a_9)_n \delta g_n + (a_{10})_n \delta g_{n-1} = (r_6)_n \quad (40)$$

$$(b_1)_n \delta v_n + (b_2)_n \delta v_{n-1} + (b_3)_n \delta \theta_n + (b_4)_n \delta \theta_{n-1} + (b_5)_n \delta l_n + (b_6)_n \delta l_{n-1} + (b_7)_n \delta g_n + (b_8)_n \delta g_{n-1} + (b_9)_n \delta v_n + (b_{10})_n \delta v_{n-1} = (r_7)_n \quad (41)$$

$$(c_1)_n \delta m_n + (c_2)_n \delta m_{n-1} + (c_3)_n \delta \theta_n + (c_4)_n \delta \theta_{n-1} + (c_5)_n \delta l_n + (c_6)_n \delta l_{n-1} + (c_7)_n \delta p_n + (c_8)_n \delta p_{n-1} + (c_3)_n \delta f_n + (c_4)_n \delta f_{n-1} = (r_8)_n \quad (42)$$

The specific boundary conditions that can be met without iteration are recalled. As a result, to ensure that these correct values are maintained across all iterations, we take

$$\delta f_0 = 0, \delta g_0 = 0, \delta p_0 = 0, \delta u_0 = 1, \delta \theta_0 = 1 - St \quad (43)$$

$$\delta p_n = 1, \delta \theta_n = 0, \delta u_n = 0$$

5. The Arrangement of Difference Equations in Vector Notation/Matrix

The next essential step is to organize the aforementioned difference equations in matrix form. If it is done wrong, either the matrix solution method fails because there is a singular matrix (determinant = 0) or sub-matrix, or the approach becomes extremely inefficient since there is no discernible structure in the matrix. We have,

$$A\delta = r, \quad (44)$$

$$[A] = \begin{bmatrix} [A_1][C_1] & \cdots & \cdots \\ [B_2][A_2][C_2] & & \\ \vdots & \ddots & \vdots \\ \vdots & \cdots & [B_{n-1}][A_{n-1}][C_{n-1}] \\ & & [B_n][A_n] \end{bmatrix}, [\delta] = \begin{bmatrix} [\delta_1] \\ [\delta_2] \\ \vdots \\ [\delta_{n-1}] \\ [\delta_n] \end{bmatrix}, [r] = \begin{bmatrix} [r_1] \\ [r_2] \\ \vdots \\ [r_{n-1}] \\ [r_n] \end{bmatrix}, \quad (45)$$

6. Analysis and Discussion of Results

The present analysis addresses the temperature-dependent density effect on magnetohydrodynamic convective heat transfer phenomena of electrical-conductive fluid flow along the vertical magnetized and heated plate placed in thermally stratified medium. The non-linear coupled PDE equations for the above fluid flow mechanism are formulated and then converted into non-similar formulation by applying an appropriate and well-known similarity transformation for integration. The final non-similar forms are integrated numerically by employing Keller Box scheme method. The transformed algebraic equations are

plotted graphically and numerically on the MATLAB software package. The behavior of physical quantities such as velocity graph, magnetic field graph, and temperature graph along with their slopes, that is skin friction, magnetic intensity, and heat transfer, under the effect of different parameters included in the flow model is discussed. The impact of physical parameters such as Prandtl number P_r , mixed convective parameter λ , stratification parameter St , density number m , magnetic-force number M , and magnetic Prandtl P_m is interpreted graphically and numerically.

Figure 2a–c are presented for the behavior of velocity graph, temperature plot, and magnetic field for different values of temperature dependent density $m = 0.0, 1.0, 2.0$, and 3.0 with some fixed parameters. It was deduced that the velocity profile was enhanced at maximum value of $m = 3.0$ but similar behavior was noted for other values of m in Figure 2a. This happens because an increasing value of density parameter tends to increase the density variation with temperature, which ultimately increases the buoyancy force. The magnetic field was increased at the lower value of $m = 0.0$ but decreased at the maximum value of $m = 3.0$ in Figure 3b. In Figure 3c, the temperature distribution was increased at lower value of $m = 0.0$ but decreased at maximum value of $m = 3.0$. The good variations in each profile were noted for each value of temperature-density parameter m . Hence, as m increased, the fluid was under two forces: the first force increased the velocity of the fluid due to increase in the buoyancy forces and the second force decreased the velocity of the fluid due to the decrease in temperature. Figure 3a–c are plotted against the various values of stratification parameter $St = 0.0, 0.1, 0.2, 0.3$ for velocity graph, temperature graph, and magnetic field. It was examined that the velocity profile increased at lower value of $St = 0.0$ but the minimum value was noted at the higher value of $St = 0.3$ in Figure 3a. It was deduced that the magnetic field was enhanced at the large value of $St = 0.3$ but decreased at the small value of $St = 0.0$ in Figure 3b. This happens because magnetic diffusion is reduced by increasing the magnetic-Prandtl number which is responsible for above-said phenomena. This behavior of the aforementioned variables supports the physical hypothesis that as the magnetic field grows stronger, more resistance is formed inside the fluid flow domain, which reduces the velocity distribution and improves the temperature profile. The temperature distribution increased at the lower value of $St = 0.0$ but decreased at the maximum value of $St = 0.3$ in Figure 3c. Figure 4a–c depict the behavior of velocity $f'(\eta)$, temperature $\theta(\eta)$, and magnetic field $g'(\eta)$ for different quantities of $\lambda = 0.1, 0.5, 1.5$, and 2.0 with some fixed parameters. It was deduced that the velocity profile was enhanced at larger $\lambda = 2.0$ but decreased at small value of $\lambda = 0.1$ in Figure 4a. It was examined that the magnetic field increased at lower value of $\lambda = 0.1$ while minimum value was noted at higher value of $\lambda = 2.0$ in Figure 4b. It was also noted that the temperature distribution increased at lower value of $\lambda = 0.1$ but decreased at maximum value of $\lambda = 2.0$ in Figure 4c. Physically, it was expected because larger values of λ correspond to stronger buoyancy forces which lead to an increase in the acceleration of fluid flow. Due to conducting phenomena, magnetic effects are strongly observed exactly at the surface, but far from the surface, each value is zero. The good variations in each profile were noted for each value of mix convection parameter λ . Figure 5a–c examined the behavior of velocity $f'(\eta)$, temperature $\theta(\eta)$, and magnetic field $g'(\eta)$ for different quantities of Prandtl number $P_r = 1.0, 4.0, 7.0, 10.0$ with some fixed parameters. It was shown that the velocity field increased at the highest value of $P_r = 10.0$ and decreased at $P_r = 1.0$ in Figure 5a. The magnetic field distribution increased at the lowest value of $P_r = 1.0$ but minimum value was noted at the highest value of $P_r = 10.0$ in Figure 5b. In Figure 5c, the temperature distribution was increased by decreasing the value of $P_r = 1.0$ but the temperature distribution was decreased by increasing the value of $P_r = 10.0$. This phenomenon is expected because an increase in Pr tends to increase density variation with temperature which enhances the buoyancy force. The thermal boundary layer thickness was reduced due to increase in Pr . The prominent thermal slip with good variations are obtained in temperature graph at each value of P_r . The good variations in each profile were examined for various values of Prandtl number P_r . Figure 6a–c are demonstrated

for velocity $f'(\eta)$, temperature $\theta(\eta)$, and magnetic field $g'(\eta)$ for different quantities of magnetic Prandtl number $P_m = 0.1, 0.7, 1.5, 3.0$ with some fixed parameters along the surface of a vertical geometry. In Figure 6a, it was deduced that the fluid velocity along the surface was increased for the large value of $P_m = 3.0$ but the minimum value was obtained at a lower value of $P_m = 1.0$ with good response. It was deduced that the magnetic graph increased at the large value of $P_m = 3.0$ but decreased at the small value of $P_m = 1.0$ in Figure 6b. In Figure 6c, it is depicted that the temperature distribution increased at the small value of $P_m = 1.0$ but the minimum value was noted at the large value of $P_m = 3.0$. It was also noted that temperature plots are in good form and excellent agreement for the given phenomenon. As the magnetic Prandtl number increased, viscosity of fluid increased and fluid became thicker more and consequently the boundary layer thickness decreased. Figure 7a–c are plotted for velocity, magnetic field, and fluid temperature against the various values of magnetic force parameter $M = 0.1, 5.0, 10.0$, and 20.0 with some constant parameters long the surface of the vertical geometry. In Figure 7a, it is deduced that the mixed convection effect in the velocity graph increased at the large value of $M = 20.0$ but minimum value was obtained at the lower value of $M = 0.1$ by satisfying the given boundary conditions asymptotically. This result was expected because an increase in M means enhancement in the Lorentz forces which opposes the flow and velocity of the fluid decreases. In Figure 7b, it was examined that the magnetic field profile along the surface was enhanced at the large value of $M = 20.0$ but the minimum value was obtained at small value of $M = 0.1$. This happens because the magnetic force parameter is the ratio of magnetic energy to kinetic energy, so with the increase in the magnetic force parameter P_m , the magnetic energy is increased, while the kinetic energy is reduced. An excellent and favorable phenomenon is observed in magnetic field graph. In Figure 7c, it is deduced that the temperature profile decreased at the large value of $M = 20.0$ but increased at the small value of $M = 0.1$. This given mechanism is performed with good agreement.

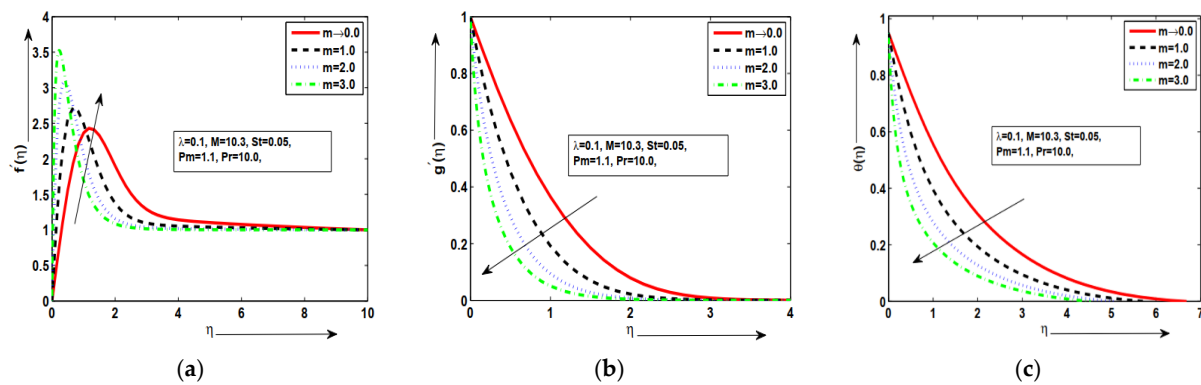


Figure 2. (a–c) The geometric profiles of $f'(\eta)$, $g'(\eta)$, and $\theta(\eta)$ against m .

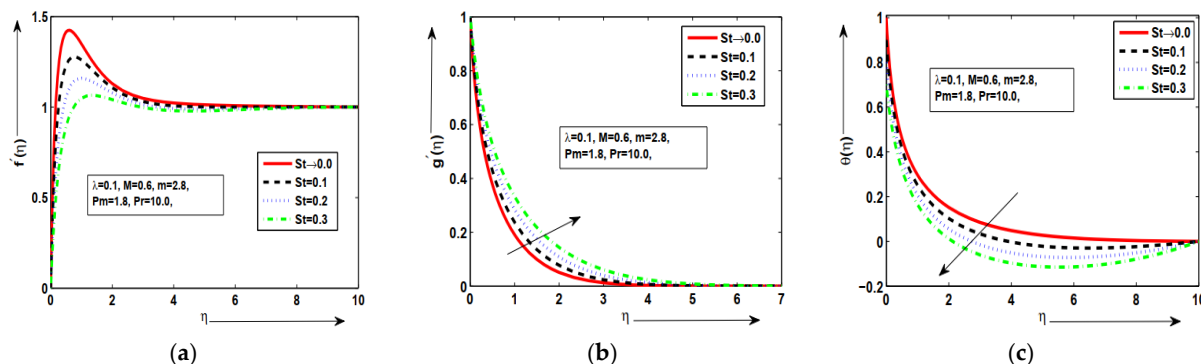


Figure 3. (a–c) The geometric profiles of $f'(\eta)$, $g'(\eta)$, and $\theta(\eta)$ against St .

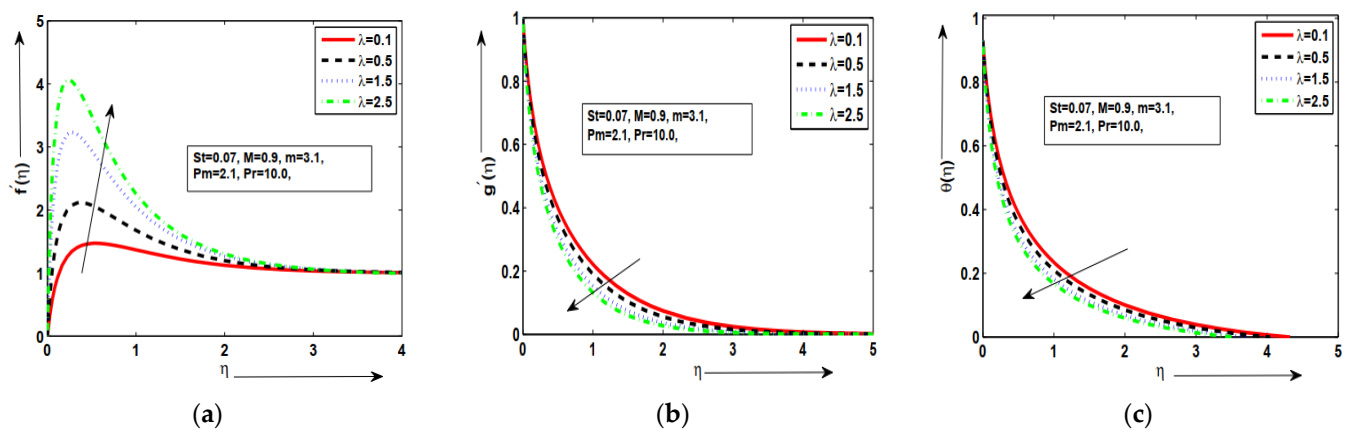


Figure 4. (a–c) The geometric profiles of $f'(\eta)$, $g'(\eta)$, and $\theta(\eta)$ against λ .

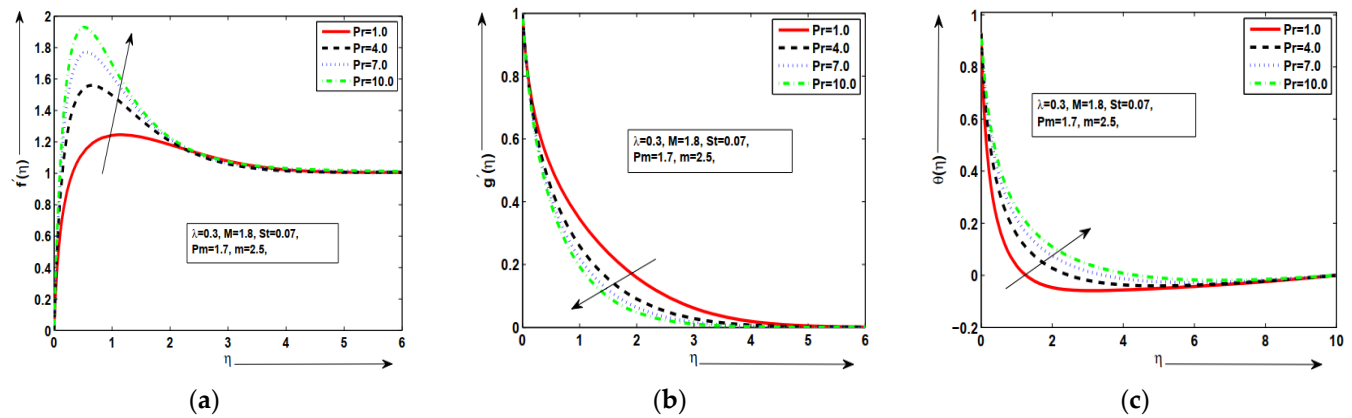


Figure 5. (a–c) The geometric profiles of $f'(\eta)$, $g'(\eta)$, and $\theta(\eta)$ against Pr .

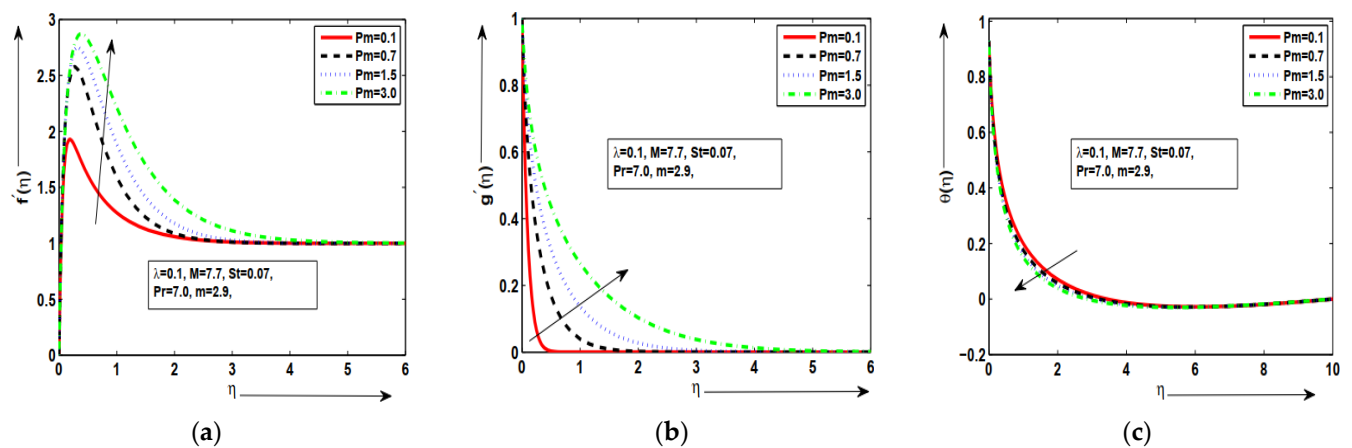


Figure 6. (a–c) The geometric profiles of $f'(\eta)$, $g'(\eta)$ and $\theta(\eta)$ against Pm .

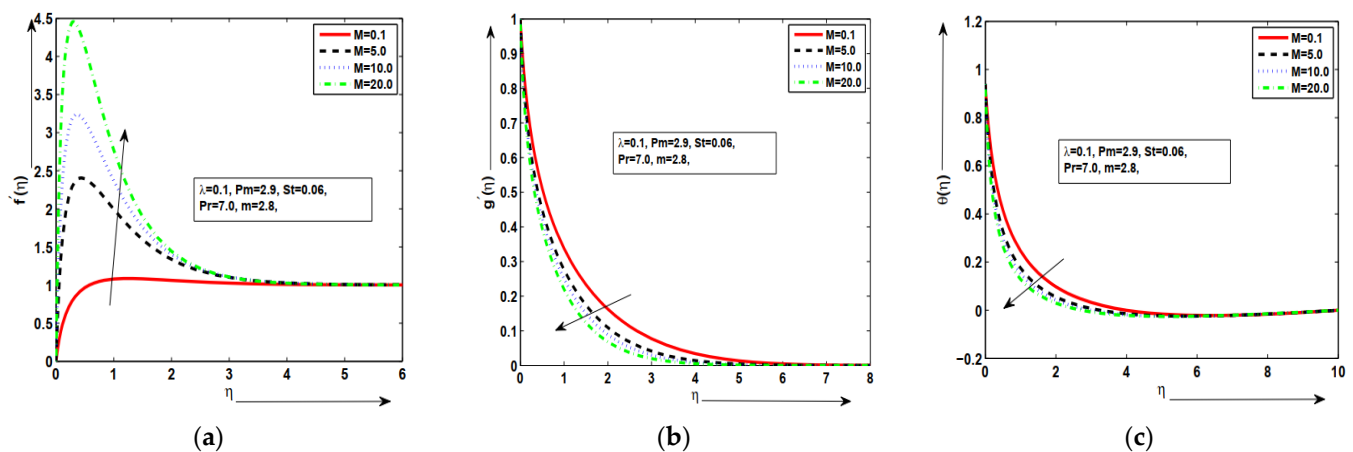


Figure 7. (a–c) The geometric profiles of $f'(\eta)$, $g'(\eta)$, and $\theta(\eta)$ against M .

From Table 1, it is concluded that the skin friction $f''(0)$ increased at $m = 0.5$ but the minimum skin friction was obtained at small $m = 0.0$ in the presence of buoyancy forces. It should be noted that $m = 0$ corresponds to the case of a small temperature difference between the surface and fluid, i.e., the buoyancy approximation becomes valid for this particular case, whereas for a high temperature difference, a considerable enhancement in the numerical values of the skin friction and heat transfer rate was inspected and can be clearly seen in Table 1. It was deduced that the magnetic intensity $-g''(0)$ was maximum at larger $m = 0.5$ but the smaller quantity of magnetic intensity was examined at smaller $m = 0.0$ in the presence of strong magnetic field. This result was expected because an increase in M means enhancement in the Lorentz forces which opposes the flow and velocity of the fluid decreases. It was also expected because as magnetic Prandtl number increases, the viscosity of fluid increases and fluid becomes thicker and consequently the boundary layer thickness decreases. It is also depicted that the heat transfer was increased at maximum $m = 0.5$ but the lower quantity of heat transfer is depicted at minimum $m = 0.0$ in the presence of magnetic force M . With the increase in value of M , the magnetic field became stronger along the surface which is a clear indication that the Lorentz force is more effective in this case. In Table 2, the influence of stratification St number with various values along a vertical plate is shown to check the behavior of physical properties of the rate of skin friction, the rate of magnetic intensity, and the rate of heat transfer. From Table 2, it is computed that the skin friction is maximum at smaller value of $St = 0.1$ but the minimum quantity of skin friction is depicted at a large value of $St = 0.4$ in the presence of Prandtl number $Pr = 7.0$. Physically, it is possible because thermal conductivity decreases as Pr increases, which has a lower magnitude of frictional forces between the viscous layers. It is depicted that the magnetic intensity is decreased at larger $St = 0.4$ but the maximum value of magnetic intensity is explored at small $St = 0.1$ in the presence of $\lambda = 6.5$. However, due to the strong buoyancy number λ which acts like a pressure gradient and dominated over the resistance, skin friction is increased and slight changes in heat and magnetic intensity are noted. It is also mentioned that the heat transfer $-\theta'(0)$ was maximum at the lower value of $St = 0.1$ but the minimum quantity of heat transfer was noticed at larger $St = 0.4$ in the presence of magnetic Prandtl number $P_m = 8.3$. The reason behind this is that increasing the value of P_m is equivalent to decreasing magnetic diffusivity and consequently the strength of magnetic field becomes loose.

Table 1. Numerical results for $f''(0)$, $-g''(0)$ and for $-\theta'(0)$ for various values of $m = 0.0, 0.1, 0.3, 0.5$, while other parameters are fixed.

m	$f''(0)$	$-g''(0)$	$-\theta'(0)$
0.0	0.695631064636244	0.800391659576938	0.330556513906720
0.1	0.728442870560860	0.843207966030059	0.352119633819730
0.3	0.809106015652243	0.939444897313997	0.400936354502018
0.5	0.913457974253570	1.051844490851958	0.458533654900388

Table 2. Numerical results for $f''(0)$, $-g''(0)$ and for $-\theta'(0)$ for various values of $St = 0.1, 0.2, 0.3, 0.4$, while other parameters are fixed.

St	$f''(0)$	$-g''(0)$	$-\theta'(0)$
0.1	0.984042850440267	0.470578779940417	0.386755927577512
0.2	0.973834647776395	0.467462868621491	0.379378708437730
0.3	0.963745902362756	0.464369050922372	0.372022244909067
0.4	0.953775319662017	0.461297160471197	0.364686553985701

7. Conclusions

The current physical problem addressed the temperature-dependent density effect on magnetohydrodynamic convective heat-transfer phenomena of electrical-conductive fluid flow along the vertical magnetized and heated plate placed in thermally stratified medium. In previous studies, the heat transfer characteristics along the non-magnetized shapes were performed numerically. Due excessive heating, these mechanisms are less interesting in engineering and industrial processes. To overcome this issue in current analysis, the surface was magnetized and fluid was electrically conducting which is responsible for reducing excessive heating along the surface. The non-linear coupled PDE equations for the above fluid flow mechanism were formulated and then converted into non-similar formulation by applying an appropriate and well-known similarity transformation for integration. The final non-similar forms were numerically integrated by employing the Keller Box scheme. The transformed algebraic equations were plotted graphically and numerically on the MATLAB software package. The main finding of the current analysis was to compute physical quantities such as velocity graph, magnetic field graph, and temperature plot along with their slopes that are skin friction, magnetic intensity and heat transfer for different parameters included in the flow model. The comparison of Keller Box analysis was computed with previous existing numerical results. Furthermore, the current issues have significant implications for the polymer industries, glass-fiber production, petroleum production, fiber spinning, plastic film production, polymer sheet extraction, heat exchangers, catalytic reactors, and the production of electronic devices. The main findings are given below:

It was deduced that the velocity profile increased for maximum temperature-dependent density but similar behavior was noted for other values of variable density. This happens because an increasing value of density parameter tends to increase the density variation with temperature, which ultimately increases the buoyancy force.

It was also noted that the temperature distribution increased for strong buoyancy forces but decreased for lower buoyancy forces. Physically, it was expected because larger values of λ correspond to stronger buoyancy forces which leads to increase the acceleration of fluid flow.

It was examined that the magnetic field profile along the surface was enhanced for strong magnetic forces but minimum value was obtained at lower magnetic forces. This happens because the magnetic force parameter is the ratio of magnetic energy to kinetic energy, so with the increase in the magnetic force parameter P_m , the magnetic energy is increased, while the kinetic energy is reduced.

It was computed that the skin friction was maximum for small thermal stratification but the minimum quantity of skin friction was depicted for large stratification in the presence of Prandtl number $P_r = 7.0$. Physically, it is possible because thermal conductivity decreases as P_r increases, which has a lower magnitude of frictional forces between the viscous layers.

It was also mentioned that the heat transfer is increased for lower stratification, but the minimum quantity of heat transfer is noticed in the presence of magnetic Prandtl number. The reason behind this is that increasing the value of P_m is equivalent to decreasing magnetic diffusivity and consequently the strength of magnetic field becomes loose.

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