

Research Article

Modified EDAS Method for MAGDM Based on MSM Operators with 2-Tuple Linguistic T -Spherical Fuzzy Sets

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Received 27 February 2022; Revised 27 April 2022; Accepted 18 May 2022; Published 7 July 2022

Academic Editor: Salman Ahmad

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The coronavirus (COVID-19) pandemic, which began in China and is fast spreading around the world, has increased the number of cases and deaths. Governments have suffered substantial damage and losses not only in the health sector but also in a variety of other areas. In this situation, it is critical to determine the most crucial vaccine that doctors and specialists should implement. In order to evaluate the many vaccines to control the COVID-19 epidemic, a decision problem based on the decisions of many experts, with some contradicting and multiple criteria, should be taken into account. This decision process is characterized as a multiattribute group decision-making (MAGDM) problem that includes uncertainty in this study. T -spherical fuzzy sets are utilized for this, allowing decision-experts to make evaluations over a larger area and better deal with complicated data. The T -spherical fuzzy set is a useful tool for dealing with uncertainty and ambiguity, especially where additional answers of the type “yes,” “no,” “abstain,” and “refusal” are required, and the 2-tuple linguistic terms are useful for the qualitative evaluation of uncertain data. From the perspective of the uncertainty surrounding the problems of MAGDM, we propose the notion of 2-tuple linguistic T -spherical fuzzy numbers (2TL T -SFNs) generated with the integration of T -spherical fuzzy numbers and 2-tuple linguistic terms. Then, the assessment based on distance from average solution (EDAS) for the ranking of alternatives based on the 2TL T -SFNs is investigated as a new decision-making strategy. This study provides the following significant contributions: (1) the procedure for constructing a 2TL T -SFNs is described, together with their aggregation operators, ranking criteria, relevant attributes, and some operational laws. (2) The traditional Maclaurin symmetric mean (MSM) operator is useful for modeling attribute interrelationships and aggregating 2TL T -SF information to tackle the MAGDM problems. A few recent MSM and dual MSM operators are being built to evaluate the 2TL T -SF information. Thus, 2-tuple linguistic T -spherical fuzzy Maclaurin symmetric mean (2TL T -SFMSM) operator, 2-tuple linguistic T -spherical fuzzy weighted Maclaurin symmetric mean (2TL T -SFWMSM) operator, 2-tuple linguistic T -spherical fuzzy dual Maclaurin symmetric mean (2TL T -SFDMSM) operator, and 2-tuple linguistic T -spherical fuzzy weighted dual Maclaurin symmetric mean (2TL T -SFDWMSM) operator are proposed. (3) We incorporate the 2TL T -SFNs into the EDAS approach and develop a new 2TL T -SF-EDAS method for solving the MAGDM problems based on the proposed aggregation operators in a 2TL T -SF environment. A case study for the selection of an optimal vaccine to control the outbreak of the COVID-19 epidemic is also presented to show the validity of the proposed methodology. Furthermore, the comparative analysis with existing approaches shows the advantages and superiority of the proposed framework.

1. Introduction

1.1. Background. The COVID-19 is an extremely pathogenic illness that may quickly spread. On December 31, 2019, the main case was discovered in Wuhan, China's capital of Hubei Province. The term COVID-19 originates from the

Latin word “Corona,” which translates as a circle of light or nimbus, around the head of the virus. The symptoms of this virus are similar to those of pneumonia and swine flu. It was first discovered in Beijing and subsequently spread across the globe, infecting about 248 million individuals. As of April 21, 2022, over 6,210,719 lives have been lost. In a

report, the World Health Organization (WHO) proclaimed the outbreak to be a global epidemic. Since the symptoms of this fatal illness are identical to influenza, coughing, and fever, it is extremely hard to detect its existence. From the moment it enters the living organism, this virus begins to exhibit symptoms between 7 and 14 days later. Because of the lack of a vaccine, social distance has been the most frequently used method for its prevention and management [1]. Globally, public health issues are measured by the number of affected and suspecting individuals. When a healthy individual comes into intimate touch with an infected individual or his/her possessions, the virus is transmitted to the healthy person. Only appropriate testing enables an infected individual to know whether they are exposed to the virus; this enables them to get the treatment they require and to take preventative actions to decrease the chances of infecting others. Recently, scholars and specialists have focused their concentration on the investigation of COVID-19 propagation as shown in Table 1. The COVID-19 pandemic is disrupting immunization activities in multiple ways: (1) additional burden on health systems; (2) reduced availability of health personnel for supply chain and services; and (3) decreased demand for vaccination (need for physical distancing and/or community reluctance). Vaccination is also essential for an effective epidemic strategy. As of April 21, 2022, there have been 505,035,185 confirmed cases of COVID-19, including 6,210,719 deaths, reported to the WHO. As of April 18, 2022, a total of 11,324,805,837 vaccine doses have been administered. Further 5,100,316,294 persons were vaccinated with at least one dose, and 4,579,350,070 persons were fully vaccinated.

1.2. Literature Review and Motivation. When dealing with a limited number of alternatives on predetermined attributes, multiple attribute decision-making (MADM) [18–22] has always been employed as an efficient strategy for assessment in order to select the most acceptable one among them. The expression of the attributes is a significant issue in the decision-making procedure. In the area of modern MADM research of risk management, a prominent and informative research topic is MAGDM. In the MAGDM environment, several decision experts (DEs) choose the appropriate alternative from a set of defined options based on assessment feedback for multiple attributes. This is a significant branch of research and investigations that finds broad applicability in everyday practice [23–27]. Moreover, because decision phenomena are intrinsically complicated, numerous genuine issues involve a lot of interpretations. As a result, it is challenging for DEs to provide quantitative judgments of attributes. The spherical fuzzy set (SFS) [28] serves as an efficient approach for dealing with complicated fuzzy data, and it is specified by four factors: a membership degree (MD), an abstinence degree (AD), a nonmembership degree (NMD), and a refusal degree (RD). For example, in a limited space of communication L , a SFS A has the following framework: $A = \{ \langle \ell, p_A(\ell), n_A(\ell), l_A(\ell) \rangle | \ell \in L \}$, where p_A indicates MD, n_A indicates AD, and l_A indicates NMD with a restriction that $0 \leq p^2(\ell) + n^2(\ell) + l^2(\ell) \leq 1$. Almost

immediately after its introduction, SFS evolved as an important tool for dealing with data that is imprecise or ambiguous in some way. In contrast, Mahmood et al. [29] observed that SFS can be extended in a way that is really valuable $T = \{ \langle \ell, p(\ell), n(\ell), l(\ell) \rangle | \ell \in L \}$ with a restriction that $0 \leq p^q(\ell) + n^q(\ell) + l^q(\ell) \leq 1$. Such a powerful extension of the SFS is known as the T -spherical fuzzy set (T -SFS). The distinction between SFSs and T -SFSs is mostly defined by the range of their memberships, abstentions, and nonmemberships. The preceding discussion of SFSs and T -SFSs has shown that T -SFSs have a stronger capability than SFSs to tackle problems in MADM scenarios when there is uncertainty. The aggregation operators and different decision-making approaches in the T -SFS environment have been effectively addressed by many researchers [30–33].

However, maximum decision problems are ambiguous and fuzzy, and somehow it is difficult to convey the attributes involved in these decision problems by crisp numbers as for qualitative data, which could be explicitly represented by 2TL terms such as improved, excellent, or pathetic. Zadeh [34] suggested the definition of linguistic variables (LVs), there have been a lot of accomplishments in research on linguistic MADM issues, especially the aggregation to address the linguistic MADM concerns, and operators for these 2TL terms have also been mentioned. Furthermore, LVs can improve the accuracy and versatility of conventional quantitative methods, and they have been observed to be widely consistent with other theories in MADM or MAGDM issues. In recent publications, several scholars researched the problems of group decision-making in which both weights of attributes and weights of DEs have been taken in the form of linguistic terms. Then, they established the linguistic evaluation operational rules, developed a few latest operators, and indicated a method based on MAGDM that depends on actual linguistic knowledge. The 2TL representation model is first proposed by Herrera and Martinez [35]. Several 2TL aggregation operators and decision-making approaches have been proposed. Ju et al. [36] proposed the q -rung orthopair fuzzy 2TL (q -ROFTL) weighted averaging and weighted geometric operators to develop an approach to solve the MAGDM problems. Furthermore, the q -ROFTL Muirhead mean and the dual Muirhead mean operators were also presented by them. Rong et al. [37] introduced the complex q -ROFTL-MSM operator and the complex q -ROFTL dual MSM operator along with several properties of the developed operators. Liu et al. [38] proposed the q -ROFL family of point aggregation operators for linguistic q -ROFL sets. Wang et al. [39] proposed the interval 2TLIFNs to better describe the fuzziness of human thinking and to avoid information loss/distortion during information aggregation phases. Verma and Aggarwal [40] used 2-tuple intuitionistic fuzzy linguistic values to represent the payoffs of the matrix game. Deng et al. [41] combined the Hamy mean (HM) operator, weighted HM operator, dual HM operator, and dual weighted HM operator with 2TL Pythagorean fuzzy numbers to propose the 2TL Pythagorean fuzzy HM operator, 2TL Pythagorean fuzzy weighted HM operator, 2TL Pythagorean fuzzy dual HM operator, and 2TL Pythagorean

TABLE 1: Literature review of different case studies for the COVID-19 outbreak.

Authors	Year	Case studies
West et al. [2]	2021	Rapid review of decision-making for place of care and death in older people
Resnick et al. [3]	2021	Gerontological society of America COVID-19 task force
Chai et al. [4]	2021	Emergency decision-making for treatment of patients with COVID-19
Cardenas [5]	2021	Harnessing strategic policy on COVID-19 vaccination rollout in the Philippines
Marti and Puertas [6]	2021	European countries' vulnerability to COVID-19
Jiang et al. [7]	2021	Impact of the COVID-19 pandemic on patient preferences
Guhathakurata et al. [8]	2021	South Asian countries are less fatal concerning COVID-19
Basch et al. [9]	2021	YouTube videos and informed decision-making about COVID-19 vaccination
Griffiths et al. [10]	2021	Decision-making around admission to intensive care in the UK pre-COVID-19
Li et al. [11]	2021	A consensus model to manage the noncooperative behaviors of individuals during the COVID-19 outbreak
Metlay and Armstrong [12]	2021	Clinical decision-making during the COVID-19 pandemic
Bauer et al. [13]	2021	Research in the time of COVID-19 and beyond
Wan et al. [14]	2021	Emergency assistance of COVID-19
Parviainen et al. [15]	2021	Building a ship while sailing. It is epistemic humility and the temporality of nonknowledge in political decision-making on COVID-19
Li and Giabbanelli [16]	2021	Identifying synergistic interventions to address COVID-19 using a large-scale agent-based model
Kocher et al. [17]	2021	Paucity and disparity of publicly available sex-disaggregated data for the COVID-19 epidemic hamper evidence-based decision-making

fuzzy dual weighted HM operator. Naz and Akram [18, 42] developed a new decision-making approach to deal with the MADM problems based on graph theory. Furthermore, Akram et al. [43–46] introduced several decision-making methods under a generalized fuzzy scenario.

Aggregation operators are mathematical functions that are used to combine information. Maclaurin et al. [47] proposed the MSM operator, which is a prominent aggregation operator to aggregate multi-input data. Subsequently, Detemple and Robertson [48] extended the MSM operator. It is capable of capturing relationships between numerous input arguments, whereas Bonferroni mean and Heronian mean operators are capable of capturing relationships within two given arguments. The MSM operator utilized a linguistic fuzzy set (FS) to resolve different decision-making problems [49]. Garg and Arora [50] extended the MSM operators to the intuitionistic FSs based on Archimedean t -conorm and t -norm. Dong and Geng [51] extended the MSM operators to trapezoid IF linguistic (TIFL) numbers to propose the TIFL-MSM operator, TIFL generalized MSM operator, TIFL weighted MSM operator, and TIFL weighted generalized MSM operator. Inspired by the MSM operator, Qin and Liu [52] developed the dual MSM operator. To aggregate uncertain information, Darko and Liang [53] expanded the MSM and the dual MSM into the dual hesitant fuzzy environment. Wang et al. [54] proposed the idea of the partitioned dual MSM operator stimulated by the partitioned MSM. Furthermore, they extended the partitioned dual MSM operator to introduce the IF-partitioned dual MSM operator and the weighted IF-partitioned dual MSM operator.

In addition, there are two basic types of approaches for making decisions that are frequently used. The first is the information aggregation operators through which data can be compiled into a single consistent value. The conventional MADM method is the second method, which mainly

includes TOPSIS, VIKOR, TODIM, EDAS, and MABAC methods. The EDAS was initially introduced by Keshavarz Ghorabae et al. [55] to tackle multiple issues in the MADM. The EDAS approach is particularly successful if the contradictory requirements in the MADM problem are present. The classical distances for the EDAS technique are also computed, analogous to the VIKOR technique and TOPSIS technique. The EDAS approach must be computed on the basic principle of AS (average solution) as both NDAS (negative distance from average solution) and PDAS (positive distance from average solution). The ideal option must have the greatest PDAS value and the lowest NDAS value [56]. Wei et al. [57] expanded the EDAS approach to the MAGDM with probabilistic linguistic term sets (LTs) and utilized a numerical illustration concerning the green supplier to validate the viability of the extending approach. Zhao et al. [58] developed an improved TODIM approach based on cumulative prospect theory and 2TL neutrosophic sets as a novel approach to MAGDM issues such as network security assessment team evaluation. In the trapezoidal neutrosophic number environment, Abdel-Basset et al. [59] used a hybrid MADM methodology, the decision-making trial, and evaluation laboratory method to determine the respective significance of the measurements and their subindicators, and EDAS to rank the alternatives. Ye et al. [60] established a new MADM method under the IF environment by using the idea of the PROMETHEE II and EDAS methods. Tan and Zhang [61] investigated a MADM technique based on information entropy and the EDAS approach in a refined single-valued neutrosophic set environment for decision-making issues with attributes and subattributes when the attribute weight is unknown.

The motivational factors for writing this article are as follows: (1) traditional T -SFS approaches fail to perceive vagueness utilizing the 2TL terms, which has a greater potential to modify linguistic forms and may also prevent

data error reduction during coping with communicative judgment concerns. Therefore, we initially introduced the 2TL T -SFS with corresponding basic notions, which would expand T -SFS theoretical frameworks and offer a viable framework for experts to convey outcome measures; (2) when collecting the judgment experts' preferences, data processing is critical. Furthermore, the association of the indicated attributes must be considered in a variety of realistic problems. Because of the MSM operator's amazing capability, various 2TL T -SFMSM operators are developed to deal with imprecise data; (3) assessment but also the choice of the case of the best vaccine has regarded as a crucial and active field. Due to the extreme uncertainty and the variability of cases of best vaccine, many assessment approaches must be investigated in order to properly examine the cases of best vaccine; (4) by considering the variability of cases of best vaccine, a MAGDM innovation known as 2TL T -SF-EDAS method for vaccine assessment in the world based on the 2TL T -SFWMSM and 2TL T -SFWDMSM operators is established.

1.3. Contributions and Organization. The purpose of this study, according to the above motivation, is to build a cognitive judgment framework for evaluating vaccines. To accomplish this goal, we initially identify a better data representation technique for displaying complicated social data. So, we must figure out how to construct the judgment process and identify the best vaccine for treating COVID-19 outbreaks. Since the interactions among the related attributes and LVs can be easily processed by the MSM operator to show the fuzzy data, it is worthy to deal with linguistic details with the extension of the MSM operator. The innovation of this research study is shown in the following five points:

- (1) We introduce the 2TL T -SFS as a new advancement in FS theory to communicate complexities in data. The 2TL T -SFS combines both the advantages of 2TL terms and T -SFSs, which increases the versatility of the T -SFS.
- (2) We develop a family of MSM aggregation operators for 2TL T -SFS, such as the 2TL T -SFMSM operator, the 2TL T -SFWMSM operator, the 2TL T -SFDMSM operator, and the 2TL T -SFWDMSM operator to deal with group decision-making problems in which the attributes have interrelationships.
- (3) Under the current conditions formal definitions, certain theorems, properties, and specific cases of the proposed information aggregation operators are deduced.
- (4) The 2TL T -SF-EDAS method is proposed based on the 2TL T -SFWMSM and 2TL T -SFWDMSM operators to rank the alternatives. A novel MAGDM model is used to fuse the evaluation preferences of DEs.
- (5) An illustrative example for choosing the best vaccine to treat COVID-19 is presented to show the usefulness and effectiveness of the proposed approach.

To achieve this goal, the structure of this study is arranged as follows: Section 2 briefly recalls some fundamental concepts relevant to the 2TL representation model, the description of T -SFSs, MSM, and dual MSM operators with weighted forms. Section 3 presents the definition of 2TL T -SFS with operational laws. Section 4 defines a new family of MSM aggregation operators including 2TL T -SFMSM, 2TL T -SFWMSM, 2TL T -SFDMSM, and 2TL T -SFWDMSM operators with the most preferable properties and specific cases. In Section 5, a strategy for MAGDM is developed under the 2TL T -SF environment based on 2TL T -SFWMSM and 2TL T -SFWDMSM operators. In Section 6, a numerical instance, parameter, and comparative analysis are given to illustrate the effectiveness and superiority of the developed method. Finally, Section 7 presents the conclusions, advantages, limitations, and future directions.

The structure of this study is graphically arranged in Figure 1.

2. Preliminaries

In this section, some correlative basic concepts of the 2TL term, T -SFS, MSM, weighted MSM, dual MSM, and weighted dual MSM operators are recapped to facilitate the next sections.

2.1. 2-Tuple Linguistic Representation Model

Definition 1 (see [62]). Let there exist a linguistic term set (LTS) $S = \{s_\varepsilon | \varepsilon = 0, 1, \dots, \tau\}$ with odd cardinality, where s_ε indicates a possible linguistic term for a linguistic variable. For instance, an LTS S having seven terms can be described as follows: $S^7 = \{s_0^7: \text{none}, s_1^7: \text{very low}, s_2^7: \text{low}, s_3^7: \text{medium}, s_4^7: \text{high}, s_5^7: \text{very high}, \text{and } s_6^7: \text{perfect}\}$.

If $s_\varepsilon, s_k \in S$, then the LTS meets the following characteristics:

- (i) The set is ordered: $s_\varepsilon > s_k$, if and only if $\varepsilon > k$.
- (ii) Max operator: $\max(s_\varepsilon, s_k) = s_\varepsilon$, if and only if $\varepsilon \geq k$.
- (iii) Min operator: $\min(s_\varepsilon, s_k) = s_\varepsilon$, if and only if $\varepsilon \leq k$.
- (iv) Negative operator: $\text{Neg}(s_\varepsilon) = s_k$ such that $k = \tau - \varepsilon$.

The 2TL representation model is based on the idea of symbolic translation, introduced by Herrera and Martinez [35], which is useful for representing the linguistic assessment information by means of a 2-tuple $(s_\varepsilon, v_\varepsilon)$, where s_ε is a linguistic label from predefined LTS S and v_ε is the value of symbolic translation, and $v_\varepsilon \in [-0.5t, n0.5)$.

Definition 2 (see [35]). Let q be the result of an aggregation of the indices of a set of labels assessed in a LTS S , i.e., the result of a symbolic aggregation operation, $q \in [1, \tau]$, where τ is the cardinality of S . Let $\varepsilon = \text{round}(q)$ and $v = q - \varepsilon$ be two values, such that, $\varepsilon \in [1, \tau]$ and $v \in [-0.5t, n0.5)$; then, v is called a symbolic translation.

Definition 3 (see [35]). [35] Let $S = \{s_\varepsilon | \varepsilon = 0, 1, \dots, \tau\}$ be a LTS, and $q \in [1, \tau]$ is a number value representing the aggregation result of linguistic symbolic. Then, the function Δ

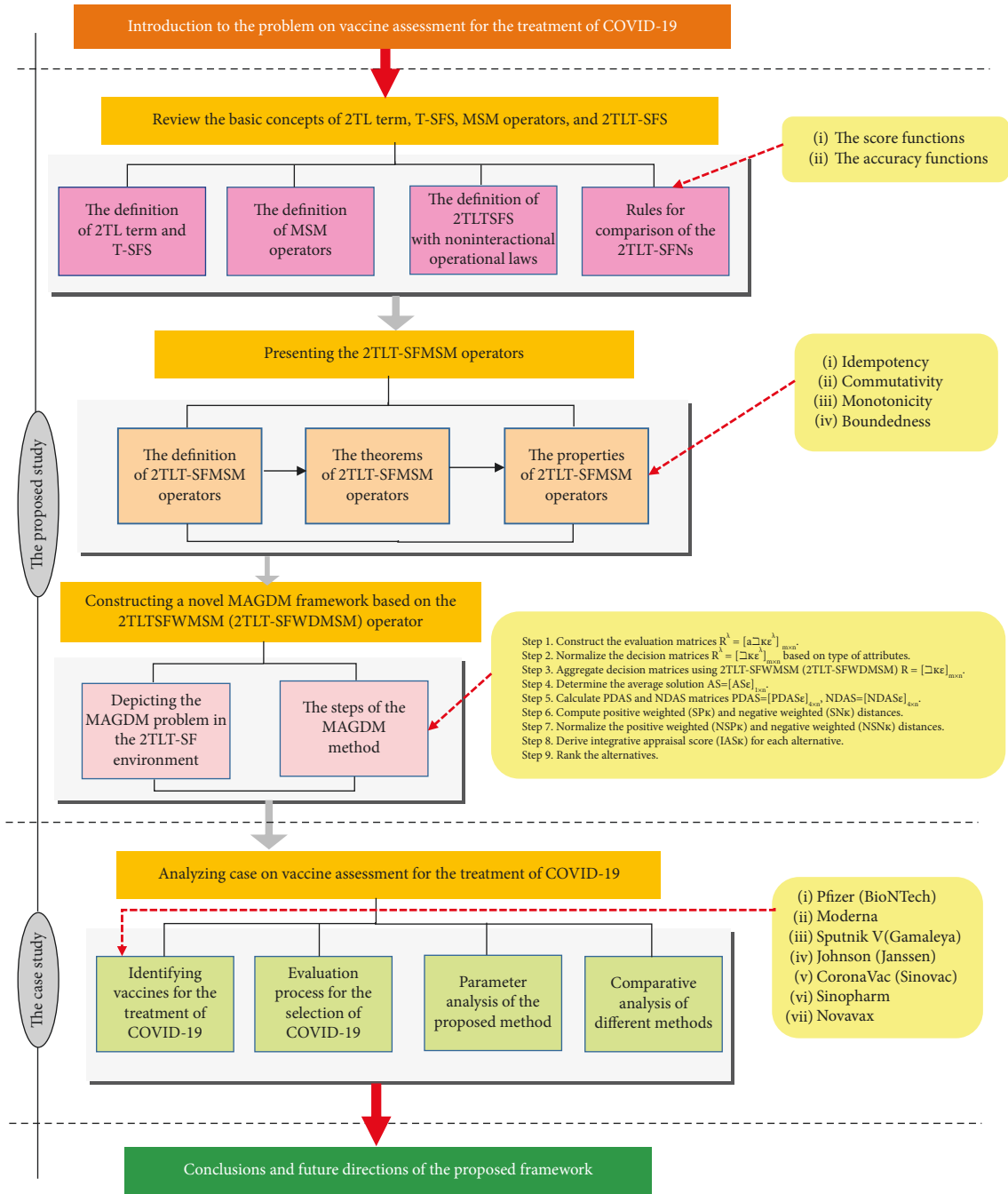


FIGURE 1: The graphical interpretation of the proposed framework in this article.

used to obtain the 2TL information equivalent to q is defined as follows:

$$\Delta: [1, \tau] \longrightarrow S \times [-0.5, 0.5],$$

$$\Delta(q) = \begin{cases} s_\varepsilon, & \varepsilon = \text{round}(q), \\ v = q - \varepsilon, & v \in [-0.5, 0.5]. \end{cases} \quad (1)$$

Definition 4 (see [35]). Let $S = \{s_\varepsilon | \varepsilon = 0, 1, \dots, \tau\}$ be a LTS and $(s_\varepsilon, v_\varepsilon)$ be a 2-tuple, then there exists a function Δ^{-1} that

restores the 2-tuple to its equivalent numerical value $q \in [1, \tau] \subset R$, where

$$\Delta^{-1}: S \times [-0.5, 0.5] \longrightarrow [1, \tau],$$

$$\Delta^{-1}(s_\varepsilon, v) = \varepsilon + v = q. \quad (2)$$

2.2. T-Spherical Fuzzy Set. Mahmood et al. [29] defined the T-spherical fuzzy set as an extension of q -ROFS and SFS as follows:

Definition 5 (see [29]). For any universal set L , a T -SFS is of the form

$$T = \{\langle \ell, p(\ell), n(\ell), l(\ell) \rangle | \ell \in L\}, \quad (3)$$

where $p, n, l: L \rightarrow [0, 1]$ represents the MD, AD and NMD, respectively, with the condition $0 \leq p^q(\ell) + n^q(\ell) + l^q(\ell) \leq 1$ for the positive number $q \geq 1$, and $r(\ell) = \sqrt[q]{1 - (p^q(\ell) + n^q(\ell) + l^q(\ell))}$ is known as the degree of refusal of ℓ in T . To express information conveniently, the triplet (p, n, l) is known as a T -spherical fuzzy number (T -SFN).

A T -SFN is a generalized form of an existing fuzzy framework, and it reduces to the following:

- (i) Spherical fuzzy number (SFN), by taking q as 2
- (ii) Picture fuzzy number (PFN), by taking q as 1
- (iii) q -rung orthopair fuzzy number (q -ROFN), by taking n as zero
- (iv) Pythagorean fuzzy number (PyFN), by taking n as zero and q as 2
- (v) Intuitionistic fuzzy number (IFN), by taking n as zero and q as 1
- (vi) Fuzzy number (FN), by taking n and l as zero and q as 1

2.3. The MSM Operator and Its Weighted Forms. Let $\mathbf{a}_\varepsilon$ ($\varepsilon = 1, 2, \dots, n$) be any set of nonnegative numbers. Then, the following operators are defined as follows:

- (1) MSM [47]: $\text{MSM}^{(t)}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n) = (\sum_{1 \leq \kappa_1 < \dots < \kappa_t \leq n} (\prod_{\varepsilon=1}^t \mathbf{a}_{\kappa_\varepsilon}) / C_n^t)^{(1/t)}$
- (2) Weighted MSM [47]: $\text{WMSM}_\xi^{(t)}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n) = (\sum_{1 \leq \kappa_1 < \dots < \kappa_t \leq n} (\prod_{\varepsilon=1}^t (\mathbf{a}_{\kappa_\varepsilon})^{\xi_{\kappa_\varepsilon}}) / C_n^t)^{(1/t)}$
- (3) Dual MSM [52]: $\text{DMSM}^{(t)}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n) = (1/t) (\prod_{1 \leq \kappa_1 < \dots < \kappa_t \leq n} (\sum_{\varepsilon=1}^t \mathbf{a}_{\kappa_\varepsilon})^{(1/C_n^t)})$
- (4) Weighted dual MSM [52]: $\text{WDMSM}_\xi^{(t)}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n) = (1/t) (\prod_{1 \leq \kappa_1 < \dots < \kappa_t \leq n} (\sum_{\varepsilon=1}^t \xi_{\kappa_\varepsilon} \mathbf{a}_{\kappa_\varepsilon})^{(1/C_n^t)})$

where t is a parameter, $t = 1, 2, \dots, n$, $\kappa_1, \kappa_2, \dots, \kappa_t$ is t -integer values taken from the set $\{1, 2, \dots, n\}$ of ε integer values, C_n^t represents the binomial coefficient, and $C_n^t = (n! / t!(n-t)!)$.

3. 2-Tuple Linguistic T -Spherical Fuzzy Set

We introduce the 2TL T -SFS with its operational rules as a new advancement of FS theory, in this part. Inspired by the ideas of 2TL terms and T -SFS, we develop the new concept of 2TL T -SFS by combining both the advantages of 2TL terms and T -SFS. The newly proposed set has flexibility due to the q th power of MD, AD, and NMD. The mathematical representation of 2TL T -SFS is described as follows:

Definition 6. Let $S = \{s_\varepsilon | \varepsilon = 0, 1, \dots, \tau\}$ be a LTS with odd cardinality. If $((s_p, \wp), (s_n, \aleph), (s_l, \xi))$ is defined for $s_p, s_n, s_l \in S$, $\wp, \aleph, \xi \in [-0.5t, n0.5)$, where $(s_p, \wp)$, $(s_n, \aleph)$, and (s_l, ξ) represent the MD, AD and NMD by 2TLs, a 2TL T -spherical fuzzy set is defined as follows:

$$\mathfrak{T} = \{\langle x, ((s_p(x), \wp(x)), (s_n(x), \aleph(x)), (s_l(x), \xi(x))) \rangle | x \in X\}, \quad (4)$$

where $0 \leq \Delta^{-1}(s_p, \wp) \leq \tau$, $0 \leq \Delta^{-1}(s_n, \aleph) \leq \tau$, $0 \leq \Delta^{-1}(s_l, \xi) \leq \tau$, and $0 \leq (\Delta^{-1}(s_p, \wp))^q + (\Delta^{-1}(s_n, \aleph))^q + (\Delta^{-1}(s_l, \xi))^q \leq \tau^q$.

The conversion of a linguistic term into a linguistic 2-tuple consists of adding a value 0 as symbolic translation is as follows:

$$\Delta(s_{p_\varepsilon}, s_{n_\varepsilon}, s_{l_\varepsilon}) = ((s_{p_\varepsilon}, 0), (s_{n_\varepsilon}, 0), (s_{l_\varepsilon}, 0)). \quad (5)$$

To compare any two 2TL T -SFNs, their score value and accuracy value are defined as follows:

Definition 7. Let $Y^* = ((s_p, \wp), (s_n, \aleph), (s_l, \xi))$ be a 2TL T -SFN. Then, the score function g^* of a 2TL T -SFN Y^* can be represented as

$$g^*(Y^*) = \Delta\left(\frac{\tau}{2} \left(1 + \left(\frac{\Delta^{-1}(s_p, \wp)}{\tau}\right)^q - \left(\frac{\Delta^{-1}(s_l, \xi)}{\tau}\right)^q\right)\right), \quad g^*(Y^*) \in [0, \tau], \quad (6)$$

and its accuracy function $\sqsupset$ is defined as

$$\sqsupset(Y^*) = \Delta\left(\tau \left(\left(\frac{\Delta^{-1}(s_p, \wp)}{\tau}\right)^q + \left(\frac{\Delta^{-1}(s_l, \xi)}{\tau}\right)^q\right)\right), \quad \sqsupset(Y^*) \in [0, \tau]. \quad (7)$$

3.1. Operational Laws for 2TL T-SFNs Based on Algebraic Operations. The novel operational laws based on 2TL T-SFNs, such as addition, multiplication, scalar multiplication, power, and ranking rules can be described as follows.

Definition 8. Let $Y^* = ((s_p, \wp), (s_n, \aleph), (s_l, \xi))$, $Y_1^* = ((s_{p_1}, \wp_1), (s_{n_1}, \aleph_1), (s_{l_1}, \xi_1))$, and $Y_2^* = ((s_{p_2}, \wp_2), (s_{n_2}, \aleph_2), (s_{l_2}, \xi_2))$ be three 2TL T-SFNs, $q \geq 1$, then

- (1) $Y_1^* \oplus Y_2^* = \left(\Delta(\tau \sqrt[q]{1 - (1 - (\Delta^{-1}(s_{p_1}, \wp_1)/\tau)^q)} (1 - (\Delta^{-1}(s_{p_2}, \wp_2)/\tau)^q)), \Delta(\tau (\Delta^{-1}(s_{n_1}, \aleph_1)/\tau) (\Delta^{-1}(s_{n_2}, \aleph_2)/\tau)), \Delta(\tau (\Delta^{-1}(s_{l_1}, \xi_1)/\tau) (\Delta^{-1}(s_{l_2}, \xi_2)/\tau)) \right)$;
- (2) $Y_1^* \otimes Y_2^* = \left(\Delta(\tau (\Delta^{-1}(s_{p_1}, \wp_1)/\tau) (\Delta^{-1}(s_{p_2}, \wp_2)/\tau)) \Delta(\tau \sqrt[q]{1 - (1 - (\Delta^{-1}(s_{n_1}, \aleph_1)/\tau)^q) (1 - (\Delta^{-1}(s_{n_2}, \aleph_2)/\tau)^q)}), \Delta(\tau \sqrt[q]{1 - (1 - (\Delta^{-1}(s_{l_1}, \xi_1)/\tau)^q) (1 - (\Delta^{-1}(s_{l_2}, \xi_2)/\tau)^q)})) \right)$;
- (3) $\lambda Y^* = \left(\Delta(\tau \sqrt[q]{1 - (1 - (\Delta^{-1}(s_p, \wp)/\tau)^q)^\lambda}}, \Delta(\tau (\Delta^{-1}(s_n, \aleph)/\tau)^\lambda), \Delta(\tau (\Delta^{-1}(s_l, \xi)/\tau)^\lambda) \right), \lambda > 0$;
- (4) $Y^{*\lambda} = \left(\Delta(\tau (\Delta^{-1}(s_p, \wp)/\tau)^\lambda), \Delta(\tau \sqrt[q]{1 - (1 - (\Delta^{-1}(s_l, \xi)/\tau)^q)^\lambda}}, \Delta(\tau \sqrt[q]{1 - (1 - (\Delta^{-1}(s_l, \xi)/\tau)^q)^\lambda}) \right), \lambda > 0$.

Definition 9. Let $Y_1^* = ((s_{p_1}, \wp_1), (s_{n_1}, \aleph_1), (s_{l_1}, \xi_1))$ and $Y_2^* = ((s_{p_2}, \wp_2), (s_{n_2}, \aleph_2), (s_{l_2}, \xi_2))$ be two 2TL T-SFNs, then these two 2TL T-SFNs can be compared according to the following rules:

- (1) If $g^*(Y_1^*) > g^*(Y_2^*)$, then $Y_1^* > Y_2^*$;
- (2) If $g^*(Y_1^*) = g^*(Y_2^*)$, then
 - (i) If $\sqsupset(Y_1^*) > \sqsupset(Y_2^*)$, then $Y_1^* > Y_2^*$;
 - (ii) If $\sqsupset(Y_1^*) = \sqsupset(Y_2^*)$, then $Y_1^* \sim Y_2^*$.

4. The 2TL T-SF Maclaurin Symmetric Mean Aggregation Operators

In this section, we expand the application criteria of the MSM operator to the 2TL T-SF environment and introduce several novel aggregation operators based on 2TL T-SF operations to aggregate data. This section is concerned with the introduction of four novel aggregation operators including the 2TL T-SFMSM operator, the 2TL T-SFWMSM operator, the 2TL T-SFDMMSM operator, and the 2TL T-SFWDMSM operator. Moreover, we analyze their properties and certain specific cases. The proposed aggregation operators satisfy the basic properties of aggregation including idempotency, commutativity, monotonicity, and boundedness.

4.1. The 2TL T-SFMSM Operator. Utilizing the Definition 6 and the novel operational rules of Definition 8, we develop the definition of 2-tuple linguistic T-spherical fuzzy Maclaurin symmetric mean (2TL T-SFMSM) operator as follows.

Definition 10. Let $h_\varepsilon = ((s_{p_\varepsilon}, \wp_\varepsilon), (s_{n_\varepsilon}, \aleph_\varepsilon), (s_{l_\varepsilon}, \xi_\varepsilon))$ ($\varepsilon = 1, 2, \dots, n$) be any set of 2TL T-SFNs; then, we define the 2TL T-SFMSM operator as follows:

$$2TLT-SFMSM^{(t)}(h_1, h_2, \dots, h_n) = \left(\frac{\oplus_{1 \leq \kappa_1 \leq \dots \leq \kappa_t \leq n} \left(\otimes_{\varepsilon=1}^t h_{\kappa_\varepsilon} \right)^{(1/C_n^t)}}{C_n^t} \right)^{(1/t)}. \quad (8)$$

Theorem 1. Let $h_\varepsilon = ((s_{p_\varepsilon}, \wp_\varepsilon), (s_{n_\varepsilon}, \aleph_\varepsilon), (s_{l_\varepsilon}, \xi_\varepsilon))$ ($\varepsilon = 1, 2, \dots, n$) be any set of 2TL T-SFNs; then, their aggregated value by using the 2TL T-SFMSM operator is also a 2TL T-SFN, and

$$2TLT-SFMSM^{(t)}(h_1, h_2, \dots, h_n) = \left(\Delta \left(\tau \left(\sqrt[q]{1 - \left(\prod_{1 \leq \kappa_1 \leq \dots \leq \kappa_t \leq n} \left(1 - \left(\prod_{\varepsilon=1}^t \left(\frac{\Delta^{-1}(s_{p_{\kappa_\varepsilon}}, \wp_{\kappa_\varepsilon}})}{\tau} \right)^q \right) \right) \right)^{(1/C_n^t)} \right)^{(1/t)} \right), \right. \\ \Delta \left(\tau \left(\sqrt[q]{1 - \left(1 - \left(\prod_{1 \leq \kappa_1 \leq \dots \leq \kappa_t \leq n} \left(1 - \prod_{\varepsilon=1}^t \left(1 - \left(\frac{\Delta^{-1}(s_{n_{\kappa_\varepsilon}}, \aleph_{\kappa_\varepsilon}})}{\tau} \right)^q \right) \right) \right) \right)^{(1/C_n^t)} \right)^{(1/t)} \right), \\ \left. \Delta \left(\tau \left(\sqrt[q]{1 - \left(1 - \left(\prod_{1 \leq \kappa_1 \leq \dots \leq \kappa_t \leq n} \left(1 - \prod_{\varepsilon=1}^t \left(1 - \left(\frac{\Delta^{-1}(s_{l_{\kappa_\varepsilon}}, \xi_{\kappa_\varepsilon}})}{\tau} \right)^q \right) \right) \right) \right)^{(1/C_n^t)} \right)^{(1/t)} \right) \right). \quad (9)$$

Proof. By utilizing the novel operational laws of 2TL T -SFNs (see Definition 8), we have

$$\begin{aligned} \otimes_{\varepsilon=1}^t \tilde{h}_{\kappa_\varepsilon} &= \left(\Delta \left(\tau \left(\prod_{\varepsilon=1}^t \left(\frac{\Delta^{-1}(s_{p_{\kappa_\varepsilon}}, \wp_{\kappa_\varepsilon})}{\tau} \right) \right) \right), \Delta \left(\tau \left(\sqrt[q]{1 - \prod_{\varepsilon=1}^t \left(1 - \left(\frac{\Delta^{-1}(s_{n_{\kappa_\varepsilon}}, \mathfrak{N}_{\kappa_\varepsilon})}{\tau} \right)^q} \right) \right) \right) \right) \\ \oplus_{1 \leq \kappa_1 \leq \dots < \kappa_t \leq n} (\otimes_{\varepsilon=1}^t \tilde{h}_{\kappa_\varepsilon}) &= \left(\Delta \left(\tau \left(\sqrt[q]{1 - \prod_{1 \leq \kappa_1 \leq \dots < \kappa_t \leq n} \left(1 - \left(\prod_{\varepsilon=1}^t \left(\frac{\Delta^{-1}(s_{p_{\kappa_\varepsilon}}, \wp_{\kappa_\varepsilon})}{\tau} \right)^q} \right) \right) \right) \right), \right. \\ &\quad \Delta \left(\tau \left(\prod_{1 \leq \kappa_1 \leq \dots < \kappa_t \leq n} \sqrt[q]{1 - \prod_{\varepsilon=1}^t \left(1 - \left(\frac{\Delta^{-1}(s_{n_{\kappa_\varepsilon}}, \mathfrak{N}_{\kappa_\varepsilon})}{\tau} \right)^q} \right) \right) \right) \\ &\quad \left. \Delta \left(\tau \left(\prod_{1 \leq \kappa_1 \leq \dots < \kappa_t \leq n} \sqrt[q]{1 - \prod_{\varepsilon=1}^t \left(1 - \left(\frac{\Delta^{-1}(s_{l_{\kappa_\varepsilon}}, \mathfrak{F}_{\kappa_\varepsilon})}{\tau} \right)^q} \right) \right) \right) \right). \end{aligned} \quad (10)$$

Thus, we obtain

$$\begin{aligned} \frac{1}{C_n^t} \oplus_{1 \leq \kappa_1 \leq \dots < \kappa_t \leq n} (\otimes_{\varepsilon=1}^t \tilde{h}_{\kappa_\varepsilon}) &= \left(\Delta \left(\tau \left(\sqrt[q]{1 - \left(\prod_{1 \leq \kappa_1 \leq \dots < \kappa_t \leq n} \left(1 - \left(\prod_{\varepsilon=1}^t \left(\frac{\Delta^{-1}(s_{p_{\kappa_\varepsilon}}, \wp_{\kappa_\varepsilon})}{\tau} \right)^q} \right) \right)^{(1/C_n^t)}} \right) \right), \right. \\ &\quad \Delta \left(\tau \left(\left(\prod_{1 \leq \kappa_1 \leq \dots < \kappa_t \leq n} \sqrt[q]{1 - \prod_{\varepsilon=1}^t \left(1 - \left(\frac{\Delta^{-1}(s_{n_{\kappa_\varepsilon}}, \mathfrak{N}_{\kappa_\varepsilon})}{\tau} \right)^q} \right) \right)^{(1/C_n^t)}} \right) \right) \\ &\quad \left. \Delta \left(\tau \left(\left(\prod_{1 \leq \kappa_1 \leq \dots < \kappa_t \leq n} \sqrt[q]{1 - \prod_{\varepsilon=1}^t \left(1 - \left(\frac{\Delta^{-1}(s_{l_{\kappa_\varepsilon}}, \mathfrak{F}_{\kappa_\varepsilon})}{\tau} \right)^q} \right) \right)^{(1/C_n^t)}} \right) \right) \right). \end{aligned} \quad (11)$$

Accordingly,

$$2TLT - SFMSM^{(t)}(h_1, h_2, \dots, h_n) = \left(\Delta \left(\tau \sqrt[q]{1 - \left(\prod_{1 \leq \kappa_1 \leq \dots < \kappa_t \leq n} \left(1 - \left(\prod_{\varepsilon=1}^t \left(\frac{\Delta^{-1}(s_{p_{\kappa_\varepsilon}}, \wp_{\kappa_\varepsilon})}{\tau} \right)^q \right) \right)^{(1/C_n^t)} (1/t)} \right) \right), \right. \\ \left. \Delta \left(\tau \sqrt[q]{1 - \left(1 - \left(\prod_{1 \leq \kappa_1 \leq \dots < \kappa_t \leq n} \left(1 - \prod_{\varepsilon=1}^t \left(1 - \left(\frac{\Delta^{-1}(s_{n_{\kappa_\varepsilon}}, \aleph_{\kappa_\varepsilon})}{\tau} \right)^q \right) \right) \right)^{(1/C_n^t)} (1/t)} \right) \right), \right. \\ \left. \Delta \left(\tau \sqrt[q]{1 - \left(1 - \left(\prod_{1 \leq \kappa_1 \leq \dots < \kappa_t \leq n} \left(1 - \prod_{\varepsilon=1}^t \left(1 - \left(\frac{\Delta^{-1}(s_{l_{\kappa_\varepsilon}}, \mathfrak{E}_{\kappa_\varepsilon})}{\tau} \right)^q \right) \right) \right)^{(1/C_n^t)} (1/t)} \right) \right) \right) \quad (12)$$

□

Theorem 2. Let $h_\varepsilon = ((s_{p_\varepsilon}, \wp_\varepsilon), (s_{n_\varepsilon}, \aleph_\varepsilon), (s_{l_\varepsilon}, \mathfrak{E}_\varepsilon))$ and $h'_\varepsilon = ((s'_{p_\varepsilon}, \wp'_\varepsilon), (s'_{n_\varepsilon}, \aleph'_\varepsilon), (s'_{l_\varepsilon}, \mathfrak{E}'_\varepsilon))$ ($\varepsilon = 1, 2, \dots, n$) be two sets of 2TL T-SFNs; then, the 2TL T-SFMSM operator has the following properties:

- (1) (Idempotency) If all h_ε ($\varepsilon = 1, 2, \dots, n$) are equal, i.e., $h_\varepsilon = h$ for all ε , then

$$2TLT - SFMSM^{(t)}(h_1, h_2, \dots, h_n) = h. \quad (13)$$

- (2) (Commutativity) Let h_ε ($\varepsilon = 1, 2, \dots, n$) be any set of 2TL T-SFNs, and h_ε ($\varepsilon = 1, 2, \dots, n$) be a permutation of h_ε ($\varepsilon = 1, 2, \dots, n$), then

$$2TLT - SFMSM^{(t)}(h_1, h_2, \dots, h_n) = 2TLT - SFMSM^{(t)}(\acute{h}_1, \acute{h}_2, \dots, \acute{h}_n). \quad (14)$$

- (3) (Monotonicity) Let h_ε and h'_ε ($\varepsilon = 1, 2, \dots, n$) be two sets of 2TL T-SFNs; if $(s_{p_\varepsilon}, \wp_\varepsilon) \geq (s'_{p_\varepsilon}, \wp'_\varepsilon)$, $(s_{n_\varepsilon}, \aleph_\varepsilon) \leq (s'_{n_\varepsilon}, \aleph'_\varepsilon)$, and $(s_{l_\varepsilon}, \mathfrak{E}_\varepsilon) \leq (s'_{l_\varepsilon}, \mathfrak{E}'_\varepsilon)$ for all ε , then

$$2TLT - SFMSM^{(t)}(h_1, h_2, \dots, h_n) \geq 2TLT - SFMSM^{(t)}(\acute{h}_1, \acute{h}_2, \dots, \acute{h}_n). \quad (15)$$

- (4) (Boundedness) Let h_ε ($\varepsilon = 1, 2, \dots, n$) be any set of 2TL T-SFNs, suppose

$$\begin{aligned} h^- &= \min_{\varepsilon} h_\varepsilon \\ &= \left(\min_{\varepsilon} (s_{p_\varepsilon}, \wp_\varepsilon), \max_{\varepsilon} (s_{n_\varepsilon}, \aleph_\varepsilon), \max_{\varepsilon} (s_{l_\varepsilon}, \mathfrak{E}_\varepsilon) \right), \\ h^+ &= \max_{\varepsilon} h_\varepsilon \\ &= \left(\max_{\varepsilon} (s_{p_\varepsilon}, \wp_\varepsilon), \min_{\varepsilon} (s_{n_\varepsilon}, \aleph_\varepsilon), \min_{\varepsilon} (s_{l_\varepsilon}, \mathfrak{E}_\varepsilon) \right). \end{aligned} \quad (16)$$

Then,

$$\hbar^- \leq 2TLT - SFMSM^{(t)}(\hbar_1, \hbar_2, \dots, \hbar_n) \leq \hbar^+. \quad (17)$$

Now, with regard to parameter t we can describe certain specific cases of the 2TL T -SFMSM operator.

Case 1. When $t = 1$, the 2TL T -SFMSM operator converts to the 2TL T -SF average operator as follows:

$$\begin{aligned} 2TLT - SFMSM^{(1)}(\hbar_1, \hbar_2, \dots, \hbar_n) &= \Delta \left(\tau \sqrt[q]{1 - \left(\prod_{1 \leq \kappa_1 \leq n} \left(1 - \left(\prod_{\varepsilon=1}^1 \left(\frac{\Delta^{-1}(s_{p_{\kappa_\varepsilon}}, \wp_{\kappa_\varepsilon})}{\tau} \right)^q \right) \right)^{(1/C_n^1)} \right)^{(1/1)}} \right) \\ &= \Delta \left(\tau \sqrt[q]{1 - \left(1 - \left(\prod_{1 \leq \kappa_1 \leq n} \left(1 - \prod_{\varepsilon=1}^1 \left(1 - \left(\frac{\Delta^{-1}(s_{n_{\kappa_\varepsilon}}, \aleph_{\kappa_\varepsilon})}{\tau} \right)^q \right) \right) \right)^{(1/C_n^1)} \right)^{(1/1)}} \right) \\ &= \Delta \left(\tau \sqrt[q]{1 - \left(1 - \left(\prod_{1 \leq \kappa_1 \leq n} \left(1 - \prod_{\varepsilon=1}^1 \left(1 - \left(\frac{\Delta^{-1}(s_{l_{\kappa_\varepsilon}}, \mathfrak{E}_{\kappa_\varepsilon})}{\tau} \right)^q \right) \right) \right)^{(1/C_n^1)} \right)^{(1/1)}} \right) \\ &= \Delta \left(\tau \sqrt[q]{1 - \left(\prod_{1 \leq \kappa_1 \leq n} \left(1 - \left(\frac{\Delta^{-1}(s_{p_{\kappa_1}}, \wp_{\kappa_1})}{\tau} \right)^q \right) \right)^{(1/n)}} \right) \\ &= \Delta \left(\tau \sqrt[q]{1 - \left(\prod_{1 \leq \kappa_1 \leq n} \left(1 - \left(\frac{\Delta^{-1}(s_{n_{\kappa_1}}, \aleph_{\kappa_1})}{\tau} \right)^q \right) \right)^{(1/n)}} \right) \\ &= \Delta \left(\tau \sqrt[q]{1 - \left(\prod_{1 \leq \kappa_1 \leq n} \left(1 - \left(\frac{\Delta^{-1}(s_{l_{\kappa_1}}, \mathfrak{E}_{\kappa_1})}{\tau} \right)^q \right) \right)^{(1/n)}} \right) \\ &= \Delta \left(\tau \sqrt[q]{1 - \left(\prod_{1 \leq \kappa_1 \leq n} \left(1 - \left(\frac{\Delta^{-1}(s_{p_{\kappa_1}}, \wp_{\kappa_1})}{\tau} \right)^q \right) \right)^{(1/n)}} \right) \\ &= \Delta \left(\tau \sqrt[q]{1 - \left(\prod_{1 \leq \kappa_1 \leq n} \left(1 - \left(\frac{\Delta^{-1}(s_{n_{\kappa_1}}, \aleph_{\kappa_1})}{\tau} \right)^q \right) \right)^{(1/n)}} \right) \\ &= \Delta \left(\tau \sqrt[q]{1 - \left(\prod_{1 \leq \kappa_1 \leq n} \left(1 - \left(\frac{\Delta^{-1}(s_{l_{\kappa_1}}, \mathfrak{E}_{\kappa_1})}{\tau} \right)^q \right) \right)^{(1/n)}} \right) \\ &= \Delta \left(\tau \sqrt[q]{1 - \left(\prod_{\kappa=1}^n \left(1 - \left(\frac{\Delta^{-1}(s_{p_{\kappa}}, \wp_{\kappa})}{\tau} \right)^q \right) \right)^{(1/n)}} \right), \\ &= \Delta \left(\tau \sqrt[q]{1 - \left(\prod_{\kappa=1}^n \left(\frac{\Delta^{-1}(s_{n_{\kappa}}, \aleph_{\kappa})}{\tau} \right)^{\frac{1}{n}} \right) \right), \Delta \left(\tau \sqrt[q]{1 - \left(\prod_{\kappa=1}^n \left(\frac{\Delta^{-1}(s_{l_{\kappa}}, \mathfrak{E}_{\kappa})}{\tau} \right)^{(1/n)} \right) \right). \end{aligned} \quad (18)$$

Case 2. When $t = 2$, the 2TL T -SFMSM operator converts to the 2TL T -SF Bonferroni mean operator as follows:

$$\begin{aligned}
 & 2TLT-SFMSM^{(2)}(\hat{h}_1, \hat{h}_2, \dots, \hat{h}_n) \\
 &= \left(\Delta \left(\tau \left(\sqrt[q]{1 - \left(\prod_{1 \leq \kappa_1 < \kappa_2 \leq n} \left(1 - \left(\prod_{\varepsilon=1}^2 \left(\frac{\Delta^{-1}(s_{p_{\kappa_\varepsilon}}, \wp_{\kappa_\varepsilon})}{\tau} \right)^q \right) \right) \right)^{(1/C_n^2)} \right)^{(1/2)} \right) \right), \\
 &= \left(\Delta \left(\tau \left(\sqrt[q]{1 - \left(1 - \left(\prod_{1 \leq \kappa_1 < \kappa_2 \leq n} \left(1 - \prod_{\varepsilon=1}^2 \left(1 - \left(\frac{\Delta^{-1}(s_{n_{\kappa_\varepsilon}}, \mathfrak{N}_{\kappa_\varepsilon})}{\tau} \right)^q \right) \right) \right)^{(1/C_n^2)} \right)^{(1/2)} \right) \right), \\
 &= \left(\Delta \left(\tau \left(\sqrt[q]{1 - \left(1 - \left(\prod_{1 \leq \kappa_1 < \kappa_2 \leq n} \left(1 - \prod_{\varepsilon=1}^2 \left(1 - \left(\frac{\Delta^{-1}(s_{l_{\kappa_\varepsilon}}, \mathfrak{E}_{\kappa_\varepsilon})}{\tau} \right)^q \right) \right) \right)^{(1/C_n^2)} \right)^{(1/2)} \right) \right), \\
 &= \left(\Delta \left(\tau \left(\sqrt[q]{1 - \left(\prod_{1 \leq \kappa_1 < \kappa_2 \leq n} \left(1 - \left(\left(\frac{\Delta^{-1}(s_{p_{\kappa_1}}, \wp_{\kappa_1})}{\tau} \right) \left(\frac{\Delta^{-1}(s_{p_{\kappa_2}}, \wp_{\kappa_2})}{\tau} \right) \right)^q \right) \right)^{(2/n(n-1))} \right)^{(1/2)} \right), \\
 &= \left(\Delta \left(\tau \left(\sqrt[q]{1 - \left(1 - \left(\prod_{1 \leq \kappa_1 < \kappa_2 \leq n} \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{n_{\kappa_1}}, \mathfrak{N}_{\kappa_1})}{\tau} \right)^q \right) \left(1 - \left(\frac{\Delta^{-1}(s_{n_{\kappa_2}}, \mathfrak{N}_{\kappa_2})}{\tau} \right)^q \right) \right) \right)^{(2/n(n-1))} \right)^{(1/2)} \right), \\
 &= \left(\Delta \left(\tau \left(\sqrt[q]{1 - \left(1 - \left(\prod_{1 \leq \kappa_1 < \kappa_2 \leq n} \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{l_{\kappa_1}}, \mathfrak{E}_{\kappa_1})}{\tau} \right)^q \right) \left(1 - \left(\frac{\Delta^{-1}(s_{l_{\kappa_2}}, \mathfrak{E}_{\kappa_2})}{\tau} \right)^q \right) \right) \right)^{(2/n(n-1))} \right)^{(1/2)} \right), \\
 &= \left(\Delta \left(\tau \left(\sqrt[q]{1 - \left(\prod_{\kappa_1, \kappa_2=1, \kappa_1 \neq \kappa_2}^n \left(1 - \left(\left(\frac{\Delta^{-1}(s_{p_{\kappa_1}}, \wp_{\kappa_1})}{\tau} \right) \left(\frac{\Delta^{-1}(s_{p_{\kappa_2}}, \wp_{\kappa_2})}{\tau} \right) \right)^q \right) \right)^{(1/2), (2/n(n-1))} \right)^{(1/2)} \right), \\
 &= \left(\Delta \left(\tau \left(\sqrt[q]{1 - \left(1 - \left(\prod_{\kappa_1, \kappa_2=1, \kappa_1 \neq \kappa_2}^n \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{n_{\kappa_1}}, \mathfrak{N}_{\kappa_1})}{\tau} \right)^q \right) \left(1 - \left(\frac{\Delta^{-1}(s_{n_{\kappa_2}}, \mathfrak{N}_{\kappa_2})}{\tau} \right)^q \right) \right) \right)^{(1/2), (2/n(n-1))} \right)^{(1/2)} \right), \\
 &= \left(\Delta \left(\tau \left(\sqrt[q]{1 - \left(1 - \left(\prod_{\kappa_1, \kappa_2=1, \kappa_1 \neq \kappa_2}^n \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{l_{\kappa_1}}, \mathfrak{E}_{\kappa_1})}{\tau} \right)^q \right) \left(1 - \left(\frac{\Delta^{-1}(s_{l_{\kappa_2}}, \mathfrak{E}_{\kappa_2})}{\tau} \right)^q \right) \right) \right)^{(1/2), (2/n(n-1))} \right)^{(1/2)} \right) \right) \quad (19)
 \end{aligned}$$

$$\begin{aligned}
& \Delta \left(\tau \left(\sqrt[q]{1 - \left(\prod_{\kappa_1, \kappa_2=1, \kappa_1 \neq \kappa_2}^n \left(1 - \left(\left(\frac{\Delta^{-1}(s_{p_{\kappa_1}}, \wp_{\kappa_1}}{\tau} \right) \left(\frac{\Delta^{-1}(s_{p_{\kappa_2}}, \wp_{\kappa_2}}{\tau} \right) \right)^q \right) \right)^{(1/n(n-1))}} \right)^{(1/2)} \right) \right) \\
&= \Delta \left(\tau \left(\sqrt[q]{1 - \left(1 - \left(\prod_{\kappa_1, \kappa_2=1, \kappa_1 \neq \kappa_2}^n \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{n_{\kappa_1}}, t \wp_{\kappa_1}}{\tau} \right)^q \right) \left(1 - \left(\frac{\Delta^{-1}(s_{n_{\kappa_2}}, \wp_{\kappa_2}}{\tau} \right)^q \right) \right) \right)^{(1/n(n-1))}} \right)^{(1/2)} \right) \right) \\
& \Delta \left(\tau \left(\sqrt[q]{1 - \left(1 - \left(\prod_{\kappa_1, \kappa_2=1, \kappa_1 \neq \kappa_2}^n \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{l_{\kappa_1}}, \wp_{\kappa_1}}{\tau} \right)^q \right) \left(1 - \left(\frac{\Delta^{-1}(s_{l_{\kappa_2}}, \wp_{\kappa_2}}{\tau} \right)^q \right) \right) \right)^{(1/n(n-1))}} \right)^{(1/2)} \right) \right) \\
&= 2TLT - SFMSM^{(1,1)}(h_1, h_2, \dots, h_n).
\end{aligned} \tag{19a}$$

Case 3. When $t = n$, the 2TL T -SFMSM operator converts to the 2TL T -SF geometric mean operator as follows:

$$2TLT - SFMSM^{(n)}(h_1, h_2, \dots, h_n)$$

$$\begin{aligned}
& \Delta \left(\tau \left(\sqrt[q]{1 - \left(\prod_{1 \leq \kappa_1 < \kappa_n \leq n} \left(1 - \left(\prod_{\varepsilon=1}^n \left(\frac{\Delta^{-1}(s_{p_{\kappa_\varepsilon}}, \wp_{\kappa_\varepsilon}}{\tau} \right) \right)^q \right) \right)^{(1/C_n^n)}} \right)^{(1/n)} \right) \\
&= \Delta \left(\tau \left(\sqrt[q]{1 - \left(1 - \left(\prod_{1 \leq \kappa_1 < \kappa_n \leq n} \left(1 - \prod_{\varepsilon=1}^n \left(1 - \left(\frac{\Delta^{-1}(s_{n_{\kappa_\varepsilon}}, \wp_{\kappa_\varepsilon}}{\tau} \right)^q \right) \right) \right)^{(1/C_n^n)}} \right)^{(1/n)} \right) \right) \\
& \Delta \left(\tau \left(\sqrt[q]{1 - \left(1 - \left(\prod_{1 \leq \kappa_1 < \kappa_n \leq n} \left(1 - \prod_{\varepsilon=1}^n \left(1 - \left(\frac{\Delta^{-1}(s_{l_{\kappa_\varepsilon}}, \wp_{\kappa_\varepsilon}}{\tau} \right)^q \right) \right) \right)^{(1/C_n^n)}} \right)^{(1/n)} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \Delta \left(\tau \left(\sqrt[q]{1 - \left(1 - \left(\prod_{\varepsilon=1}^n \left(\frac{\Delta^{-1}(s_{p_{\kappa_\varepsilon}}, \wp_{\kappa_\varepsilon})}{\tau} \right)^q \right)^{(1/C_n^n)} \right)^{(1/n)}} \right) \right) \\
&= \Delta \left(\tau \left(\sqrt[q]{1 - \left(1 - \left(1 - \prod_{\varepsilon=1}^n \left(1 - \left(\frac{\Delta^{-1}(s_{n_{\kappa_\varepsilon}}, \mathfrak{N}_{\kappa_\varepsilon})}{\tau} \right)^q \right)^{(1/C_n^n)} \right)^{(1/n)}} \right) \right) \right) \quad (\text{let } \kappa_\varepsilon = \kappa) \\
& \Delta \left(\tau \left(\sqrt[q]{1 - \left(1 - \left(1 - \prod_{\varepsilon=1}^n \left(1 - \left(\frac{\Delta^{-1}(s_{l_{\kappa_\varepsilon}}, \mathfrak{L}_{\kappa_\varepsilon})}{\tau} \right)^q \right)^{(1/C_n^n)} \right)^{(1/n)}} \right) \right) \right) \\
&= \left(\Delta \left(\tau \left(\prod_{\kappa=1}^n \left(\frac{\Delta^{-1}(s_{p_\kappa}, \wp_\kappa)}{\tau} \right)^{(1/n)} \right) \right), \Delta \left(\tau \left(\sqrt[q]{1 - \left(\prod_{\kappa=1}^n \left(1 - \left(\frac{\Delta^{-1}(s_{n_\kappa}, \mathfrak{N}_\kappa)}{\tau} \right)^q \right)^{(1/n)} \right) \right) \right) \right) \\
& \Delta \left(\tau \left(\sqrt[q]{1 - \left(\prod_{\kappa=1}^n \left(1 - \left(\frac{\Delta^{-1}(s_{l_\kappa}, \mathfrak{L}_\kappa)}{\tau} \right)^q \right)^{(1/n)} \right) \right) \right) \right). \quad (20)
\end{aligned}$$

4.2. The 2TL T-SFWMSM Operator. There is no attention paid to the correlation among every given information and the significance of every individual given information in the proposed 2TL T-SFMSM operator; instead, just the input factor t is taken into account. The significance of aggregating information is considered in order to handle real-world issues, and we present the 2TL T-SFWMSM operator to account for this. Utilizing the Definition 6 and the novel operational rules of Definition 8, we develop the definition of 2-tuple linguistic T-spherical fuzzy weighted Maclaurin symmetric mean (2TL T-SFWMSM) operator as follows:

Definition 11. Let $h_\varepsilon = ((s_{p_\varepsilon}, \wp_\varepsilon), (s_{n_\varepsilon}, \mathfrak{N}_\varepsilon), (s_{l_\varepsilon}, \mathfrak{L}_\varepsilon))$ ($\varepsilon = 1, 2, \dots, n$) be any set of 2TL T-SFNs, $\xi = (\xi_1, \xi_2, \dots, \xi_n)^T$ be the weight vector of h_ε ($\varepsilon = 1, 2, \dots, n$), and $\xi_\varepsilon > 0$, $\sum_{\varepsilon=1}^n \xi_\varepsilon = 1$. The 2TL T-SFWMSM operator is defined below as follows:

$$\begin{aligned}
& 2TLT - SFWMSM_\xi^{(t)}(h_1, h_2, \dots, h_n) \\
&= \left(\frac{\oplus_{1 \leq \kappa_1 \leq \dots \leq \kappa_t \leq n} \left(\otimes_{\varepsilon=1}^t (h_{\kappa_\varepsilon})^{\xi_{\kappa_\varepsilon}} \right)}{C_n^t} \right)^{(1/t)}. \quad (21)
\end{aligned}$$

By utilizing the novel operational laws of 2TL T-SFNs (see Definition 8), we can obtain Theorem 3.

Theorem 3. Let $1 \leq t \leq n$ ($t \in \mathbb{Z}$), $h_\varepsilon = ((s_{p_\varepsilon}, \wp_\varepsilon), (s_{n_\varepsilon}, \mathfrak{N}_\varepsilon), (s_{l_\varepsilon}, \mathfrak{L}_\varepsilon))$ ($\varepsilon = 1, 2, \dots, n$) be any set of 2TL T-SFNs, $\xi = (\xi_1, \xi_2, \dots, \xi_n)^T$ be the weight vector of h_ε ($\varepsilon = 1, 2, \dots, n$), and $\xi_\varepsilon > 0$, $\sum_{\varepsilon=1}^n \xi_\varepsilon = 1$. Then, the aggregated value by using the 2TL T-SFWMSM operator is also a 2TL T-SFN, and

$$\begin{aligned}
& 2TLT - SFWMSM_{\xi}^{(t)}(h_1, h_2, \dots, h_n) \\
&= \left(\Delta \left(\tau \sqrt[q]{1 - \left(\prod_{1 \leq \kappa_1 \leq \dots \leq \kappa_t \leq n} \left(1 - \left(\prod_{\varepsilon=1}^t \left(\frac{\Delta^{-1}(s_{p_{\kappa_\varepsilon}}, \wp_{\kappa_\varepsilon}}{\tau} \right)^{q_{\xi_{\kappa_\varepsilon}}}} \right) \right) \right)^{(1/C_n^t)} \right)^{(1/t)} \right), \\
&\quad \Delta \left(\tau \sqrt[q]{1 - \left(1 - \left(\prod_{1 \leq \kappa_1 \leq \dots \leq \kappa_t \leq n} \left(1 - \prod_{\varepsilon=1}^t \left(1 - \left(\frac{\Delta^{-1}(s_{n_{\kappa_\varepsilon}}, \aleph_{\kappa_\varepsilon}}{\tau} \right)^q \right)^{\xi_{\kappa_\varepsilon}} \right) \right)^{(1/C_n^t)} \right)^{(1/t)} \right), \\
&\quad \Delta \left(\tau \sqrt[q]{1 - \left(1 - \left(\prod_{1 \leq \kappa_1 \leq \dots \leq \kappa_t \leq n} \left(1 - \prod_{\varepsilon=1}^t \left(1 - \left(\frac{\Delta^{-1}(s_{l_{\kappa_\varepsilon}}, \mathfrak{L}_{\kappa_\varepsilon}}{\tau} \right)^q \right)^{\xi_{\kappa_\varepsilon}} \right) \right)^{(1/C_n^t)} \right)^{(1/t)} \right) \right). \quad (22)
\end{aligned}$$

The proof is the same as Theorem 1, and it is omitted here.

Theorem 4. Let $h_\varepsilon = ((s_{p_\varepsilon}, \wp_\varepsilon), (s_{n_\varepsilon}, \aleph_\varepsilon), (s_{l_\varepsilon}, \mathfrak{L}_\varepsilon))$ and $h'_\varepsilon = ((s'_{p_\varepsilon}, \wp'_\varepsilon), (s'_{n_\varepsilon}, \aleph'_\varepsilon), (s'_{l_\varepsilon}, \mathfrak{L}'_\varepsilon))$ ($\varepsilon = 1, 2, \dots, n$) be two sets of 2TL T-SFNs; then, the 2TL T-SFWMSM operator has the following properties:

- (1) (Commutativity) If h_ε ($\varepsilon = 1, 2, \dots, n$) is any set of 2TL T-SFNs, and $\tilde{h}_\varepsilon$ ($\varepsilon = 1, 2, \dots, n$) is any permutation of h_ε ($\varepsilon = 1, 2, \dots, n$), then

$$\begin{aligned}
& 2TLT - SFWMSM_{\xi}^{(t)}(h_1, h_2, \dots, h_n) \\
&= 2TLT - SFWMSM_{\xi}^{(t)}(\tilde{h}_1, \tilde{h}_2, \dots, \tilde{h}_n). \quad (23)
\end{aligned}$$

- (2) (Monotonicity) Let $h_\varepsilon, \tilde{h}_\varepsilon$ ($\varepsilon = 1, 2, \dots, n$) be two sets of 2TL T-SFNs; if $(s_{p_\varepsilon}, \wp_\varepsilon) \geq (s'_{p_\varepsilon}, \wp'_\varepsilon)$, $(s_{n_\varepsilon}, \aleph_\varepsilon) \leq (s'_{n_\varepsilon}, \aleph'_\varepsilon)$, and $(s_{l_\varepsilon}, \mathfrak{L}_\varepsilon) \leq (s'_{l_\varepsilon}, \mathfrak{L}'_\varepsilon)$, for all ε , then

$$\begin{aligned}
& 2TLT - SFWMSM_{\xi}^{(t)}(h_1, h_2, \dots, h_n) \\
&\geq 2TLT - SFWMSM_{\xi}^{(t)}(h'_1, h'_2, \dots, h'_n). \quad (24)
\end{aligned}$$

- (3) (Boundedness) Let h_ε ($\varepsilon = 1, 2, \dots, n$) be any set of 2TL T-SFNs, suppose

$$\begin{aligned}
& h^- = \min_{\varepsilon} h_{\varepsilon} \\
&= \left(\min_{\varepsilon} (s_{p_\varepsilon}, \wp_\varepsilon), \max_{\varepsilon} (s_{n_\varepsilon}, \aleph_\varepsilon), \max_{\varepsilon} (s_{l_\varepsilon}, \mathfrak{L}_\varepsilon) \right), \\
& h^+ = \max_{\varepsilon} h_{\varepsilon} \\
&= \left(\max_{\varepsilon} (s_{p_\varepsilon}, \wp_\varepsilon), \min_{\varepsilon} (s_{n_\varepsilon}, \aleph_\varepsilon), \min_{\varepsilon} (s_{l_\varepsilon}, \mathfrak{L}_\varepsilon) \right). \quad (25)
\end{aligned}$$

Then,

$$h^- \leq 2TLT - SFWMSM_{\xi}^{(t)}(h_1, h_2, \dots, h_n) \leq h^+. \quad (26)$$

Now, with regard to parameter t we can describe certain specific cases of the 2TL T-SFWMSM operator.

Case 1. When $t = 1$, the 2TL T-SFWMSM operator converts to the 2TL T-SF weighted average (2TL T-SFWA) operator.

$$\begin{aligned}
& 2TLT - SFWA_{\xi}^{(1)}(h_1, h_2, \dots, h_n) \\
&= \left(\Delta \left(\tau \sqrt[q]{1 - \left(\prod_{\varepsilon=1}^n \left(1 - \left(\frac{\Delta^{-1}(s_{p_{\kappa_\varepsilon}}, \wp_{\kappa_\varepsilon}}{\tau} \right)^{q_{\xi_{\kappa_\varepsilon}}}} \right) \right)^{(1/n)}} \right), \Delta \left(\tau \sqrt[q]{1 - \left(1 - \left(\prod_{\varepsilon=1}^n \left(1 - \left(\frac{\Delta^{-1}(s_{n_{\kappa_\varepsilon}}, \aleph_{\kappa_\varepsilon}}{\tau} \right)^q \right)^{\xi_{\kappa_\varepsilon}} \right) \right)^{(1/n)}} \right), \right. \\
&\quad \left. \Delta \left(\tau \sqrt[q]{1 - \left(1 - \left(\prod_{\varepsilon=1}^n \left(1 - \left(\frac{\Delta^{-1}(s_{l_{\kappa_\varepsilon}}, \mathfrak{L}_{\kappa_\varepsilon}}{\tau} \right)^q \right)^{\xi_{\kappa_\varepsilon}} \right) \right)^{(1/n)}} \right) \right). \quad (27)
\end{aligned}$$

Case 2. When $t = 2$, the 2TL T -SFWMSM operator converts to the 2TL T -SF weighted Bonferroni mean (2TL T -SFWBM) operator.

$$\begin{aligned}
 & 2TLT - SFWBM^{(1,1)}(h_1, h_2, \dots, h_n) \\
 &= \Delta \left(\tau \left(\sqrt[q]{1 - \left(\prod_{1 \leq \kappa_1 < \kappa_2 \leq n} \left(1 - \left(\left(\frac{\Delta^{-1}(s_{p_{\kappa_1}}, \wp_{\kappa_1}})}{\tau} \right)^{\xi_{\kappa_1}} \left(\frac{\Delta^{-1}(s_{p_{\kappa_2}}, \wp_{\kappa_2}})}{\tau} \right)^{\xi_{\kappa_2}} \right)^q \right)} \right)^{(2/n(n-1))} \right) \right) \\
 &= \Delta \left(\tau \left(\prod_{1 \leq \kappa_1 < \kappa_2 \leq n} \sqrt[q]{1 - \left(\left(1 - \left(\frac{\Delta^{-1}(s_{n_{\kappa_1}}, \aleph_{\kappa_1}})}{\tau} \right)^q \right)^{\xi_{\kappa_1}} \left(1 - \left(\frac{\Delta^{-1}(s_{n_{\kappa_2}}, \aleph_{\kappa_2}})}{\tau} \right)^q \right)^{\xi_{\kappa_2}}} \right)} \right)^{(2/n(n-1))} \right) \\
 &= \Delta \left(\tau \left(\prod_{1 \leq \kappa_1 < \kappa_2 \leq n} \sqrt[q]{1 - \left(\left(1 - \left(\frac{\Delta^{-1}(s_{l_{\kappa_1}}, \mathfrak{E}_{\kappa_1}})}{\tau} \right)^q \right)^{\xi_{\kappa_1}} \left(1 - \left(\frac{\Delta^{-1}(s_{l_{\kappa_2}}, \mathfrak{E}_{\kappa_2}})}{\tau} \right)^q \right)^{\xi_{\kappa_2}}} \right)} \right)^{(2/n(n-1))} \right). \quad (28)
 \end{aligned}$$

Case 3. When $t = n$, the 2TL T -SFWMSM operator converts to the q -ROF weighted geometric (2TL T -SFWG) operator.

$$\begin{aligned}
 & 2TLT - SFWG^{(n)}(h_1, h_2, \dots, h_n) \\
 &= \Delta \left(\tau \left(\prod_{\varepsilon=1}^n \left(\frac{\Delta^{-1}(s_{p_{\kappa_\varepsilon}}, \wp_{\kappa_\varepsilon}})}{\tau} \right)^{\xi_{\varepsilon}} \right)^{(1/n)}, \Delta \left(\tau \left(\sqrt[q]{1 - \prod_{\varepsilon=1}^n \left(1 - \left(\frac{\Delta^{-1}(s_{n_{\kappa_\varepsilon}}, \aleph_{\kappa_\varepsilon}})}{\tau} \right)^q \right)^{\xi_{\varepsilon}}} \right) \right)^{(1/n)}, \right. \\
 &\quad \left. \Delta \left(\tau \left(\sqrt[q]{1 - \prod_{\varepsilon=1}^n \left(1 - \left(\frac{\Delta^{-1}(s_{l_{\kappa_\varepsilon}}, \mathfrak{E}_{\kappa_\varepsilon}})}{\tau} \right)^q \right)^{\xi_{\varepsilon}}} \right) \right)^{(1/n)} \right). \quad (29)
 \end{aligned}$$

4.3. The 2TL T -SFDMSM Operator. Utilizing the Definition 6 and the novel operational rules of Definition 8, we develop the definition of 2-tuple linguistic T -spherical fuzzy dual Maclaurin symmetric mean (2TL T -SFDMSM) operator as follows:

Definition 12. Let $h_\varepsilon = ((s_{p_\varepsilon}, \wp_\varepsilon), (s_{n_\varepsilon}, \aleph_\varepsilon), (s_{l_\varepsilon}, \mathfrak{E}_\varepsilon))$ ($\varepsilon = 1, 2, \dots, n$) be any set of 2TL T -SFNs; then, we define the 2TL T -SFDMSM operator as follows:

$$\begin{aligned}
 & 2TLT - SFDM SM^{(t)}(h_1, h_2, \dots, h_n) \\
 &= \frac{1}{t} \left(\otimes_{1 \leq \kappa_1 < \dots < \kappa_t \leq n} \left(\oplus_{\varepsilon=1}^t h_{\kappa_\varepsilon} \right)^{(1/C_n^t)} \right). \quad (30)
 \end{aligned}$$

Theorem 5. Let $h_\varepsilon = ((s_{p_\varepsilon}, \wp_\varepsilon), (s_{n_\varepsilon}, \aleph_\varepsilon), (s_{l_\varepsilon}, \mathfrak{E}_\varepsilon))$ ($\varepsilon = 1, 2, \dots, n$) be any set of 2TL T -SFNs; then, their aggregated value by using 2TL T -SFDMSM operator is also a 2TL T -SFN, and

$$2TLT-SF\ DM\ SM^{(t)}(\hbar_1, \hbar_2, \dots, \hbar_n)$$

$$= \left(\begin{array}{c} \Delta \left(\tau \left(\sqrt[q]{1 - \left(1 - \prod_{1 \leq \kappa_1 \leq \dots \leq \kappa_t \leq n} \left(1 - \prod_{\varepsilon=1}^t \left(1 - \left(\frac{\Delta^{-1}(s_{p_{\kappa_\varepsilon}}, \wp_{\kappa_\varepsilon}}{\tau} \right)^q \right) \right) \right) \right)^{(1/C_n^t)} \right)^{(1/t)} \right) \\ \Delta \left(\tau \left(\sqrt[q]{1 - \prod_{1 \leq \kappa_1 \leq \dots \leq \kappa_t \leq n} \left(1 - \left(\prod_{\varepsilon=1}^t \frac{\Delta^{-1}(s_{n_{\kappa_\varepsilon}}, \mathfrak{N}_{\kappa_\varepsilon}}{\tau} \right)^q \right) \right)^{(1/C_n^t)} \right)^{(1/t)} \right) \\ \Delta \left(\tau \left(\sqrt[q]{1 - \prod_{1 \leq \kappa_1 \leq \dots \leq \kappa_t \leq n} \left(1 - \left(\prod_{\varepsilon=1}^t \frac{\Delta^{-1}(s_{l_{\kappa_\varepsilon}}, \mathfrak{L}_{\kappa_\varepsilon}}{\tau} \right)^q \right) \right)^{(1/C_n^t)} \right)^{(1/t)} \right) \end{array} \right). \quad (31)$$

Proof. By utilizing the novel operational laws of 2TLT-SFNs (see Definition 8), we have

$$\begin{aligned} \oplus_{\varepsilon=1}^t \hbar_{\kappa_\varepsilon} &= \left(\Delta \left(\tau \left(\sqrt[q]{1 - \prod_{\varepsilon=1}^t \left(1 - \left(\frac{\Delta^{-1}(s_{p_{\kappa_\varepsilon}}, \wp_{\kappa_\varepsilon}}{\tau} \right)^q \right) \right) \right) \right), \\ &\quad \Delta \left(\tau \left(\prod_{\varepsilon=1}^t \frac{\Delta^{-1}(s_{n_{\kappa_\varepsilon}}, \mathfrak{N}_{\kappa_\varepsilon}}{\tau} \right) \right), \Delta \left(\tau \left(\prod_{\varepsilon=1}^t \frac{\Delta^{-1}(s_{l_{\kappa_\varepsilon}}, \mathfrak{L}_{\kappa_\varepsilon}}{\tau} \right) \right) \right), \\ (\oplus_{\varepsilon=1}^t \hbar_{\kappa_\varepsilon})^{(1/C_n^t)} &= \left(\begin{array}{c} \Delta \left(\tau \left(\left(\sqrt[q]{1 - \prod_{\varepsilon=1}^t \left(1 - \left(\frac{\Delta^{-1}(s_{p_{\kappa_\varepsilon}}, \wp_{\kappa_\varepsilon}}{\tau} \right)^q \right) \right) \right)^{(1/C_n^t)} \right) \right) \\ \Delta \left(\tau \left(\sqrt[q]{1 - \left(1 - \left(\prod_{\varepsilon=1}^t \frac{\Delta^{-1}(s_{n_{\kappa_\varepsilon}}, \mathfrak{N}_{\kappa_\varepsilon}}{\tau} \right)^q \right) \right)^{(1/C_n^t)} \right) \right) \\ \Delta \left(\tau \left(\sqrt[q]{1 - \left(1 - \left(\prod_{\varepsilon=1}^t \frac{\Delta^{-1}(s_{l_{\kappa_\varepsilon}}, \mathfrak{L}_{\kappa_\varepsilon}}{\tau} \right)^q \right) \right)^{(1/C_n^t)} \right) \right) \end{array} \right). \quad (32) \end{aligned}$$

Thus, we obtain

$$\otimes_{1 \leq \kappa_1 < \dots < \kappa_t \leq n} \left(\bigoplus_{\varepsilon=1}^t \hbar_{\kappa_\varepsilon} \right)^{(1/C_n^t)} = \left(\begin{array}{c} \Delta \left(\tau \left(\prod_{1 \leq \kappa_1 < \dots < \kappa_t \leq n} \left(\sqrt[q]{1 - \prod_{\varepsilon=1}^t \left(1 - \left(\frac{\Delta^{-1}(s_{p_{\kappa_\varepsilon}}, \wp_{\kappa_\varepsilon}}{\tau} \right)^q} \right)} \right)^{(1/C_n^t)} \right) \right) \\ \Delta \left(\tau \left(\sqrt[q]{1 - \prod_{1 \leq \kappa_1 < \dots < \kappa_t \leq n} \left(1 - \left(\prod_{\varepsilon=1}^t \frac{\Delta^{-1}(s_{n_{\kappa_\varepsilon}}, \aleph_{\kappa_\varepsilon}}{\tau} \right)^q} \right)} \right)^{(1/C_n^t)} \right) \\ \Delta \left(\tau \left(\sqrt[q]{1 - \prod_{1 \leq \kappa_1 < \dots < \kappa_t \leq n} \left(1 - \left(\prod_{\varepsilon=1}^t \frac{\Delta^{-1}(s_{l_{\kappa_\varepsilon}}, \mathfrak{E}_{\kappa_\varepsilon}}{\tau} \right)^q} \right)} \right)^{(1/C_n^t)} \right) \end{array} \right). \quad (33)$$

Accordingly,

$$2TLT-SFDM SM^{(t)}(\hbar_1, \hbar_2, \dots, \hbar_n) = \left(\begin{array}{c} \Delta \left(\tau \left(\sqrt[q]{1 - \left(1 - \prod_{1 \leq \kappa_1 < \dots < \kappa_t \leq n} \left(1 - \prod_{\varepsilon=1}^t \left(1 - \left(\frac{\Delta^{-1}(s_{p_{\kappa_\varepsilon}}, \wp_{\kappa_\varepsilon}}{\tau} \right)^q} \right)} \right)^{(1/C_n^t)} \right)} \right)^{(1/t)} \right) \\ \Delta \left(\tau \left(\sqrt[q]{1 - \prod_{1 \leq \kappa_1 < \dots < \kappa_t \leq n} \left(1 - \left(\prod_{\varepsilon=1}^t \frac{\Delta^{-1}(s_{n_{\kappa_\varepsilon}}, \aleph_{\kappa_\varepsilon}}{\tau} \right)^q} \right)} \right)^{(1/C_n^t)} \right)^{(1/t)} \\ \Delta \left(\tau \left(\sqrt[q]{1 - \prod_{1 \leq \kappa_1 < \dots < \kappa_t \leq n} \left(1 - \left(\prod_{\varepsilon=1}^t \frac{\Delta^{-1}(s_{l_{\kappa_\varepsilon}}, \mathfrak{E}_{\kappa_\varepsilon}}{\tau} \right)^q} \right)} \right)^{(1/C_n^t)} \right)^{(1/t)} \end{array} \right). \quad (34)$$

Theorem 6. Let $\hbar_\varepsilon = ((s_{p_\varepsilon}, \wp_\varepsilon), (s_{n_\varepsilon}, \aleph_\varepsilon), (s_{l_\varepsilon}, \mathfrak{E}_\varepsilon))$ and $\hbar'_\varepsilon = ((s'_{p_\varepsilon}, \wp'_\varepsilon), (s'_{n_\varepsilon}, \aleph'_\varepsilon), (s'_{l_\varepsilon}, \mathfrak{E}'_\varepsilon))$ ($\varepsilon = 1, 2, \dots, n$) be two sets of 2TL T-SFNs; then, the 2TL T-SFDMSM operator has the following properties:

- (1) (Idempotency) If all $\hbar_\varepsilon$ ($\varepsilon = 1, 2, \dots, n$) are equal, i.e., $\hbar_\varepsilon = \hbar$ for all ε , then

$$2TLT-SFDM SM^{(t)}(\hbar_1, \hbar_2, \dots, \hbar_n) = \hbar. \quad (35)$$

- (2) (Commutativity) Let $\hbar_\varepsilon$ ($\varepsilon = 1, 2, \dots, n$) be any set of 2TL T-SFNs and $\hbar_\varepsilon$ ($\varepsilon = 1, 2, \dots, n$) be any permutation of $\hbar_\varepsilon$ ($\varepsilon = 1, 2, \dots, n$); thus,

$$2TLT-SFDM SM^{(t)}(\hbar_1, \hbar_2, \dots, \hbar_n) = 2TLT-SFDM SM^{(t)}(\hbar'_1, \hbar'_2, \dots, \hbar'_n). \quad (36)$$

- (3) (Monotonicity) Let $\hbar_\varepsilon$ and $\hbar'_\varepsilon$ ($\varepsilon = 1, 2, \dots, n$) be two sets of 2TL T-SFNs; if $((s_{p_\varepsilon}, \wp_\varepsilon) \geq (s'_{p_\varepsilon}, \wp'_\varepsilon))$,

$(s_{n_\varepsilon}, \aleph_\varepsilon) \leq (s'_{n_\varepsilon}, \aleph'_\varepsilon)$, and $(s_{l_\varepsilon}, \mathfrak{E}_\varepsilon) \leq (s'_{l_\varepsilon}, \mathfrak{E}'_\varepsilon)$, for all ε , then

$$2TLT - SF \text{ DM } SM^{(t)}(h_1, h_2, \dots, h_n) \geq 2TLT - SF \text{ DM } SM^{(t)}(\dot{h}_1, \dot{h}_2, \dots, \dot{h}_n). \quad (37)$$

(4) (Boundedness). Let $\dot{h}_\varepsilon (\varepsilon = 1, 2, \dots, n)$ be any set of 2TL T-SFNs, suppose

$$\begin{aligned} \dot{h}^- &= \min_{\varepsilon} \dot{h}_{\varepsilon} \\ &= \left(\max_{\varepsilon} (s_{p_{\varepsilon}}, \wp_{\varepsilon}), \min_{\varepsilon} (s_{n_{\varepsilon}}, \aleph_{\varepsilon}), \min_{\varepsilon} (s_{l_{\varepsilon}}, \mathfrak{L}_{\varepsilon}) \right), \\ \dot{h}^+ &= \max_{\varepsilon} \dot{h}_{\varepsilon} \\ &= \left(\max_{\varepsilon} (s_{p_{\varepsilon}}, \wp_{\varepsilon}), \min_{\varepsilon} (s_{n_{\varepsilon}}, \aleph_{\varepsilon}), \min_{\varepsilon} (s_{l_{\varepsilon}}, \mathfrak{L}_{\varepsilon}) \right). \end{aligned} \quad (38)$$

Then,

$$\dot{h}^- \leq 2TLT - SF \text{ DM } SM^{(t)}(h_1, h_2, \dots, h_n) \leq \dot{h}^+. \quad (39)$$

Now, with regard to parameter t we can describe certain specific cases of the 2TL T-SFDMSM operator.

Case 1. When $t = 1$, the 2TL T-SFDMSM operator converts to the 2TL T-SF geometric operator.

Case 2. When $t = 2$, the 2TL T-SFDMSM operator converts to the 2TL T-SF geometric Bonferroni mean operator.

Case 3. When $t = n$, the 2TL T-SFDMSM operator converts to the 2TL T-SF average operator.

4.4. The 2TL T-SFWDMSM Operator. Utilizing the Definition 6 and the novel operational rules of Definition 8, we develop the definition of 2-tuple linguistic T-spherical fuzzy weighted dual Maclaurin symmetric mean (2TL T-SFWDMSM) operator as follows:

Definition 13. Let $\dot{h}_{\varepsilon} = ((s_{p_{\varepsilon}}, \wp_{\varepsilon}), (s_{n_{\varepsilon}}, \aleph_{\varepsilon}), (s_{l_{\varepsilon}}, \mathfrak{L}_{\varepsilon})) (\varepsilon = 1, 2, \dots, n)$ be any set of 2TL T-SFNs, and $\xi = (\xi_1, \xi_2, \dots, \xi_n)^T$ be the weight vector of $\dot{h}_{\varepsilon} (\varepsilon = 1, 2, \dots, n)$, and $\xi_{\varepsilon} > 0$, $\sum_{\varepsilon=1}^n \xi_{\varepsilon} = 1$. The 2TL T-SFWDMSM operator is defined as follows:

$$2TLT - SFW \text{ DM } SM_{\xi}^{(t)}(h_1, h_2, \dots, h_n) = \frac{1}{t} \left(\otimes_{1 \leq \kappa_1 < \dots < \kappa_t \leq n} \left(\oplus_{\varepsilon=1}^t (\xi_{\kappa_{\varepsilon}} \dot{h}_{\kappa_{\varepsilon}}) \right)^{1/C_n^t} \right). \quad (40)$$

By utilizing the novel operational laws of 2TL T-SFNs (see Definition 8), we can obtain Theorem 7.

Theorem 7. Let $1 \leq t \leq n (t \in \mathbb{Z})$, $\dot{h}_{\varepsilon} = ((s_{p_{\varepsilon}}, \wp_{\varepsilon}), (s_{n_{\varepsilon}}, \aleph_{\varepsilon}), (s_{l_{\varepsilon}}, \mathfrak{L}_{\varepsilon})) (\varepsilon = 1, 2, \dots, n)$ be any set of 2TL T-SFNs, $\xi =$

$(\xi_1, \xi_2, \dots, \xi_n)^T$ be the weight vector of $\dot{h}_{\varepsilon} (\varepsilon = 1, 2, \dots, n)$, and $\xi_{\varepsilon} > 0$, $\sum_{\varepsilon=1}^t \xi_{\varepsilon} = 1$. Therefore, aggregated value by using 2TL T-SFWDMSM operator is also a 2TL T-SFN, and

$$\begin{aligned} &2TLT - SFW \text{ DM } SM_{\xi}^{(t)}(h_1, h_2, \dots, h_n) \\ &= \left(\Delta \left(\tau \left(\sqrt[q]{1 - \left(1 - \prod_{1 \leq \kappa_1 \leq \dots < \kappa_t \leq n} \left(1 - \prod_{\varepsilon=1}^t \left(1 - \left(\frac{\Delta^{-1}(s_{p_{\kappa_{\varepsilon}}}, \wp_{\kappa_{\varepsilon}})}{\tau} \right)^q \right)^{\xi_{\kappa_{\varepsilon}}} \right)^{(1/C_n^t)} \right)^{(1/t)} \right) \right) \right. \\ &\quad \Delta \left(\tau \left(\sqrt[q]{1 - \prod_{1 \leq \kappa_1 \leq \dots < \kappa_t \leq n} \left(1 - \left(\prod_{\varepsilon=1}^t \left(\frac{\Delta^{-1}(s_{n_{\kappa_{\varepsilon}}}, \aleph_{\kappa_{\varepsilon}})}{\tau} \right)^{\xi_{\kappa_{\varepsilon}}} \right)^q \right)^{(1/C_n^t)} \right)^{(1/t)} \right) \right. \\ &\quad \left. \Delta \left(\tau \left(\sqrt[q]{1 - \prod_{1 \leq \kappa_1 \leq \dots < \kappa_t \leq n} \left(1 - \left(\prod_{\varepsilon=1}^t \left(\frac{\Delta^{-1}(s_{l_{\kappa_{\varepsilon}}}, \mathfrak{L}_{\kappa_{\varepsilon}})}{\tau} \right)^{\xi_{\kappa_{\varepsilon}}} \right)^q \right)^{(1/C_n^t)} \right)^{(1/t)} \right) \right) \right). \end{aligned} \quad (41)$$

The proof is the same as Theorem 5, and it is omitted here.

Theorem 8. Let $\tilde{h}_\varepsilon = ((s_{p_\varepsilon}, \wp_\varepsilon), (s_{n_\varepsilon}, \aleph_\varepsilon), (s_{l_\varepsilon}, \xi_\varepsilon))$ and $\tilde{h}'_\varepsilon = ((s'_{p_\varepsilon}, \wp'_\varepsilon), (s'_{n_\varepsilon}, \aleph'_\varepsilon), (s'_{l_\varepsilon}, \xi'_\varepsilon))$ ($\varepsilon = 1, 2, \dots, n$) be two sets of 2TL T-SFNs; then, the 2TL T-SFWDMSM operator has the following properties:

- (1) (Commutativity) Let $\tilde{h}_\varepsilon$ ($\varepsilon = 1, 2, \dots, n$) be any set of 2TL T-SFNs, and $\tilde{h}_\varepsilon$ ($\varepsilon = 1, 2, \dots, n$) be any permutation of $\tilde{h}_\varepsilon$ ($\varepsilon = 1, 2, \dots, n$), then

$$2TLT - SFW DM SM_\xi^{(t)}(\tilde{h}_1, \tilde{h}_2, \dots, \tilde{h}_n) = 2TLT - SFW DM SM_\xi^{(t)}(\tilde{h}'_1, \tilde{h}'_2, \dots, \tilde{h}'_n). \quad (42)$$

- (2) (Monotonicity) Let $\tilde{h}_\varepsilon$ and $\tilde{h}'_\varepsilon$ ($\varepsilon = 1, 2, \dots, n$) be two sets of 2TL T-SFNs; if $(s_{p_\varepsilon}, \wp_\varepsilon) \geq (s'_{p_\varepsilon}, \wp'_\varepsilon)$, $(s_{n_\varepsilon}, \aleph_\varepsilon) \leq (s'_{n_\varepsilon}, \aleph'_\varepsilon)$, and $(s_{l_\varepsilon}, \xi_\varepsilon) \leq (s'_{l_\varepsilon}, \xi'_\varepsilon)$, for all ε , then

$$2TLT - SFW MSM_\xi^{(t)}(\tilde{h}_1, \tilde{h}_2, \dots, \tilde{h}_n) \geq 2TLT - SFW MSM_\xi^{(t)}(\tilde{h}'_1, \tilde{h}'_2, \dots, \tilde{h}'_n). \quad (43)$$

- (3) (Boundedness). Let $\tilde{h}_\varepsilon$ ($\varepsilon = 1, 2, \dots, n$) be any set of 2TL T-SFNs, suppose

$$\begin{aligned} \tilde{h}^- &= \min_\varepsilon \tilde{h}_\varepsilon \\ &= \left(\max_\varepsilon (s_{p_\varepsilon}, \wp_\varepsilon), \min_\varepsilon (s_{n_\varepsilon}, \aleph_\varepsilon), \min_\varepsilon (s_{l_\varepsilon}, \xi_\varepsilon) \right), \\ \tilde{h}^+ &= \max_\varepsilon \tilde{h}_\varepsilon \\ &= \left(\max_\varepsilon (s_{p_\varepsilon}, \wp_\varepsilon), \min_\varepsilon (s_{n_\varepsilon}, \aleph_\varepsilon), \min_\varepsilon (s_{l_\varepsilon}, \xi_\varepsilon) \right). \end{aligned} \quad (44)$$

Then,

$$\tilde{h}^- \leq 2TLT - SFW MSM_\xi^{(t)}(\tilde{h}_1, \tilde{h}_2, \dots, \tilde{h}_n) \leq \tilde{h}^+. \quad (45)$$

Now, with regard to parameter t we can describe certain specific cases of the 2TL T-SFWDMSM operator.

Case 1. When $t = 1$, the 2TL T-SFWDMSM operator converts to the 2TL T-SFWG operator.

Case 2. When $t = 2$, the 2TL T-SFWDMSM operator converts to the 2TL T-SF weighted geometric Bonferroni mean operator.

Case 3. When $t = n$, the 2TL T-SFWDMSM operator converts to the 2TL T-SFWA operator.

5. EDAS Method for MAGDM in 2TL T-SF Environment

In this section, the EDAS method is modified to solve a MAGDM problem within a 2TL T-SF environment to build the ranking procedure. According to the work of Zhang [63], group decision-making problems can be solved from two angles: (a) aggregation stage and (b) exploitation stage. In the aggregation stage, collective evaluation values are obtained from the individual evaluation values of the alternatives. In the exploitation stage, the optimal alternative(s) is selected according to the priorities of the cumulative evaluation values. Therefore, we employ the 2TL T-SFWMSM and 2TL T-SFWDMSM operators to combine the individual decision matrices into a group decision matrix. By utilizing the 2TL T-SFWMSM (2TL T-SFWDMSM) operator, the overall value of alternative based on attribute is calculated.

5.1. Proposed 2-Tuple Linguistic T-Spherical Fuzzy EDAS Approach. To solve the group decision-making problem, we choose a set of m alternatives $\mathfrak{A} = \{\mathfrak{A}_1, \mathfrak{A}_2, \dots, \mathfrak{A}_m\}$, n attributes $\tilde{h} = \{\tilde{h}_1, \tilde{h}_2, \dots, \tilde{h}_n\}$, and λ experts $E = \{e_1, e_2, \dots, e_\lambda\}$, and let $\xi = (\xi_1, \xi_2, \dots, \xi_n)^T$ and $\omega = (\omega_1, \omega_2, \dots, \omega_\lambda)^T$ be the weighting vector of the attributes and weighting vector of the experts satisfying $\xi_\varepsilon \in [0, 1]$, $\omega_\ell \in [0, 1]$, $\sum_{\varepsilon=1}^n \xi_\varepsilon = 1$, and $\sum_{\ell=1}^\lambda \omega_\ell = 1$, respectively. The technique of implementing the 2TL T-SF-EDAS approach is described in the following phases.

Phase 1. We establish the 2TL T-SF evaluation matrix $R^\lambda = [\mathfrak{A}_{\kappa\varepsilon}^\lambda]_{m \times n} = ((s_{p_{\kappa\varepsilon}}, \wp^\lambda), (s_{n_{\kappa\varepsilon}}, \aleph^\lambda), (s_{l_{\kappa\varepsilon}}, \xi^\lambda))$ as follows:

$$\begin{aligned} R^\lambda &= [\mathfrak{A}_{\kappa\varepsilon}^\lambda]_{m \times n} \\ &= \begin{bmatrix} ((s_{p_{11}}, \wp)^\lambda, (s_{n_{11}}, \aleph)^\lambda, (s_{l_{11}}, \xi)^\lambda) & ((s_{p_{12}}, \wp)^\lambda, (s_{n_{12}}, \aleph)^\lambda, (s_{l_{12}}, \xi)^\lambda) & \dots & ((s_{p_{1n}}, \wp)^\lambda, (s_{n_{1n}}, \aleph)^\lambda, (s_{l_{1n}}, \xi)^\lambda) \\ ((s_{p_{21}}, \wp)^\lambda, (s_{n_{21}}, \aleph)^\lambda, (s_{l_{21}}, \xi)^\lambda) & ((s_{p_{22}}, \wp)^\lambda, (s_{n_{22}}, \aleph)^\lambda, (s_{l_{22}}, \xi)^\lambda) & \dots & ((s_{p_{2n}}, \wp)^\lambda, (s_{n_{2n}}, \aleph)^\lambda, (s_{l_{2n}}, \xi)^\lambda) \\ \vdots & \vdots & \ddots & \vdots \\ ((s_{p_{m1}}, \wp)^\lambda, (s_{n_{m1}}, \aleph)^\lambda, (s_{l_{m1}}, \xi)^\lambda) & ((s_{p_{m2}}, \wp)^\lambda, (s_{n_{m2}}, \aleph)^\lambda, (s_{l_{m2}}, \xi)^\lambda) & \dots & ((s_{p_{mn}}, \wp)^\lambda, (s_{n_{mn}}, \aleph)^\lambda, (s_{l_{mn}}, \xi)^\lambda) \end{bmatrix}. \end{aligned} \quad (46)$$

where $\mathfrak{A}_{\kappa\varepsilon}^\lambda = ((s_{p_{\kappa\varepsilon}}, \wp)^\lambda, (s_{n_{\kappa\varepsilon}}, \aleph)^\lambda, (s_{l_{\kappa\varepsilon}}, \xi)^\lambda)$ ($\kappa = 1, 2, \dots, m, \varepsilon = 1, 2, \dots, n$) represents the 2TL T-SF information of alternatives $\mathfrak{A}_\kappa$ on attributes $\tilde{h}_\varepsilon$ by DEs e_λ .

Phase 2. We normalize the evaluation matrix $R^\lambda = [\mathfrak{A}_{\kappa\varepsilon}^\lambda]_{m \times n}$ by using a formula determined by the kind of each attribute.

For benefit attributes,

$$N_{\kappa\epsilon} = \mathfrak{I}_{\kappa\epsilon} = \left((s_{p_{\kappa\epsilon}}, \wp_{\kappa\epsilon}), (s_{n_{\kappa\epsilon}}, \mathfrak{N}_{\kappa\epsilon}), (s_{l_{\kappa\epsilon}}, \mathfrak{E}_{\kappa\epsilon}) \right), \quad \kappa = 1, 2, \dots, m, \epsilon = 1, 2, \dots, n. \quad (47)$$

For cost attributes,

$$N_{\kappa\epsilon} = \mathfrak{I}_{\kappa\epsilon}^c = \left((s_{l_{\kappa\epsilon}}, \mathfrak{E}_{\kappa\epsilon}), (s_{n_{\kappa\epsilon}}, \mathfrak{N}_{\kappa\epsilon}), (s_{p_{\kappa\epsilon}}, \wp_{\kappa\epsilon}) \right), \quad \kappa = 1, 2, \dots, m, \epsilon = 1, 2, \dots, n. \quad (48)$$

Phase 3. According to the 2TL T -SFWMSM or 2TL T -SFWDMSM operator, we utilize overall $\mathfrak{I}_{\kappa\epsilon}^\lambda$ to $\mathfrak{I}_{\kappa\epsilon}$,

and the fused 2TL T -SFN matrix $R = [\mathfrak{I}_{\kappa\epsilon}]_{m \times n}$ is given as follows:

$$R = [\mathfrak{I}_{\kappa\epsilon}]_{m \times n} = \begin{bmatrix} ((s_{p_{11}}, \wp), (s_{n_{11}}, \mathfrak{N}), (s_{l_{11}}, \mathfrak{E})) & ((s_{p_{12}}, \wp), (s_{n_{12}}, \mathfrak{N}), (s_{l_{12}}, \mathfrak{E})) & \dots & ((s_{p_{1n}}, \wp), (s_{n_{1n}}, \mathfrak{N}), (s_{l_{1n}}, \mathfrak{E})) \\ ((s_{p_{21}}, \wp), (s_{n_{21}}, \mathfrak{N}), (s_{l_{21}}, \mathfrak{E})) & ((s_{p_{22}}, \wp), (s_{n_{22}}, \mathfrak{N}), (s_{l_{22}}, \mathfrak{E})) & \dots & ((s_{p_{2n}}, \wp), (s_{n_{2n}}, \mathfrak{N}), (s_{l_{2n}}, \mathfrak{E})) \\ \vdots & \vdots & \ddots & \vdots \\ ((s_{p_{m1}}, \wp), (s_{n_{m1}}, \mathfrak{N}), (s_{l_{m1}}, \mathfrak{E})) & ((s_{p_{m2}}, \wp), (s_{n_{m2}}, \mathfrak{N}), (s_{l_{m2}}, \mathfrak{E})) & \dots & ((s_{p_{mn}}, \wp), (s_{n_{mn}}, \mathfrak{N}), (s_{l_{mn}}, \mathfrak{E})) \end{bmatrix}. \quad (49)$$

where $\mathfrak{I}_{\kappa\epsilon} = ((s_{p_{\kappa\epsilon}}, \wp), (s_{n_{\kappa\epsilon}}, \mathfrak{N}), (s_{l_{\kappa\epsilon}}, \mathfrak{E}))$ ($\kappa = 1, 2, \dots, m, \epsilon = 1, 2, \dots, n$) indicates the aggregated 2TL T -SF information of alternatives $\mathfrak{I}_{\kappa}$ ($\kappa = 1, 2, \dots, m$) on attributes h_{ϵ} ($\epsilon = 1, 2, \dots, n$).

Phase 4. We determine the AS of all the alternatives under each attribute.

$$AS = (AS_1, AS_2, \dots, AS_n), \quad (50)$$

where

$$AS_{\epsilon} = ((s_{p_{\epsilon}}, \wp_{\epsilon}), (s_{n_{\epsilon}}, \mathfrak{N}_{\epsilon}), (s_{l_{\epsilon}}, \mathfrak{E}_{\epsilon})) = (1/m)\mathfrak{I}_{\epsilon 1} \oplus (1/m)\mathfrak{I}_{\epsilon 2} \oplus \dots \oplus (1/m)\mathfrak{I}_{\epsilon n} = \left(\Delta \left(\tau \sqrt[q]{1 - \prod_{\kappa=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{p_{\kappa\epsilon}}, \wp_{\kappa\epsilon})}{\tau} \right)^q} \right)^{(1/m)} \right), \Delta \left(\tau \left(\prod_{\kappa=1}^m \left(\frac{\Delta^{-1}(s_{n_{\kappa\epsilon}}, \mathfrak{N}_{\kappa\epsilon})}{\tau} \right)^{(1/m)} \right) \right), \Delta \left(\tau \left(\prod_{\kappa=1}^m \left(\frac{\Delta^{-1}(s_{l_{\kappa\epsilon}}, \mathfrak{E}_{\kappa\epsilon})}{\tau} \right)^{(1/m)} \right) \right) \right)_{1 \times n} \quad (\epsilon = 1, 2, \dots, n). \quad (51)$$

Phase 5. According to different types of attributes, we calculate the PDAS matrix and the NDAS matrix.

$$\begin{aligned} PDAS &= (P \ DA \ S_{\kappa\epsilon})_{m \times n}, \\ NDAS &= (N \ DA \ S_{\kappa\epsilon})_{m \times n}, \\ PDAS_{\kappa\epsilon} &= \frac{\max(0, (\Xi^*(\mathfrak{I}_{\kappa\epsilon}) - \Xi^*(AS_{\epsilon})))}{\Xi^*(AS_{\epsilon})}, \\ NDAS_{\kappa\epsilon} &= \frac{\max(0, (\Xi^*(AS_{\epsilon}) - \Xi^*(\mathfrak{I}_{\kappa\epsilon})))}{\Xi^*(AS_{\epsilon})}, \end{aligned} \quad (52)$$

where $\Xi^*(\mathfrak{I}_{\kappa\epsilon})$ and $\Xi^*(AS_{\epsilon})$ are the score functions of fused 2TL T -SFN matrix and AS matrix, respectively.

Phase 6. We compute the positive weighted distance SP_{κ} ($\kappa = 1, 2, \dots, m$) and the negative weighted distance SN_{κ} ($\kappa = 1, 2, \dots, m$) as follows:

$$\begin{aligned} SP_{\kappa} &= \sum_{\epsilon=1}^n \xi_{\epsilon} PDAS_{\kappa\epsilon}, \\ SN_{\kappa} &= \sum_{\epsilon=1}^n \xi_{\epsilon} NDAS_{\kappa\epsilon}, \end{aligned} \quad (53)$$

where $\xi_{\epsilon} \in [0, 1]$ and $\sum_{\epsilon=1}^n \xi_{\epsilon} = 1$.

Phase 7. We normalize the SP_{κ} ($\kappa = 1, 2, \dots, m$) and SN_{κ} ($\kappa = 1, 2, \dots, m$) by the following equation:

$$\begin{aligned} NSP_{\kappa} &= \frac{SP_{\kappa}}{\max(SP_1, SP_2, \dots, SP_m)}, \\ SN_{\kappa} &= 1 - \frac{SN_{\kappa}}{\max(SN_1, SN_2, \dots, SN_m)}, \end{aligned} \quad (54)$$

where $\max(SP_1, SP_2, \dots, SP_m)$ and $\max(SN_1, SN_2, \dots, SN_m)$ are the maximum distances.

Phase 8. We derive the integrative appraisal score (IAS_{κ}) ($\kappa = 1, 2, \dots, m$) by the following equation:

$$IAS_{\kappa} = \frac{1}{2} (NSP_{\kappa} + NSN_{\kappa}), \quad (55)$$

where $IAS_{\kappa} \in [0, 1]$.

Phase 9. We derive the ordering in accordance with the results of (IAS_{κ}) ($\kappa = 1, 2, \dots, m$). The larger the (IAS_{κ}) ($\kappa = 1, 2, \dots, m$), the better the alternative is.

6. Numerical Illustration

The most appropriate alternative is selected based on the combination of weighted attributes and the data provided by the DEs in the MAGDM environment. To validate our model, we tackle the problem of selecting the best vaccine to treat the COVID-19 outbreak.

6.1. The Problem Description. In today's environment, the COVID-19 is among the top causes of mortality other than

older diseases, which is quickly becoming a universal threat. On December 31, 2019, the main case was discovered in Wuhan, China's capital of Hubei Province. It is considered a medical emergency and characterized by fever, flu, chest pain, throat pain, migraine, and other symptoms. There is no effective and adequate vaccine for the treatment of the COVID-19. In medical research, vaccines to control the COVID-19 outbreak are inherently generic drugs to treat influenza, sore throat, a weak body's immune, fever, and so on. In medical care, medical professionals incorporate a variety of vaccines for treating the COVID-19 outbreak. For countries affected by the COVID-19 outbreak, the benefits of conducting safe and effective vaccines against the risks of increasing transmission of the COVID-19 are weighted. Deciding on whether to, and how to select and implement a mass vaccine is a complex decision-making process and is dependent on the country-specific context. Figure 2 shows the key principles for the implementation of the COVID-19 vaccine.

Following an initial assessment, let $\{\mathfrak{I}_1, \mathfrak{I}_2, \dots, \mathfrak{I}_7\}$ be a set of seven vaccines and let $\{h_1, h_2, h_3, h_4\}$ be a set of four symptoms with a weighting vector $\xi = (0.17, 0.31, 0.27, 0.25)^T$. It is supposed that seven vaccines are evaluated by four experts $E = \{e_1, e_2, e_3, e_4\}$, with a weighting vector $\varpi = (0.2, 0.4, 0.3, 0.1)^T$ for choosing the best vaccine to treat COVID-19. In order to quantify each LTS $S = \{s_0 = \text{none}, s_1 = \text{very low}, s_2 = \text{low}, s_3 = \text{medium}, s_4 = \text{high}, s_5 = \text{very high}, s_6 = \text{perfect}, s_7 = \text{slightly perfect}, \text{ and } s_8 = \text{extremely perfect}\}$, four experts E_e ($e = 1, 2, 3, 4$) provide their opinions. Based on their experience, each decision expert has an opinion on the selection of the best vaccine. These vaccines are as follows:

- (1) Pfizer (BioNTech) ($\mathfrak{I}_1$);
- (2) Moderna ($\mathfrak{I}_2$);
- (3) Sputnik V (Gamaleya) ($\mathfrak{I}_3$);
- (4) Johnson (Janssen) ($\mathfrak{I}_4$);
- (5) CoronaVac (Sinovac) ($\mathfrak{I}_5$);
- (6) Sinopharm ($\mathfrak{I}_6$);
- (7) Novavax ($\mathfrak{I}_7$).

The seven described vaccines are evaluated according to four symptoms, including the following:

- (1) Fever (h_1);
- (2) Influenza (h_2);
- (3) Throat pain (h_3);
- (4) Cough (h_4).

In order to avoid the risk of mistreatment and over-diagnosis in the treatment of COVID-19 disease, experts should evaluate the effective qualities of vaccines concerning all symptoms in conjunction with their interaction in the vaccine center and identify the most suitable vaccine to cure the symptoms, according to the guidelines. Each decision expert uses the 2TL T -SFNs to assess each vaccine's ability to control the effect of COVID-19 for each symptom. Following the experts' recommendations, the 2TL T -SFNs for the selection of the best vaccine are recorded in Table 2.

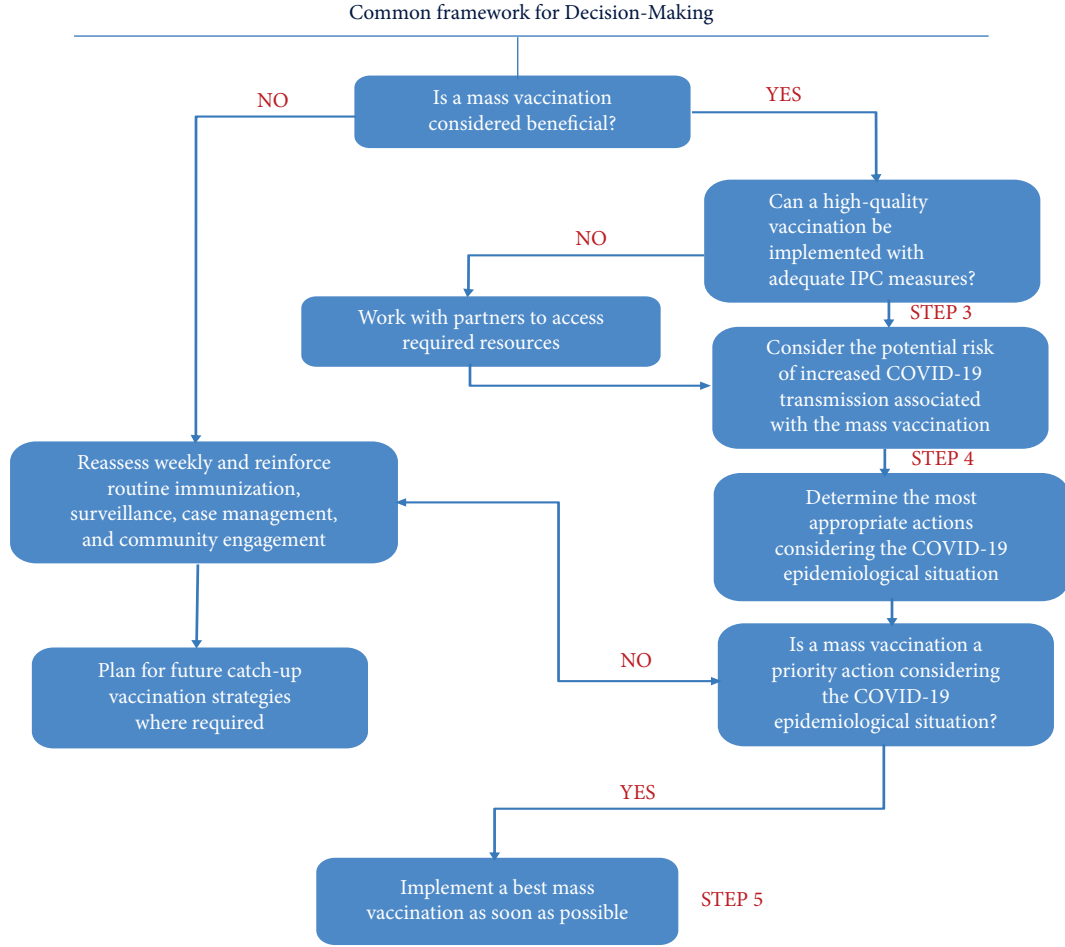


FIGURE 2: The graphical interpretation for the implementation of the COVID-19 vaccine.

TABLE 2: The assessing matrix with 2TL T -SFNs by four experts.

Decision experts	Alternatives	Attributes			
		$\tilde{h}_1$	$\tilde{h}_2$	$\tilde{h}_3$	$\tilde{h}_4$
e_1	λ_1	$((s_6, 0), t(s_3, 0)n, q(s_2, 0))$	$((s_5, 0), t(s_2, 0)n, q(s_4, 0))$	$((s_7, 0), t(s_2, 0)n, q(s_1, 0))$	$((s_3, 0), t(s_3, 0)n, q(s_3, 0))$
	λ_2	$((s_2, 0), t(s_3, 0)n, q(s_6, 0))$	$((s_4, 0), t(s_2, 0)n, q(s_5, 0))$	$((s_4, 0), t(s_4, 0)n, q(s_3, 0))$	$((s_7, 0), t(s_2, 0)n, q(s_1, 0))$
	λ_3	$((s_5, 0), t(s_2, 0)n, q(s_4, 0))$	$((s_6, 0), t(s_0, 0)n, q(s_2, 0))$	$((s_2, 0), t(s_4, 0)n, q(s_6, 0))$	$((s_2, 0), t(s_1, 0)n, q(s_6, 0))$
	λ_4	$((s_4, 0), t(s_4, 0)n, q(s_3, 0))$	$((s_3, 0), t(s_4, 0)n, q(s_4, 0))$	$((s_3, 0), t(s_3, 0)n, q(s_2, 0))$	$((s_2, 0), t(s_3, 0)n, q(s_3, 0))$
	λ_5	$((s_1, 0), t(s_0, 0)n, q(s_7, 0))$	$((s_2, 0), t(s_4, 0)n, q(s_5, 0))$	$((s_3, 0), t(s_3, 0)n, q(s_3, 0))$	$((s_2, 0), t(s_3, 0)n, q(s_6, 0))$
	λ_6	$((s_2, 0), t(s_1, 0)n, q(s_6, 0))$	$((s_3, 0), t(s_4, 0)n, q(s_5, 0))$	$((s_5, 0), t(s_0, 0)n, q(s_5, 0))$	$((s_2, 0), t(s_0, 0)n, q(s_6, 0))$
	λ_7	$((s_3, 0), t(s_2, 0)n, q(s_5, 0))$	$((s_2, 0), t(s_4, 0)n, q(s_6, 0))$	$((s_2, 0), t(s_4, 0)n, q(s_6, 0))$	$((s_0, 0), t(s_4, 0)n, q(s_6, 0))$
e_2	λ_1	$((s_6, 0), t(s_4, 0)n, q(s_1, 0))$	$((s_4, 0), t(s_5, 0)n, q(s_0, 0))$	$((s_2, 0), t(s_4, 0)n, q(s_6, 0))$	$((s_7, 0), t(s_2, 0)n, q(s_2, 0))$
	λ_2	$((s_3, 0), t(s_4, 0)n, q(s_5, 0))$	$((s_3, 0), t(s_3, 0)n, q(s_3, 0))$	$((s_4, 0), t(s_4, 0)n, q(s_5, 0))$	$((s_5, 0), t(s_4, 0)n, q(s_4, 0))$
	λ_3	$((s_7, 0), t(s_2, 0)n, q(s_0, 0))$	$((s_2, 0), t(s_3, 0)n, q(s_3, 0))$	$((s_4, 0), t(s_2, 0)n, q(s_5, 0))$	$((s_4, 0), t(s_4, 0)n, q(s_3, 0))$
	λ_4	$((s_3, 0), t(s_3, 0)n, q(s_4, 0))$	$((s_5, 0), t(s_3, 0)n, q(s_3, 0))$	$((s_2, 0), t(s_2, 0)n, q(s_2, 0))$	$((s_3, 0), t(s_1, 0)n, q(s_1, 0))$
	λ_5	$((s_1, 0), t(s_1, 0)n, q(s_6, 0))$	$((s_3, 0), t(s_4, 0)n, q(s_4, 0))$	$((s_5, 0), t(s_2, 0)n, q(s_4, 0))$	$((s_4, 0), t(s_2, 0)n, q(s_3, 0))$
	λ_6	$((s_0, 0), t(s_3, 0)n, q(s_7, 0))$	$((s_2, 0), t(s_0, 0)n, q(s_6, 0))$	$((s_5, 0), t(s_0, 0)n, q(s_4, 0))$	$((s_3, 0), t(s_4, 0)n, q(s_5, 0))$
	λ_7	$((s_1, 0), t(s_3, 0)n, q(s_5, 0))$	$((s_2, 0), t(s_4, 0)n, q(s_6, 0))$	$((s_0, 0), t(s_4, 0)n, q(s_6, 0))$	$((s_5, 0), t(s_1, 0)n, q(s_3, 0))$
e_3	λ_1	$((s_3, 0), t(s_4, 0)n, q(s_5, 0))$	$((s_6, 0), t(s_3, 0)n, q(s_1, 0))$	$((s_2, 0), t(s_2, 0)n, q(s_5, 0))$	$((s_7, 0), t(s_2, 0)n, q(s_2, 0))$
	λ_2	$((s_4, 0), t(s_2, 0)n, q(s_4, 0))$	$((s_7, 0), t(s_2, 0)n, q(s_3, 0))$	$((s_3, 0), t(s_3, 0)n, q(s_3, 0))$	$((s_5, 0), t(s_2, 0)n, q(s_3, 0))$
	λ_3	$((s_7, 0), t(s_3, 0)n, q(s_0, 0))$	$((s_6, 0), t(s_4, 0)n, q(s_1, 0))$	$((s_0, 0), t(s_5, 0)n, q(s_5, 0))$	$((s_2, 0), t(s_4, 0)n, q(s_5, 0))$
	λ_4	$((s_6, 0), t(s_5, 0)n, q(s_0, 0))$	$((s_5, 0), t(s_3, 0)n, q(s_2, 0))$	$((s_3, 0), t(s_2, 0)n, q(s_2, 0))$	$((s_5, 0), t(s_5, 0)n, q(s_1, 0))$
	λ_5	$((s_2, 0), t(s_3, 0)n, q(s_3, 0))$	$((s_1, 0), t(s_7, 0)n, q(s_2, 0))$	$((s_2, 0), t(s_4, 0)n, q(s_2, 0))$	$((s_6, 0), t(s_2, 0)n, q(s_2, 0))$
	λ_6	$((s_3, 0), t(s_0, 0)n, q(s_7, 0))$	$((s_3, 0), t(s_5, 0)n, q(s_4, 0))$	$((s_3, 0), t(s_0, 0)n, q(s_3, 0))$	$((s_5, 0), t(s_1, 0)n, q(s_6, 0))$
	λ_7	$((s_0, 0), t(s_4, 0)n, q(s_6, 0))$	$((s_1, 0), t(s_3, 0)n, q(s_5, 0))$	$((s_2, 0), t(s_4, 0)n, q(s_6, 0))$	$((s_4, 0), t(s_4, 0)n, q(s_4, 0))$

TABLE 2: Continued.

Decision experts	Alternatives	Attributes			
		$\hat{h}_1$	$\hat{h}_2$	$\hat{h}_3$	$\hat{h}_4$
e_4	λ_1	$((s_2, 0), t(s_6, 0)n, q(s_1, 0))$	$((s_7, 0), t(s_3, 0)n, q(s_1, 0))$	$((s_4, 0), t(s_4, 0)n, q(s_4, 0))$	$((s_3, 0), t(s_4, 0)n, q(s_4, 0))$
	λ_2	$((s_1, 0), t(s_1, 0)n, q(s_7, 0))$	$((s_2, 0), t(s_2, 0)n, q(s_6, 0))$	$((s_0, 0), t(s_0, 0)n, q(s_6, 0))$	$((s_3, 0), t(s_2, 0)n, q(s_4, 0))$
	λ_3	$((s_2, 0), t(s_2, 0)n, q(s_5, 0))$	$((s_6, 0), t(s_3, 0)n, q(s_2, 0))$	$((s_4, 0), t(s_6, 0)n, q(s_1, 0))$	$((s_3, 0), t(s_3, 0)n, q(s_3, 0))$
	λ_4	$((s_0, 0), t(s_1, 0)n, q(s_7, 0))$	$((s_2, 0), t(s_2, 0)n, q(s_4, 0))$	$((s_4, 0), t(s_4, 0)n, q(s_3, 0))$	$((s_7, 0), t(s_2, 0)n, q(s_1, 0))$
	λ_5	$((s_6, 0), t(s_3, 0)n, q(s_3, 0))$	$((s_4, 0), t(s_4, 0)n, q(s_4, 0))$	$((s_3, 0), t(s_3, 0)n, q(s_4, 0))$	$((s_5, 0), t(s_1, 0)n, q(s_4, 0))$
	λ_6	$((s_1, 0), t(s_3, 0)n, q(s_7, 0))$	$((s_3, 0), t(s_0, 0)n, q(s_3, 0))$	$((s_3, 0), t(s_2, 0)n, q(s_7, 0))$	$((s_5, 0), t(s_2, 0)n, q(s_5, 0))$
	λ_7	$((s_2, 0), t(s_4, 0)n, q(s_6, 0))$	$((s_3, 0), t(s_2, 0)n, q(s_5, 0))$	$((s_1, 0), t(s_3, 0)n, q(s_5, 0))$	$((s_3, 0), t(s_0, 0)n, q(s_7, 0))$

6.2. The Outcomes of a Case Study

6.2.1. Decision-Making Procedure Based on the 2TL T-SFWMSM Operator

Phase 1. According to the 2TL T-SFWMSM aggregation operator, we utilize overall $\lambda_{\kappa\epsilon}^1$ to $\lambda_{\kappa\epsilon}$, and the fused 2TL T-SFN matrix $R = [\lambda_{\kappa\epsilon}]_{m \times n}$ is shown in Table 3. (Suppose $q = 4$, $\tau = 8$, $\bar{\omega} = (0.2, 0.4, 0.3, 0.1)^T$).

Phase 2. We determine the scores of fused 2TL T-SFN matrix (see Table 4).

Phase 3. We compute the AS matrix $AS = [AS_\epsilon]_{4 \times 3}$ by utilizing the attribute weights ($\xi = (0.17, 0.31, 0.27, 0.25)^T$). The results of AS matrix $AS = [AS_\epsilon]_{4 \times 3}$ are shown in Table 5.

Phase 4. According to the scores of fused 2TL T-SFN matrix and AS matrix, we calculate the PDAS and NDAS. The results of PDAS = $[PDAS_\kappa]_{7 \times 4}$ and the results of NDAS = $[NDAS_\kappa]_{7 \times 4}$ are shown in Table 6.

Phase 5. According to the outcomes of PDAS and NDAS, we calculate the assessing values of the SP_κ ($\kappa = 1, 2, \dots, 7$) and the SN_κ ($\kappa = 1, 2, \dots, 7$). The results of SP and SN are shown in Table 7.

Phase 6. According to the outcomes of SP and SN, we calculate the NSP_κ ($\kappa = 1, 2, \dots, 7$) and the NSN_κ ($\kappa = 1, 2, \dots, 7$). The results of NSP and NSN are shown in Table 7.

Phase 7. We derive the integrative appraisal score (IAS_κ) ($\kappa = 1, 2, \dots, 7$) by the following equation:

$$IAS_\kappa = \frac{1}{2} (NSP_\kappa + NSN_\kappa), \quad (56)$$

where $IAS_\kappa \in [0, 1]$. The computed results are shown below as follows:

$$\begin{aligned} IAS_1 &= 1.0000, \\ IAS_2 &= 0.5741, \\ IAS_3 &= 0.7111, \\ IAS_4 &= 0.5314, \\ IAS_5 &= 0.4030, \\ IAS_6 &= 0.4009, \\ IAS_7 &= 0.0000. \end{aligned} \quad (57)$$

Phase 8. We derive the ordering in accordance with the results of (IAS_κ) ($\kappa = 1, 2, \dots, 7$). The larger the IAS_κ ($\kappa = 1, 2, \dots, 7$), the better the vaccine is. The

ranking of vaccines is as follows: $\lambda_1 > \lambda_3 > \lambda_2 > \lambda_4 > \lambda_5 > \lambda_6 > \lambda_7$. So, λ_1 is the best vaccine.

6.2.2. Decision-Making Procedure Based on the 2TL T-SFWDMSM Operator

Phase 1. According to the 2TL T-SFWDMSM aggregation operator, we utilize overall $\lambda_{\kappa\epsilon}^1$ to $\lambda_{\kappa\epsilon}$, and the fused 2TL T-SFN matrix $R = [\lambda_{\kappa\epsilon}]_{m \times n}$ is shown in Table 8. (Suppose $q = 4$, $\tau = 8$, $\bar{\omega} = (0.2, 0.4, 0.3, 0.1)^T$).

Phase 2. We determine the scores of fused 2TL T-SFN matrix (see Table 9).

Phase 3. We compute the AS matrix $AS = [AS_\epsilon]_{4 \times 3}$ by utilizing the attribute weights ($\xi = (0.17, 0.31, 0.27, 0.25)^T$). The results of AS matrix $AS = [AS_\epsilon]_{4 \times 3}$ are shown in Table 10.

Phase 4. According to the scores of the fused 2TL T-SFN matrix and AS matrix, we calculate the PDAS and NDAS. The results of $PDA S = [PDA S_\kappa]_{7 \times 4}$ and the results of $NDAS = [NDAS_\kappa]_{7 \times 4}$ are shown in Table 11.

Phase 5. According to the outcomes of PDAS and NDAS, we calculate the assessing values of the SP_κ ($\kappa = 1, 2, \dots, 7$) and the SN_κ ($\kappa = 1, 2, \dots, 7$). The results of SP and SN are shown in Table 12.

Phase 6. According to the outcomes of SP and SN, we calculate the NSP_κ ($\kappa = 1, 2, \dots, 7$) and the NSN_κ ($\kappa = 1, 2, \dots, 7$). The results of NSP and NSN are shown in Table 12.

Phase 7. We derive the integrative appraisal score (IAS_κ) ($\kappa = 1, 2, \dots, 7$) by the following equation:

$$IAS_\kappa = \frac{1}{2} (NSP_\kappa + NSN_\kappa), \quad (58)$$

where $IAS_\kappa \in [0, 1]$. The computed results are shown below as follows:

$$\begin{aligned} IAS_1 &= 1.0000, \\ IAS_2 &= 0.5851, \\ IAS_3 &= 0.8080, \\ IAS_4 &= 0.9596, \\ IAS_5 &= 0.5792, \\ IAS_6 &= 0.0831, \\ IAS_7 &= 0.0000. \end{aligned} \quad (59)$$

TABLE 3: Collective 2TL T -SF assessing matrix by the 2TL T -SFWMMSM operator.

Alternatives	Collective assessment matrix			
	h_1	h_2	h_3	h_4
α_1	$((s_7, -0.0761), t(s_3, 0.0518)n, q(s_2, 0.2320))$	$((s_7, 0.1833), t(s_3, -0.2487)n, q(s_2, -0.4900))$	$((s_6, 0.3324), t(s_2, 0.3522)n, q(s_4, -0.3168))$	$((s_7, 0.2911), t(s_2, -0.0960)n, q(s_2, -0.0960))$
α_2	$((s_6, 0.1245), t(s_2, 0.2531)n, q(s_4, -0.0409))$	$((s_7, -0.1981), t(s_2, -0.2600)n, q(s_3, -0.0232))$	$((s_6, -0.3646), t(s_3, -0.4341)n, q(s_3, 0.2332))$	$((s_7, 0.1639), t(s_2, 0.1178)n, q(s_2, 0.4489))$
α_3	$((s_7, 0.3860), t(s_2, -0.3083)n, q(s_2, 0.3048))$	$((s_7, 0.0061), t(s_2, 0.2932)n, q(s_3, -0.2147))$	$((s_6, -0.1955), t(s_3, 0.1615)n, q(s_4, -0.2963))$	$((s_6, 0.1761), t(s_3, -0.4020)n, q(s_3, 0.3747))$
α_4	$((s_6, -0.1823), t(s_3, -0.1614)n, q(s_3, 0.1978))$	$((s_7, -0.2197), t(s_2, 0.2828)n, q(s_2, 0.3112))$	$((s_6, 0.1997), t(s_3, -0.0960)n, q(s_2, -0.4607))$	$((s_7, -0.3958), t(s_2, 0.4894)n, q(s_1, 0.2728))$
α_5	$((s_6, -0.4888), t(s_2, -0.3559)n, q(s_4, 0.2018))$	$((s_6, -0.1580), t(s_4, -0.0707)n, q(s_3, -0.1596))$	$((s_6, 0.4618), t(s_2, 0.2301)n, q(s_2, 0.4307))$	$((s_7, -0.2076), t(s_2, -0.3987)n, q(s_3, -0.0498))$
α_6	$((s_6, -0.4958), t(s_2, -0.2833)n, q(s_5, 0.1603))$	$((s_6, 0.1446), t(s_3, -0.2917)n, q(s_4, -0.3244))$	$((s_7, -0.2038), t(s_6, 0.0000)n, q(s_3, 0.4821))$	$((s_7, -0.4635), t(s_2, -0.1034)n, q(s_4, 0.0388))$
α_7	$((s_5, -0.1675), t(s_2, 0.4122)n, q(s_4, -0.0423))$	$((s_6, -0.4310), t(s_3, -0.4273)n, q(s_4, 0.0966))$	$((s_5, 0.2826), t(s_3, -0.2179)n, q(s_4, 0.3097))$	$((s_6, 0.0729), t(s_2, 0.3561)n, q(s_3, -0.3087))$

TABLE 4: Scores of the fused 2TL T -SFN matrix.

Alternatives	Scores			
	$\bar{h}_1$	$\bar{h}_2$	$\bar{h}_3$	$\bar{h}_4$
$\bar{\lambda}_1$	0.7775	0.8244	0.6738	0.8434
$\bar{\lambda}_2$	0.6418	0.7517	0.6098	0.8171
$\bar{\lambda}_3$	0.8598	0.7868	0.6156	0.6618
$\bar{\lambda}_4$	0.6271	0.7545	0.6797	0.7319
$\bar{\lambda}_5$	0.5746	0.6342	0.7085	0.7505
$\bar{\lambda}_6$	0.5255	0.6517	0.7424	0.6904
$\bar{\lambda}_7$	0.5366	0.5830	0.5529	0.6434

TABLE 5: The assessing AS matrix with 2TL T -SFNs.

AS'_s	The 2TL T -SFNs	Scores
AS_1	$((s_6, 0.3432), t(s_2, 0.1672)n, q(s_3, 0.4304))$	0.6807
AS_2	$((s_7, -0.3792), t(s_3, -0.4576)n, q(s_3, -0.2344))$	0.7274
AS_3	$((s_6, 0.1688), t(s_0, 0.0000)n, q(s_3, 0.0536))$	0.6662
AS_4	$((s_7, -0.2400), t(s_2, 0.1104)n, q(s_3, -0.3656))$	0.7490

TABLE 6: Assessing values of PDAS and NDAS.

Alternatives	PDAS				NDAS			
	$\bar{h}_1$	$\bar{h}_2$	$\bar{h}_3$	$\bar{h}_4$	$\bar{h}_1$	$\bar{h}_2$	$\bar{h}_3$	$\bar{h}_4$
$\bar{\lambda}_1$	0.1423	0.1333	0.0114	0.1260	0.0000	0.000	0.000	0.000
$\bar{\lambda}_2$	0.000	0.0334	0.0000	0.910	0.572	0.000	0.0847	0.0000
$\bar{\lambda}_3$	0.2631	0.0817	0.000	0.000	0.000	0.000	0.0759	0.1165
$\bar{\lambda}_4$	0.000	0.0372	0.0202	0.000	0.0788	0.000	0.000	0.0229
$\bar{\lambda}_5$	0.000	0.000	0.0636	0.0020	0.1559	0.1281	0.000	0.0782
$\bar{\lambda}_6$	0.000	0.000	0.1144	0.0000	0.2280	0.1040	0.0000	0.0782
$\bar{\lambda}_7$	0.0000	0.0000	0.0000	0.0000	0.2116	0.1985	0.1700	0.1410

TABLE 7: The assessing values of SP_κ , SN_κ , NSP_κ , and NSN_κ ($\kappa = 1, 2, \dots, 7$).

Alternatives	The outcomes of SP	The outcomes of SN	The outcomes of NSP	The outcomes of NSN
$\bar{\lambda}_1$	0.1001	0.0000	1.0000	1.0000
$\bar{\lambda}_2$	0.0331	0.0326	0.3306	0.8176
$\bar{\lambda}_3$	0.0700	0.0496	0.998	0.7223
$\bar{\lambda}_4$	0.0170	0.0191	0.1699	0.2930
$\bar{\lambda}_5$	0.0177	0.0662	0.1766	0.6294
$\bar{\lambda}_6$	0.0309	0.0906	0.3087	0.4932
$\bar{\lambda}_7$	0.0000	0.1787	0.0000	0.0000

Phase 8. We derive the ordering in accordance with the results of $(IAS_\kappa)(\kappa = 1, 2, \dots, 7)$. The larger the $IAS_\kappa(\kappa = 1, 2, \dots, 7)$, the better the vaccine is. The ranking of vaccines is as follows: $\bar{\lambda}_1 > \bar{\lambda}_4 > \bar{\lambda}_3 > \bar{\lambda}_2 > \bar{\lambda}_5 > \bar{\lambda}_6 > \bar{\lambda}_7$. So, $\bar{\lambda}_1$ is the best vaccine.

6.3. Effects of Parameters t and q on the Ranking Outcomes.

When aggregating data, it should be observed that the parameters t and q of the 2TL T -SFWMSM (2TL T -SFWDMSM) operator serve as an essential part in determining the outcome. As an initial step, in order to investigate the impact of the parameter t on the aggregation results, we vary the value of the parameter t in phase 3 of the developed MAGDM approach. The desirable outcomes of vaccine are depicted in Tables 13, 14 and Figure 3 (suppose

$q = 4$). From Table 13, we can also see that the vaccines are ranked in the order of importance and changes from $\bar{\lambda}_1 > \bar{\lambda}_3 > \bar{\lambda}_4 > \bar{\lambda}_2 > \bar{\lambda}_5 > \bar{\lambda}_6 > \bar{\lambda}_7$ to $\bar{\lambda}_1 > \bar{\lambda}_3 > \bar{\lambda}_2 > \bar{\lambda}_4 > \bar{\lambda}_5 > \bar{\lambda}_6 > \bar{\lambda}_7$, as parameter t take different values ($t = 1, 2, 3, 4$) by utilizing the 2TL T -SFWMSM operator. From Table 14, we can also see that the vaccines are ranked in the order of importance and changes from $\bar{\lambda}_1 > \bar{\lambda}_4 > \bar{\lambda}_3 > \bar{\lambda}_5 > \bar{\lambda}_2 > \bar{\lambda}_6 > \bar{\lambda}_7$ to $\bar{\lambda}_1 > \bar{\lambda}_4 > \bar{\lambda}_3 > \bar{\lambda}_2 > \bar{\lambda}_5 > \bar{\lambda}_6 > \bar{\lambda}_7$, as parameter t take different values ($t = 1, 2, 3, 4$) by utilizing the 2TL T -SFWDMSM operator. That is, parameter t indicates the degree of interrelations among symptoms due to the alternate order of vaccines. The interrelationship pattern of symptoms changes by using the 2TL T -SFWMSM (2TL T -SFWDMSM) operator, as the decision expert chooses alternate values of the t parameter. Thus, the ranking orders of vaccines differ from one another in terms of importance.

TABLE 8: Collective 2TL T -SF assessing matrix by the 2TL T -SFWDMSM operator.

Alternatives	Collective assessment matrix			
	h_1	h_2	h_3	h_4
α_1	$((s_4, -0.1719), t(s_7, -0.2636)m, q(s_6, -0.0947))$	$((s_4, -0.0083), t(s_6, 0.4832)m, q(s_5, 0.1608))$	$((s_3, 0.2630), t(s_6, 0.2296)m, q(s_7, -0.2404))$	$((s_5, -0.1553), t(s_6, 0.0097)m, q(s_6, 0.0097))$
α_2	$((s_2, 0.2612), t(s_6, 0.1158)m, q(s_7, 0.1613))$	$((s_4, -0.2933), t(s_6, -0.0648)m, q(s_6, 0.4646))$	$((s_3, -0.4341), t(s_6, -0.3646)m, q(s_7, -0.2536))$	$((s_4, 0.0709), t(s_6, 0.1029)m, q(s_6, 0.2028))$
α_3	$((s_5, -0.0928), t(s_6, -0.0701)m, q(s_0, 0.0000))$	$((s_4, -0.1434), t(s_6, -0.3419)m, q(s_6, 0.1958))$	$((s_2, 0.3188), t(s_7, -0.3132)m, q(s_7, -0.0875))$	$((s_2, 0.2038), t(s_6, 0.2907)m, q(s_7, -0.1580))$
α_4	$((s_3, -0.2272), t(s_6, 0.4765)m, q(s_6, 0.0884))$	$((s_3, 0.2726), t(s_6, 0.3402)m, q(s_6, 0.3001))$	$((s_2, 0.0659), t(s_6, 0.0097)m, q(s_6, -0.1820))$	$((s_3, 0.4301), t(s_6, 0.1993)m, q(s_5, 0.2619))$
α_5	$((s_2, 0.1803), t(s_5, -0.2596)m, q(s_7, 0.0667))$	$((s_2, -0.0354), t(s_7, 0.0721)m, q(s_7, -0.4446))$	$((s_3, -0.2909), t(s_6, 0.2962)m, q(s_6, 0.3656))$	$((s_3, 0.4457), t(s_6, -0.2383)m, q(s_6, 0.4556))$
α_6	$((s_2, -0.4846), t(s_5, 0.2733)m, q(s_8, -0.3184))$	$((s_2, -0.0786), t(s_6, 0.0000)m, q(s_7, 0.0476))$	$((s_3, 0.1748), t(s_6, 0.0000)m, q(s_7, -0.1786))$	$((s_3, -0.1038), t(s_5, -0.0422)m, q(s_7, 0.3029))$
α_7	$((s_1, -0.3596), t(s_6, 0.3729)m, q(s_7, 0.2685))$	$((s_1, 0.4492), t(s_6, 0.4957)m, q(s_7, 0.3218))$	$((s_1, 0.1740), t(s_7, -0.2970)m, q(s_7, 0.4170))$	$((s_3, -0.0336), t(s_5, -0.1865)m, q(s_7, -0.1075))$

TABLE 9: Scores of the fused 2TL T -SFN matrix.

Alternatives	Scores			
	$\bar{h}_1$	$\bar{h}_2$	$\bar{h}_3$	$\bar{h}_4$
$\bar{\lambda}_1$	0.3777	0.4444	0.2589	0.4080
$\bar{\lambda}_2$	0.1821	0.3098	0.2524	0.3528
$\bar{\lambda}_3$	0.5708	0.3471	0.2248	0.2353
$\bar{\lambda}_4$	0.3455	0.3217	0.3623	0.4233
$\bar{\lambda}_5$	0.1984	0.2764	0.3061	0.3051
$\bar{\lambda}_6$	0.0756	0.2006	0.2481	0.1613
$\bar{\lambda}_7$	0.1596	0.1498	0.1308	0.2339

TABLE 10: The assessing AS matrix with 2TL T -SFNs.

AS'_s	The 2TL T -SFNs for AS'_s	Scores
AS_1	$((s_3, -0.4968), t(s_6, 0.1024)n, q(s_7, -0.0752))$	0.2241
AS_2	$((s_3, -0.3088), t(s_6, 0.2376)n, q(s_7, -0.3856))$	0.2728
AS_3	$((s_2, 0.3576), t(s_6, 0.1288)n, q(s_7, -0.2000))$	0.2428
AS_4	$((s_3, 0.3168), t(s_6, -0.1528)n, q(s_7, -0.4032))$	0.2836

TABLE 11: The assessing values of PDAS and NDAS.

Alternatives	PDAS				NDAS			
	$\bar{h}_1$	$\bar{h}_2$	$\bar{h}_3$	$\bar{h}_4$	$\bar{h}_1$	$\bar{h}_2$	$\bar{h}_3$	$\bar{h}_4$
$\bar{\lambda}_1$	0.6856	0.6293	0.0668	0.4386	0.0000	0.000	0.000	0.000
$\bar{\lambda}_2$	0.0000	0.1360	0.0399	0.2439	0.1872	0.000	0.0847	0.0000
$\bar{\lambda}_3$	1.5469	0.2726	0.000	0.000	0.000	0.0000	0.0739	0.1702
$\bar{\lambda}_4$	0.5417	0.1794	0.4927	0.4925	0.0000	0.000	0.000	0.0000
$\bar{\lambda}_5$	0.000	0.0133	0.2611	0.0760	0.1550	0.000	0.000	0.000
$\bar{\lambda}_6$	0.000	0.000	0.0220	0.000	0.6626	0.2648	0.0000	0.4310
$\bar{\lambda}_7$	0.0000	0.0000	0.0000	0.0000	0.2874	0.4511	0.4611	0.1752

TABLE 12: The assessing values of SP_κ , SN_κ , NSP_κ , and NSN_κ ($\kappa = 1, 2, \dots, 7$).

Alternatives	The outcomes of SP	The outcomes of SN	The outcomes of NSP	The outcomes of NSN
$\bar{\lambda}_1$	0.4393	0.0000	1.0000	1.0000
$\bar{\lambda}_2$	0.1139	0.0318	0.2593	0.9108
$\bar{\lambda}_3$	0.3475	0.0625	0.7910	0.8250
$\bar{\lambda}_4$	0.4038	0.0000	0.9193	1.0000
$\bar{\lambda}_5$	0.0936	0.0196	0.2131	0.9452
$\bar{\lambda}_6$	0.0059	0.3025	0.0135	0.1526
$\bar{\lambda}_7$	0.0000	0.3570	0.0000	0.0000

TABLE 13: Score functions and ranking results by 2TL T -SFWMSM operator ($q = 4$ and $t = 1, 2, 3, 4$).

Parameter	$\Xi^*(\bar{\lambda}_1)$	$\Xi^*(\bar{\lambda}_2)$	$\Xi^*(\bar{\lambda}_3)$	$\Xi^*(\bar{\lambda}_4)$	$\Xi^*(\bar{\lambda}_5)$	$\Xi^*(\bar{\lambda}_6)$	$\Xi^*(\bar{\lambda}_7)$	Ranking
$t = 1$	1.0000	0.4652	0.6640	0.5080	0.4048	0.2954	0.0000	$\bar{\lambda}_1 > \bar{\lambda}_3 > \bar{\lambda}_4 > \bar{\lambda}_2 > \bar{\lambda}_5 > \bar{\lambda}_6 > \bar{\lambda}_7$
$t = 2$	1.0000	0.5239	0.6902	0.4883	0.3692	0.3471	0.0000	$\bar{\lambda}_1 > \bar{\lambda}_3 > \bar{\lambda}_2 > \bar{\lambda}_4 > \bar{\lambda}_5 > \bar{\lambda}_6 > \bar{\lambda}_7$
$t = 3$	1.0000	0.5741	0.7110	0.5315	0.4031	0.4010	0.0000	$\bar{\lambda}_1 > \bar{\lambda}_3 > \bar{\lambda}_2 > \bar{\lambda}_4 > \bar{\lambda}_5 > \bar{\lambda}_6 > \bar{\lambda}_7$
$t = 4$	1.0000	0.5531	0.5835	0.5751	0.5386	0.4918	0.0000	$\bar{\lambda}_1 > \bar{\lambda}_3 > \bar{\lambda}_4 > \bar{\lambda}_2 > \bar{\lambda}_5 > \bar{\lambda}_6 > \bar{\lambda}_7$

TABLE 14: Score functions and ranking results by 2TL T -SFWDMSM operator ($q = 4$ and $t = 1, 2, 3, 4$).

Parameter	$\Xi^*(\bar{\lambda}_1)$	$\Xi^*(\bar{\lambda}_2)$	$\Xi^*(\bar{\lambda}_3)$	$\Xi^*(\bar{\lambda}_4)$	$\Xi^*(\bar{\lambda}_5)$	$\Xi^*(\bar{\lambda}_6)$	$\Xi^*(\bar{\lambda}_7)$	Ranking
$t = 1$	1.0000	0.4627	0.6982	0.9287	0.5844	0.0755	0.0000	$\bar{\lambda}_1 > \bar{\lambda}_4 > \bar{\lambda}_3 > \bar{\lambda}_5 > \bar{\lambda}_2 > \bar{\lambda}_6 > \bar{\lambda}_7$
$t = 2$	1.0000	0.5815	0.7269	0.9915	0.5800	0.0901	0.0000	$\bar{\lambda}_1 > \bar{\lambda}_4 > \bar{\lambda}_3 > \bar{\lambda}_2 > \bar{\lambda}_5 > \bar{\lambda}_6 > \bar{\lambda}_7$
$t = 3$	1.0000	0.5851	0.8080	0.9596	0.5792	0.0831	0.0000	$\bar{\lambda}_1 > \bar{\lambda}_4 > \bar{\lambda}_3 > \bar{\lambda}_2 > \bar{\lambda}_5 > \bar{\lambda}_6 > \bar{\lambda}_7$
$t = 4$	1.0000	0.5562	0.7178	0.9672	0.5565	0.0743	0.0000	$\bar{\lambda}_1 > \bar{\lambda}_4 > \bar{\lambda}_3 > \bar{\lambda}_5 > \bar{\lambda}_2 > \bar{\lambda}_6 > \bar{\lambda}_7$

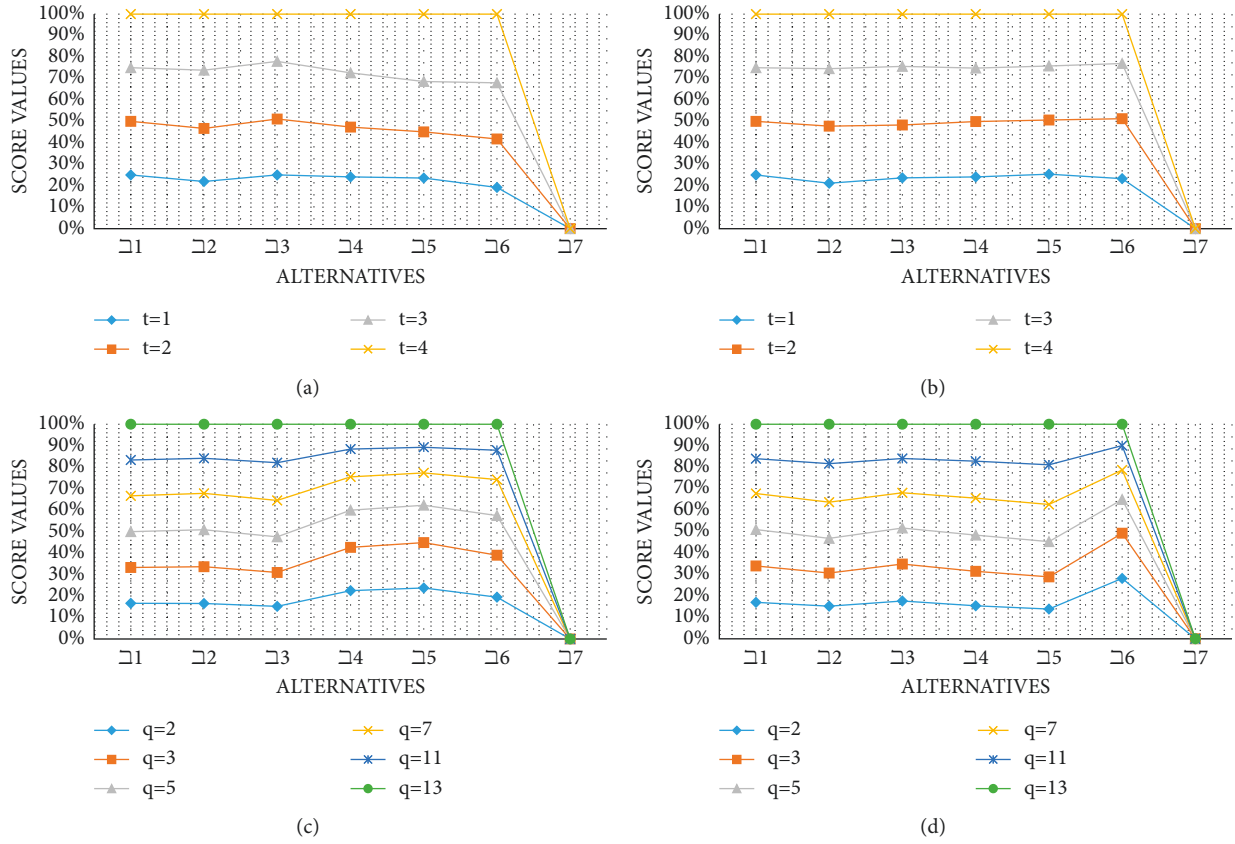


FIGURE 3: Scores of seven vaccines based on the 2TL T -SFWSM (2TL T -SFWDMSM) operator. (a) Scores of vaccines λ_κ ($\kappa = 1, 2, \dots, 7$) when $q = 4$ and $t \in (1, 4)$ based on the 2TL T -SFWSM operator. (b) Scores of vaccines λ_κ ($\kappa = 1, 2, \dots, 7$) when $q = 4$ and $t \in (1, 4)$ based on the 2TL T -SFWDMSM operator. (c) Scores of vaccines λ_κ ($\kappa = 1, 2, \dots, 7$) when $t = 3$ and $q \in (2, 3, 5, 7, 11, 13)$ based on the 2TL T -SFWSM operator. (d) Scores of vaccines λ_κ ($\kappa = 1, 2, \dots, 7$) when $t = 3$ and $q \in (2, 3, 5, 7, 11, 13)$ based on the 2TL T -SFWDMSM operator.

We examine the impact of the parameter q on the aggregated outcome by experimenting with various values of the parameter q in phase 4 of the developed MAGDM approach. The desirable outcomes of vaccines are depicted in Tables 15, 16 and Figure 3 (Suppose $t = 3$). We can also see that the vaccines are ranked in the order of importance as $\lambda_1 > \lambda_3 > \lambda_4 > \lambda_2 > \lambda_5 > \lambda_6 > \lambda_7$, when $q \in [2, 3]$, and the vaccines are ranked as $\lambda_1 > \lambda_3 > \lambda_2 > \lambda_4 > \lambda_6 > \lambda_5 > \lambda_7$, when $q \in [5, 7, 11, 13]$ by utilizing the 2TL T -SFWSM operator. We can also see that the vaccines are ranked in the order of importance as $\lambda_1 > \lambda_4 > \lambda_3 > \lambda_2 > \lambda_5 > \lambda_6 > \lambda_7$, when $q \in [2, 3]$, the vaccines are ranked as $\lambda_1 > \lambda_4 > \lambda_3 > \lambda_5 > \lambda_2 > \lambda_6 > \lambda_7$, when $q = 5$, and the vaccines are ranked as $\lambda_4 > \lambda_1 > \lambda_3 > \lambda_5 > \lambda_2 > \lambda_6 > \lambda_7$, when $q \in [7, 11, 13]$ by utilizing the 2TL T -SFWDMSM operator. So as the parameter q varies, the ranking order of vaccine shifts, while the best vaccine (λ_1) remains unchanged in the procedure. During the decision-making process, the decision expert must choose the optimal vaccine with the variation of the parameter q for effectively modeling the 2TL T -SF data. On the basis of the symptoms' evaluating values, the parameter q can be set to the lowest integer that meets the inequality's requirements as $0 \leq (\Delta^{-1}(s_p(\ell), \wp(\ell)))^q + (\Delta^{-1}(s_n(\ell), \aleph(\ell)))^q + (\Delta^{-1}(s_i(\ell), \mathfrak{E}(\ell)))^q \leq \tau^q$.

6.4. Comparative Analysis with the Existing MAGDM Methods. A new 2TL T -SFS is proposed in this study describe cognitive information, and the proposed 2TL T -SFWSM and 2TL T -SFWDMSM operators are presented to aggregate the information in the 2TL T -SF environment. Therefore, some existing decision methods cannot directly process 2TL T -SF information. We compare and analyze the 2TL T -SF-EDAS method in this subsection.

- (i) First, in order to prove the superiority and effectiveness of the 2TL T -SF-EDAS method, we use the 2TLN-EDAS method to calculate the example 6.1 and compare the calculation result with the results of Reference [55], and the comparison results are shown in Tables 17 and 18. Here, we let $q = 1$. We can see from Tables 17 and 18 that the ranking results obtained by these methods are consistent, indicating that the 2TL T -SF-EDAS method is correct and effective.
- (ii) Second, in order to prove the superiority and effectiveness of the 2TL T -SF-EDAS method, we use the 2TLN-MABAC and LS-MABAC methods to calculate the example 6.1 and compare the calculation result with the results of Reference [55], and

TABLE 15: Score functions and ranking results by 2TL T -SFWMSM operator ($t = 3$ and $q = 2, 3, 5, 7, 11, 13$).

Parameter	$\Xi^*(\lambda_1)$	$\Xi^*(\lambda_2)$	$\Xi^*(\lambda_3)$	$\Xi^*(\lambda_4)$	$\Xi^*(\lambda_5)$	$\Xi^*(\lambda_6)$	$\Xi^*(\lambda_7)$	Ranking
$q = 2$	1.0000	0.5512	0.6668	0.6462	0.5033	0.4103	0.0000	$\lambda_1 > \lambda_3 > \lambda_4 > \lambda_2 > \lambda_5 > \lambda_6 > \lambda_7$
$q = 3$	1.0000	0.5691	0.6929	0.5768	0.4475	0.4107	0.0000	$\lambda_1 > \lambda_3 > \lambda_4 > \lambda_2 > \lambda_5 > \lambda_6 > \lambda_7$
$q = 5$	1.0000	0.5729	0.7245	0.4966	0.3683	0.3869	0.0000	$\lambda_1 > \lambda_3 > \lambda_2 > \lambda_4 > \lambda_6 > \lambda_5 > \lambda_7$
$q = 7$	1.0000	0.5638	0.7445	0.4446	0.3188	0.3543	0.0000	$\lambda_1 > \lambda_3 > \lambda_2 > \lambda_4 > \lambda_6 > \lambda_5 > \lambda_7$
$q = 11$	1.0000	0.5403	0.7723	0.3688	0.2530	0.2866	0.0000	$\lambda_1 > \lambda_3 > \lambda_2 > \lambda_4 > \lambda_6 > \lambda_5 > \lambda_7$
$q = 13$	1.0000	0.5285	0.7830	0.3328	0.2271	0.2541	0.0000	$\lambda_1 > \lambda_3 > \lambda_2 > \lambda_4 > \lambda_6 > \lambda_5 > \lambda_7$

TABLE 16: Score functions and ranking results by 2TL T -SFWDMSM operator ($t = 3$ and $q = 2, 3, 5, 7, 11, 13$).

Parameter	$\Xi^*(\lambda_1)$	$\Xi^*(\lambda_2)$	$\Xi^*(\lambda_3)$	$\Xi^*(\lambda_4)$	$\Xi^*(\lambda_5)$	$\Xi^*(\lambda_6)$	$\Xi^*(\lambda_7)$	Ranking
$q = 2$	1.0000	0.5649	0.8418	0.8917	0.5076	0.1332	0.0000	$\lambda_1 > \lambda_4 > \lambda_3 > \lambda_2 > \lambda_5 > \lambda_6 > \lambda_7$
$q = 3$	1.0000	0.5726	0.8203	0.9324	0.5499	0.0994	0.0000	$\lambda_1 > \lambda_4 > \lambda_3 > \lambda_2 > \lambda_5 > \lambda_6 > \lambda_7$
$q = 5$	1.0000	0.5990	0.8000	0.9797	0.6019	0.0743	0.0000	$\lambda_1 > \lambda_4 > \lambda_3 > \lambda_5 > \lambda_2 > \lambda_6 > \lambda_7$
$q = 7$	0.9915	0.6247	0.7851	1.0000	0.6358	0.0651	0.0000	$\lambda_4 > \lambda_1 > \lambda_3 > \lambda_5 > \lambda_2 > \lambda_6 > \lambda_7$
$q = 11$	0.9570	0.6636	0.7620	1.0000	0.6767	0.0539	0.0000	$\lambda_4 > \lambda_1 > \lambda_3 > \lambda_5 > \lambda_2 > \lambda_6 > \lambda_7$
$q = 13$	0.9458	0.6811	0.7602	1.0000	0.6935	0.0470	0.0000	$\lambda_4 > \lambda_1 > \lambda_3 > \lambda_5 > \lambda_2 > \lambda_6 > \lambda_7$

TABLE 17: Evaluation outcomes by utilizing different methodologies based on the 2TL T -SFWMSM operator.

Methods	Ranking
2TL T -SF-EDAS	$\lambda_1 > \lambda_3 > \lambda_2 > \lambda_4 > \lambda_5 > \lambda_6 > \lambda_7$
Wang et al. (2TLN-EDAS) [64]	$\lambda_1 > \lambda_4 > \lambda_3 > \lambda_2 > \lambda_5 > \lambda_6 > \lambda_7$
Wang et al. (2TLN-MABAC) [65]	$\lambda_6 > \lambda_1 > \lambda_4 > \lambda_2 > \lambda_3 > \lambda_5 > \lambda_7$
Liu et al. (LS-MABAC) [66]	$\lambda_1 > \lambda_4 > \lambda_6 > \lambda_2 > \lambda_3 > \lambda_5 > \lambda_7$
Wang et al. (2TLN-CODAS) [67]	$\lambda_1 > \lambda_6 > \lambda_4 > \lambda_2 > \lambda_3 > \lambda_5 > \lambda_7$
He et al. (2TLP-CODAS) [68]	$\lambda_1 > \lambda_4 > \lambda_6 > \lambda_5 > \lambda_2 > \lambda_3 > \lambda_7$
Cheng et al. (2TL-IF-TOPSIS) [69]	$\lambda_1 > \lambda_4 > \lambda_3 > \lambda_2 > \lambda_5 > \lambda_6 > \lambda_7$

TABLE 18: Evaluation outcomes by utilizing different methodologies based on the 2TL T -SFWDMSM operator.

Methods	Ranking
2TL T -SF-EDAS	$\lambda_1 > \lambda_4 > \lambda_3 > \lambda_2 > \lambda_5 > \lambda_6 > \lambda_7$
Wang et al. (2TLN-EDAS) [64]	$\lambda_1 > \lambda_3 > \lambda_4 > \lambda_2 > \lambda_5 > \lambda_6 > \lambda_7$
Wang et al. (2TLN-MABAC) [65]	$\lambda_3 > \lambda_1 > \lambda_6 > \lambda_4 > \lambda_2 > \lambda_5 > \lambda_7$
Liu et al. (LS-MABAC) [66]	$\lambda_1 > \lambda_3 > \lambda_4 > \lambda_2 > \lambda_6 > \lambda_5 > \lambda_7$
Wang et al. (2TLN-CODAS) [67]	$\lambda_1 > \lambda_3 > \lambda_4 > \lambda_2 > \lambda_6 > \lambda_5 > \lambda_7$
He et al. (2TLP-CODAS) [68]	$\lambda_1 > \lambda_3 > \lambda_4 > \lambda_2 > \lambda_5 > \lambda_6 > \lambda_7$
Cheng et al. (2TL-IF-TOPSIS) [69]	$\lambda_1 > \lambda_3 > \lambda_4 > \lambda_2 > \lambda_5 > \lambda_6 > \lambda_7$

the comparison results are shown in Tables 17 and 18. Here, we let $q = 1$ and $q = 2$, respectively. We can see from Tables 17 and 18 that the ranking results obtained by these methods are consistent, indicating that the 2TL T -SF-EDAS method is correct and effective.

- (iii) Third, in order to prove the superiority and effectiveness of the 2TL T -SF-EDAS method, we use the 2TLN-CODAS and 2TLP-CODAS methods to calculate the example 6.1 and compare the calculation result with the results of Reference [55], and the comparison results are shown in Tables 17 and 18. Here, we let $q = 1$ and $q = 2$, respectively. We can see from Tables 17 and 18 that the ranking

results obtained by these methods are consistent, indicating that the 2TL T -SF-EDAS method is correct and effective.

- (iv) Fourth, in order to prove the superiority and effectiveness of the 2TL T -SF-EDAS method, we use the 2TL-IF-TOPSIS method to calculate the example 6.1 and compare the calculation result with the results of Reference [55], and the comparison results are shown in Tables 17 and 18. Here, we let $q = 1$. We can see from Tables 17 and 18 that the ranking results obtained by these methods are consistent, indicating that the 2TL T -SF-EDAS method is correct and effective.

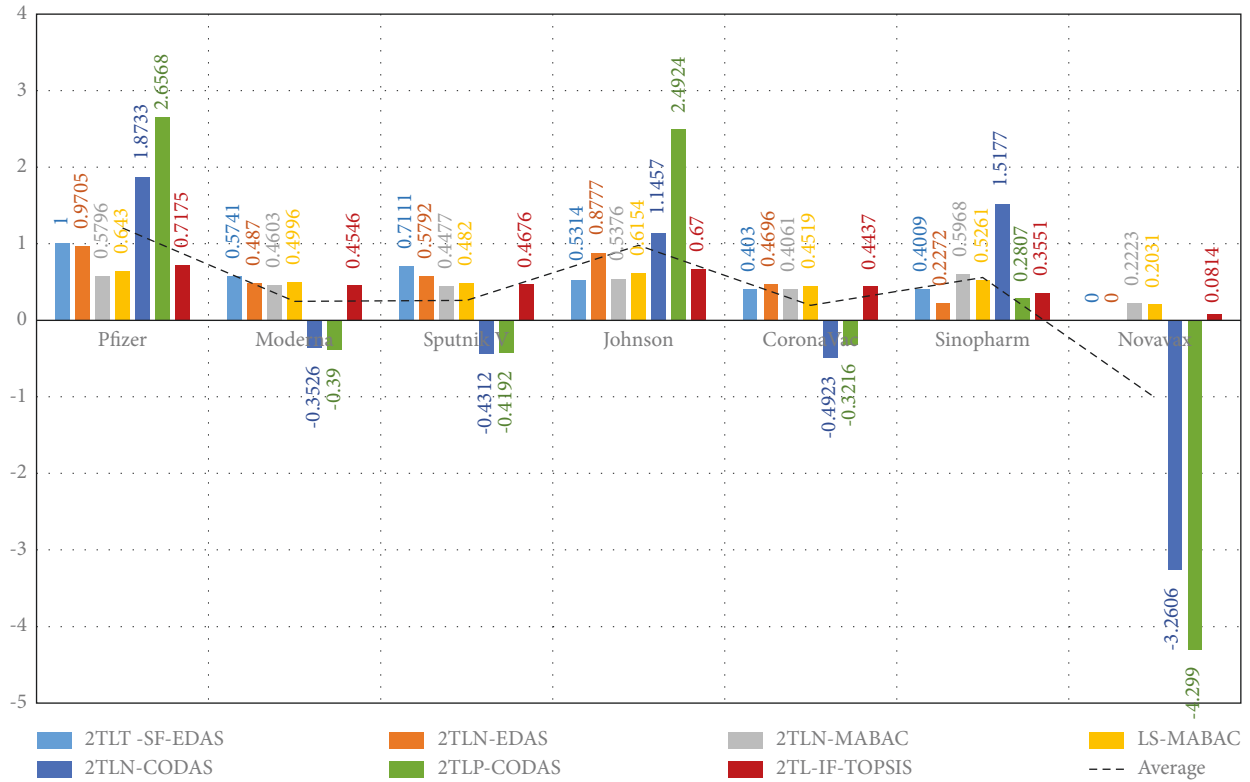
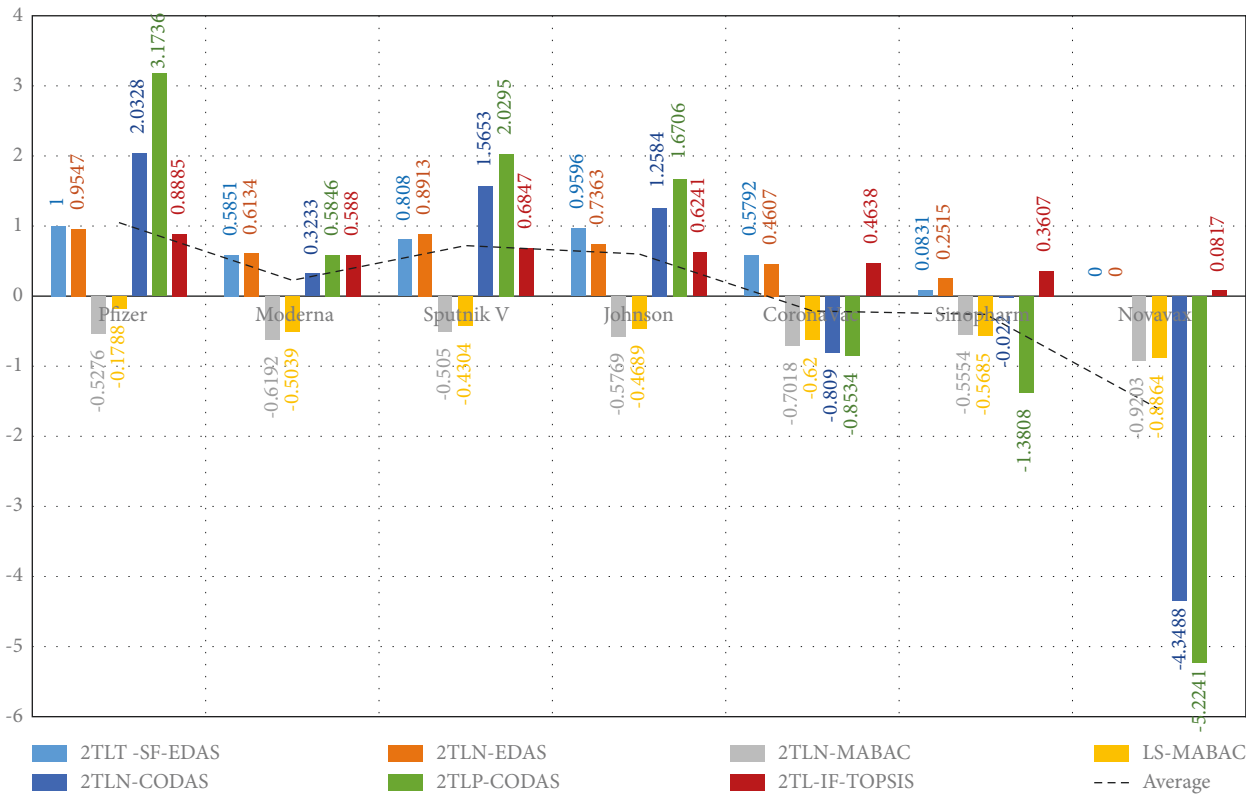
From Tables 17 and 18, we can see that using our proposed method to calculate the example 6.1 in different references, the ranking results obtained are consistent with the optimal choice among the ranking results obtained by the methods in the relevant references. Therefore, our proposed method is effective.

- (v) Lastly, according to the above analysis, the 2TL T -SF-EDAS method is significantly better than 2TLNPFSSs and 2TLNFSFs. The advantages are given below:

- (1) If $0 \leq (\Delta^{-1}(s_p(\ell), \wp(\ell))) + (\Delta^{-1}(s_n(\ell), \aleph(\ell))) + (\Delta^{-1}(s_l(\ell), \xi(\ell))) \leq \tau$, the 2TL T -SF-EDAS method can deal with the MAGDM problem in a picture fuzzy environment.
- (2) If $0 \leq (\Delta^{-1}(s_p(\ell), \wp(\ell)))^2 + (\Delta^{-1}(s_n(\ell), \aleph(\ell)))^2 + (\Delta^{-1}(s_l(\ell), \xi(\ell)))^2 \leq \tau^2$, then the 2TL T -SF-EDAS method can deal with the MAGDM problem in a spherical fuzzy environment.

Therefore, the applicability of the 2TL T -SF-EDAS method is more extensive, and it provides more space for DEs.

Further detail about the comparison outcomes is plotted in Figures 4 and 5.

FIGURE 4: Comparative outcomes with the 2TL T -SFWMSM operator.FIGURE 5: Comparative outcomes with the 2TL T -SFWDMSM operator.

7. Conclusions

Various nations and areas have encountered constraints on medical staff and medical resources such as vaccination throughout the antiepidemic procedure as a result of the unexpected circumstances caused by the COVID-19 virus. Vaccines and other critical medical devices are in low supply, making the battle against the pandemic extremely complicated. As a result, in the presence of inadequate medical services, medical workers should make an emergency choice to emphasize saving lives to the maximum degree possible. We gather information on different COVID-19 vaccines throughout the world via study. Furthermore, we used this, as an illustration, to conduct an immediate decision-making scenario to select the appropriate vaccination dosing strategy for infected individuals. So, in this research study, we constructed the novel 2TL T -SFNs, its rating criteria, aggregation operators, arithmetic operations, and certain relevant attributes for selecting the optimal vaccine to treat the COVID-19 epidemic. We also presented a family of the 2TL T -SFMSM aggregation operators for describing the interconnections among attributes and weighting values. An innovative 2TL T -SF-EDAS approach has been formulated for resolving the proposed COVID-19 MAGDM problem by merging the EDAS method in the 2TL T -SF environment for the ranking of certain vaccines.

The benefits of the proposed methodology are described as follows:

- (1) The proposed approach is based on 2TL T -SFS, which can both quantitatively and qualitatively tackle practical problems. The 2TL T -SFS is a new advancement in FS theory for handling group decision-making problems. As a result, we can conclude that the proposed approach is more efficient and has a wider range of applications due to the flexibility of the 2TL T -SFS.
- (2) The weighted MSM and weighted dual MSM operators are well known to tackle the interrelationship between attributes with the importance of weights. So, 2TL T -SFWMSM and 2TL T -SFWDMMSM operators are more powerful and adequate in dealing with MAGDM problems in the real world. As a consequence, our new framework for communicating with MAGDM problems is very efficient and effective.
- (3) This study reflected a selection of COVID-19 vaccines among the different vaccines during the pandemic period in the 2TL T -SF environment.
- (4) The 2TL T -SF-EDAS method can also provide robust and flexible information integration, enabling risk MAGDM problems that are more feasible. The existing operators and methods cannot control the certainty degrees. Our proposed model can effectively redistribute the MD, AD, and NMD in 2TL T -SFNs by different rules, allowing us to extract more detailed and objective data from the original 2TL T -SFS. Furthermore, the proposed approach is more effective and more applicable due to the parameter effect.

The number of attributes and alternatives is the study's limitation. The procedure of selecting the appropriate vaccine is essentially complicated due to the numerous attributes that must be analyzed and evaluated. These attributes are objectively and subjectively expressed. The presence of ambiguity and unpredictability in the structure of the qualitative attribute, on the other hand, makes quantitative measurement complicate. As a consequence, the main limitation of this study is the use of the EDAS technique to determine the attributes. Furthermore, the COVID-19 vaccine's limitation is that it may take weeks for its effectiveness to take effect, and it takes two weeks for your body to mount an appropriate response.

In future research, we will propose ordered operators for MSM such as power-ordered geometric MSM and power-ordered weighted geometric MSM operators. In the suggested 2TL T -SF-EDAS method, triangular 2TL T -SFSs, trapezoidal 2TL T -SFSs, or interval-valued 2TL T -SFSs can be employed instead of singleton 2TL T -SFSs for future investigation. The proposed methods apply to other types of ordinary FSs, such as type-2 FSs, hesitant FSs, and neutrosophic sets. Moreover, immunization programs will be designed to remove hurdles in vaccine price and vaccination convenience in order to increase vaccine uptake in response to the COVID-19 pandemic. Furthermore, health education and communication from credible sources will be critically studied in assuaging public concerns about vaccine safety.

Data Availability

No data were used to support this study.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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