Artificial Intelligence Review (2023) 56:10227–10274
https://doi.org/10.1007/s10462-023-10408-4

Complex q-rung orthopair fuzzy 2-tuple linguistic group decision-making framework with Muirhead mean operators

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Accepted: 1 February 2023 / Published online: 20 February 2023

Abstract

In this study, we aim to develop a method for solving multi-attribute group decision-making (MAGDM) problems with complex q-rung orthopair fuzzy 2-tuple linguistic sets. First, a complex q-rung orthopair fuzzy 2-tuple linguistic set (Cq-ROFTLS) is presented by combining the idea of 2-tuple linguistic set (2TLS) with complex q-rung orthopair fuzzy set (Cq-ROFS). Cq-ROFTLS is the best mechanism to deal with the imprecise and uncertain data in decision-making problems, characterized by 2TL complex-valued membership degree and non-membership degree and deals with two-dimensional data in one set at the same time by using extra terms known as phase terms related to periodicity and providing more space for decision makers (DMs) to explain their opinions. Muirhead mean (MM) operator can capture the interrelationships among all input arguments by changing a parameter vector. Considering the benefits of MM operator, we propose some new aggregation operators, including complex q-rung orthopair fuzzy 2-tuple linguistic Muirhead mean (Cq-ROFTLMM) operator, weighted Muirhead mean (Cq-ROFTLWMM) operator, complex q-rung orthopair fuzzy 2-tuple linguistic dual Muirhead mean (Cq-ROFTLDMM) operator, and weighted dual Muirhead mean (Cq-ROFTLWDMM) operator under Cq-ROFTL environment to expand its applied fields. Some properties and special cases related to the parameter vector are also proposed asong with the aggregation operators. Furthermore, a method to MAGDM problem is developed, and the proposed aggregation operators are used to solve the MAGDM problems related to the best choice of medication for pneumonia with Cq-ROFTL numbers (Cq-ROFTLNs). Finally, comparative analysis is presented with some existing methods.

Keywords Complex q-rung orthopair fuzzy 2-tuple linguistic set · Muirhead mean operator · Dual Muirhead mean operator · Pneumonia disease

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1 Introduction

In the field of decision-making, MAGDM is a significant branch. However, due to the complexity of the decision problems and the uncertainty of the decision environment, it is difficult to represent the relevant decision information by its exact value in the decision-making process. Recently, many techniques have been introduced for depicting and communicating fuzziness and uncertainty. Zadeh (1965) introduced the fuzzy set (FS). FS theory has obtained a lot of attention for many years, but the downside of the FS is that it just has a membership degree (MD) and ignores unsure data in real life decision-making problems. To tackle this deficiency in FS, Atanassov (1986) proposed the notion of an intuitionistic fuzzy set (IFS), which is defined by a MD and non-membership degree (NMD), with the sum of MD and NMD being less than or equal to one. The range of Pythagorean fuzzy set (PFS) (Yager 2013) is wider than IFS, in which the sum of the squares of MD and NMD is restricted to 1. Chen (2022) exploited an effective likelihood measure for determining outranking relations with Pythagorean fuzzy information. Yager (2016) proposed the q-rung orthopair fuzzy sets (q-ROFSs), a generalized form of IFSs and PFSs, in which the sum of the qth power of the MD and the NMD is limited to one. In some complex decision-making problems, however, the DMs are unable to allocate the MD and NMD by crisp numbers, and a viable alternative is to define them using linguistic variables. Lin et al. (2020) recently proposed linguistic q-ROFSs (Lq-ROFSs) and developed some aggregation operators, such as weighted averaging and weighted geometric operators, to aggregate the Lq-ROF numbers based on the Lq-ROF environment. Lq-ROFS is an extended form of the linguistic IFS (Zhang 2014) and linguistic PFS (Garg 2018). Wang et al. (2019a) proposed some novel operational laws, distance measure methods, and comparison methods within Lq-ROF framework. Utilizing q-ROFSs and uncertain linguistic variables, Liu et al. (2019) put forward a novel class of fuzzy sets known as q-rung orthopair uncertain linguistic sets and proposed uncertain linguistic aggregation operators.

Ramot et al. (2002) introduced the idea of the membership function range from real to complex numbers within the unit disc and proposed the concept of complex fuzzy set (CFS). The concept of complex intuitionistic fuzzy sets (CIFSs) is put forward by Alkouri and Salleh (2012). They also developed the idea of distance measure in CIFS information and CIF relations. Rani and Garg introduced the idea of generalized CIF Bonferroni mean aggregation operators utilizing Archimedean t-norm and t-conorm (Garg and Rani 2020), CIF power aggregation operators (Rani and Garg 2018) and presented its applications in decision-making. Mahmood et al. (2022a) constructed a multi-attribute decision-making technique for complex picture fuzzy uncertain linguistic information and employed the Bonferroni mean operators in the setting of bipolar complex fuzzy numbers (Mahmood et al. 2022b). Zeng et al. (2022) explored the complex interval valued q-rung orthopair 2-tuple linguistic set, which is a new powerful mixture to handle unreliable and vague data in realistic decision concerns. The q-ROFS theory only considers one dimension at a time, which frequently results in data loss. However, in real life, we observe such complex natural phenomena in which evaluating the second dimension to the expression of the MD and NMD becomes necessary. By developing the second dimension, complete information is projected into a set, which avoids any loss of information. The amplitude term shows the extension of an object belonging to Cq-ROFS (Liu et al. 2020b) and the phase terms are connected to periodicity. These phase terms differentiate the concept of Cq-ROFS from the concept of traditional q-ROFS theories. Du et al. (2022) developed some complex q-rung fuzzy Frank aggregation operators, such as the Cq-ROF Frank weighted averaging operator, the Cq-ROF Frank weighted geometric operator, and the Cq-ROF Frank ordered weighted averaging operator, and discussed their special cases. Recently, Feng et al. (2022a, b) presented certain concepts on generalized orthopair fuzzy sets and probabilistic generalized orthopair fuzzy sets.

Information aggregation operator has an important role in the process of decision-making, particularly in MAGDM. In practical MAGDM problems, the preferences of DMs change dynamically and there always exist various interactions among different considered multi-attributes. The MM operator, firstly suggested by Muirhead (1902), is a classical mean type aggregation operator used to aggregate numerical values in modern information fusion theory. It has a prominent capability to capture the interrelationships among all multi-input arguments of the data. The MM operator can be reduced to other existing operators by changing the parameter vector. This is the reason; it is called a universal operator. Recently, Liu et al. (2020a) incorporated the Pythagorean fuzzy linguistic numbers with the MM operator to introduce the Pythagorean fuzzy linguistic MM, the Pythagorean fuzzy linguistic weighted MM, the Pythagorean fuzzy linguistic dual MM and the Pythagorean fuzzy linguistic weighted dual MM operators. Wang et al. (2019b) extended the MM to q-ROF environment and introduced a family of q-ROFMM aggregation operators. Luo et al. (2019) proposed linguistic neutrosophic power MM operators for the safety evaluation of mines. 2TL information can be processed in a way that prevents information distortion and loss of data. Herrera and Martìnez (2000) proposed the 2-tuple fuzzy linguistic representation model as one of the most crucial approaches to cope with linguistic decision-making problems. Later, several 2TL aggregation operators and decision-making approaches were introduced. Deng et al. (2019) expanded the generalized Heronian mean aggregation operators and their weighted form with 2TL Pythagorean numbers to develop generalized 2TL Pythagorean Heronian mean aggregation operators. Wei and Gao (2020) utilized power average and power geometric operators with Pythagorean 2-tuple linguistic information to develop some Pythagorean 2-tuple linguistic power aggregation operators to solve the MADM problems. Wei et al. (2018) extended the concept of q-ROFTL sets to characterize the MD and NMD of an element to a 2TL variable with some of its operational laws and further developed some q-ROFTL Heronian mean aggregation operators. Ju et al. (2020) introduced the q-ROFTL weighted averaging operator and the q-ROFTL weighted geometric operator to develop an approach to solve MAGDM problem with q-ROFTL information. Further, the q-ROFTLMM and the q-ROFTLDMM operators are presented by them. Recently, many researchers (Akram et al. 2021a, b, c, 2022a, b; Alsalem et al. 2022; Liu and Liu 2021; Liu et al. 2021; Naz et al. 2021, 2022a, b, c; Liu et al. 2022; Peng et al. 2021a, b; Peng and Yuan 2021; Peng and Selvachandran 2019; Peng and Luo 2021) presented a lot of decision-making approaches in the context of a generalized fuzzy framework. In this paper, we develop the new concept of Cq-ROFTLSs based on the Cq-ROFSs and 2TLSs. Furthermore, the MM operator, being one of the most efficient aggregation operators, can accommodate the interrelationships among all number of arguments, whereas all of the above methods, based on the classic MM and DMM operators, are insufficient to aggregate these Cq-ROFTLNs. So how to quantify such Cq-ROFTLNs using the MM and DMM operators is an interesting topic. We generalize MM and DMM to the Cq-ROFTL environment and design some new information aggregation operators, such as Cq-ROFTLMM operator, Cq-ROFTLDMM operator and their weighted forms.

The remaining manuscript is ordered as: In Sect. 2, we introduce some basic concepts related to q-ROFTLS, Cq-ROFS and the MM operator and its dual form. Section 3 introduces the concept of Cq-ROFTLS along with the modified score function and some operational laws. In Sect. 4, we propose some Cq-ROFTLMM aggregation operators, including the Cq-ROFTLMM operator, the Cq-ROFTLWMM operator, the Cq-ROFTLDMM operator, and the Cq-ROFTLWDMM operator. Section 5 represents the model for solving the MAGDM problem. In Sect. 6, a numerical illustration for the selection of the best medication for pneumonia with Cq-ROFTLNs is presented and comparative analysis is conducted to demonstrate the validity of the proposed approach. At long last, in Sect. 7, we summarize the paper. The above content is depicted in Fig. 1.

2 Preliminaries

A brief review of some correlative concepts of 2TL representation model, q-ROFTLS, Cq-ROFS and the MM operator and its dual form will be given in this section.

2.1 2-Tuple linguistic representation model

Tai and Chen (2009) introduced the notion of 2TLS, described as follows:

Definition 1 (Tai and Chen 2009) Let $S = \{s_0, s_1, \dots, s_h\}$ be a linguistic term set (LTS), $\mathfrak{R} \in [0, 1]$ be a number value that represents the aggregation result of linguistic symbolic. The function Δ is then defined to obtain the 2-tuple linguistic information equivalent to $\mathfrak{R}$ as follows:

$$\Delta : [0, 1] \longrightarrow S \times \left[-\frac{1}{2h}, \frac{1}{2h}\right),$$
$$\Delta(\mathfrak{R}) = (s_j, \pounds) \text{ with } \begin{cases} s_j, & j = \text{round}(\mathfrak{R} \times h), \\ \pounds = \mathfrak{R} - \frac{j}{h}, & \pounds \in [-\frac{1}{2h}, \frac{1}{2h}) \end{cases}$$

where $\pounds$ is the value of the symbolic translation and s_j has the closest index label to $\mathfrak{R}$.

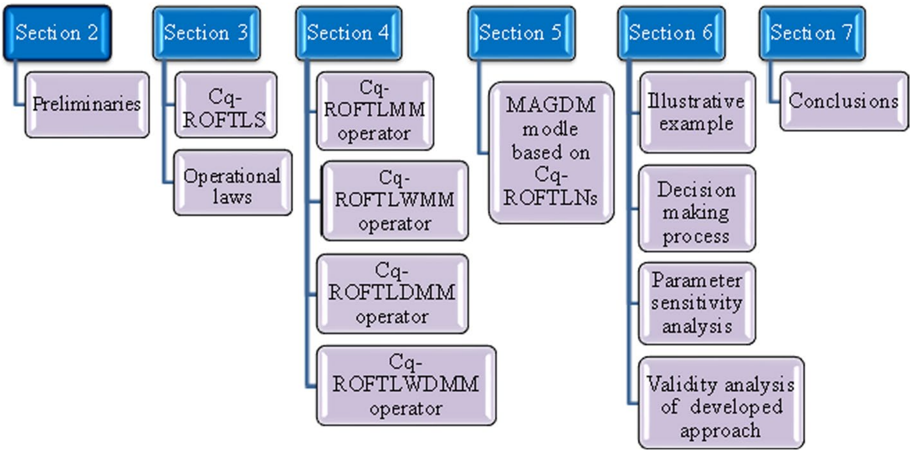

Fig. 1 The framework of the study

Definition 2 (*Tai and Chen* 2009) Let $S = \{s_0, s_1, \ldots, s_h\}$ be a LTS, $(s_j, \pounds)$ is 2TL information, then there is an inverse function Δ^{-1}, which transforms 2TL information into its equivalent numerical value $\mathfrak{R} \in [0, 1]$. The inverse function Δ^{-1} is defined as follows:

$$\Delta^{-1} : S \times \left[-\frac{1}{2h}, \frac{1}{2h}\right) \longrightarrow [0, 1],$$
$$\Delta^{-1}(s_j, \pounds) = \frac{j}{h} + \pounds = \mathfrak{R}.$$

2.2 *q*-Rung orthopair fuzzy 2-tuple linguistic set

Definition 3 (*Wei et a*l. 2018) Let L be a non-empty set of the universe, S be a LTS with odd cardinality; then the definition of q-ROFTLS is as follows:

$$\mathrm{D} = \{\langle (s_{p(\mathfrak{l})}, \pounds), (\mathfrak{I}_D(\mathfrak{l}), \aleph_D(\mathfrak{l}))\rangle | \mathfrak{l} \in L\} \tag{1}$$

, where $s_{p(\mathfrak{l})} \in S, \pounds \in [-\frac{1}{2h}, \frac{1}{2h})$, the numbers $\mathfrak{I}_D(\mathfrak{l})$ and $\aleph_D(\mathfrak{l})$ represent the MD and NMD of $\mathfrak{l}$ to the 2-tuple linguistics variable $(s_{p(\mathfrak{l})}, \pounds)$, respectively, satisfying the following conditions: $\mathfrak{I}_D(\mathfrak{l}) \in [0, 1], \aleph_D(\mathfrak{l}) \in [0, 1],\ 0 \le (\mathfrak{I}_D(\mathfrak{l}))^q + (\aleph_D(\mathfrak{l}))^q \le 1, \quad \forall\, \mathfrak{l} \in L$ and $q \ge 1$. For $\mathfrak{l} \in L,\ \pi_D(\mathfrak{l}) = \sqrt[q]{1 - (\mathfrak{I}_D(\mathfrak{l}))^q - (\aleph_D(\mathfrak{l}))^q}$ is the indeterminacy degree of the element $\mathfrak{l}$ to 2TL variable $(s_{p(\mathfrak{l})}, \pounds)$.

Definition 4 (*Liu et al.* 2020b) A Cq-ROFS $\mathfrak{T}$, is defined as:

$$\mathfrak{T} = \{\langle \mathfrak{l}, (\mathfrak{I}'_{\mathfrak{T}}(\mathfrak{l}), \aleph'_{\mathfrak{T}}(\mathfrak{l}))\rangle | \mathfrak{l} \in L\}$$

where $\mathfrak{I}'_{\mathfrak{T}}$ and $\aleph'_{\mathfrak{T}}$ are the complex-valued membership and non-membership functions, respectively and are defined as:

$$\mathfrak{I}'_{\mathfrak{T}}(\mathfrak{l}) = \mathfrak{I}_{\mathfrak{T}}(\mathfrak{l}) e^{i2\pi\omega_{\mathfrak{I}_{\mathfrak{T}}(\mathfrak{l})}}; \aleph'_{\mathfrak{T}}(\mathfrak{l}) = \aleph_{\mathfrak{T}}(\mathfrak{l}) e^{i2\pi\omega_{\aleph_{\mathfrak{T}}(\mathfrak{l})}}$$

where $0 \le \mathfrak{I}_{\mathfrak{T}}(\mathfrak{l}), \aleph_{\mathfrak{T}}(\mathfrak{l}), \mathfrak{I}^q_{\mathfrak{T}}(\mathfrak{l}) + \aleph^q_{\mathfrak{T}}(\mathfrak{l}) \le 1$ and $0 \le \omega_{\mathfrak{I}_{\mathfrak{T}}}(\mathfrak{l}), \omega_{\aleph_{\mathfrak{T}}}(\mathfrak{l}), \omega^q_{\mathfrak{I}_{\mathfrak{T}}}(\mathfrak{l}) + \omega^q_{\aleph_{\mathfrak{T}}}(\mathfrak{l}) \le 1$.

Further, $\pi_{\mathfrak{T}}(\mathfrak{l}) = \sqrt[q]{1 - (\mathfrak{I}^q_{\mathfrak{T}}(\mathfrak{l}) + \aleph^q_{\mathfrak{T}}(\mathfrak{l}))}$ and $\omega_{\pi_{\mathfrak{T}}}(\mathfrak{l}) = \sqrt[q]{1 - (\omega^q_{\mathfrak{I}_{\mathfrak{T}}}(\mathfrak{l}) + \omega^q_{\aleph_{\mathfrak{T}}}(\mathfrak{l}))}$ are complex hesitancy degree of $\mathfrak{l}$. For simplicity, $\mathcal{N} = ((\mathfrak{I}, \omega_{\mathfrak{I}}), (\aleph, \omega_{\aleph}))$ is called the Cq-ROF number (Cq-ROFN) where $0 \le \mathfrak{I}, \aleph, \mathfrak{I}^q + \aleph^q \le 1$, and $0 \le \omega_{\mathfrak{I}}, \omega_{\aleph}, \omega^q_{\mathfrak{I}} + \omega^q_{\aleph} \le 1$.

2.3 The MM and DMM operator

Muirhead's original MM operator (Muirhead 1902) is a useful tool for aggregating crisp numbers. Qin and Liu (2016) developed the DMM operator, which was inspired by the MM operator. The MM and DMM operators have the distinct advantages of being able to capture interrelationships among all aggregated arguments.

Definition 5 (*Muirhead* 1902; Qin and Liu 2016) Let $\tau_\jmath (\jmath = 1, 2, \ldots, z)$ be a set of positive numbers and $\psi = (\psi_1, \psi_2, \ldots, \psi_z) \in \mathbb{R}^z$ be a vector of parameters, then the MM and DMM operators are defined as:

$$\mathrm{MM}^{\psi}(\tau_1, \tau_2, \ldots, \tau_z) = \left(\frac{1}{z!} \sum_{\varrho \in \mathbb{S}_z} \prod_{\jmath=1}^{z} \tau_{\varrho(\jmath)}^{\psi_\jmath} \right)^{\frac{1}{\sum_{\jmath=1}^{z} \psi_\jmath}}; \tag{2}$$

$$\mathrm{DMM}^{\psi}(\tau_1, \tau_2, \ldots, \tau_z) = \frac{1}{\sum_{\jmath=1}^{z} \psi_\jmath} \left(\prod_{\varrho \in \mathbb{S}_z} \sum_{\jmath=1}^{z} \psi_\jmath \tau_{\varrho(\jmath)} \right)^{\frac{1}{z!}} \tag{3}$$

, where $\varrho(\jmath)(\jmath = 1, 2, \ldots, z)$ is a permutation of $(1, 2, \ldots, z)$ and $\mathbb{S}_z$ is the set of all permutations of $(1, 2, \ldots, z)$.

3 C*q*-ROFTLS

The phase terms of complex numbers and 2TL terms are considered as an effective strategy to introduce the concept of Cq-ROFTLS. This section employs the new definition of Cq-ROFTLS to represent the periodicity and uncertainty of problems in Cq-ROFTL environment. Later, we will modify the score function, accuracy function, and introduce some operational laws of Cq-ROFTLNs.

Inspired by the ideas of 2TLS and Cq-ROFS, we develop the new concept of Cq-ROFTLS and describe its novelty by using several examples. The mathematical representation of Cq-ROFTLS is described as follows:

Definition 6 Let $S = \{s_0, s_1, \ldots, s_h\}$ be a LTS with odd cardinality. If $\mathcal{N} = \langle (s_{p(\mathcal{N})}, \pounds), ((\mathfrak{I}_{\mathcal{N}}, \omega_{\mathfrak{I}_{\mathcal{N}}}), (\aleph_{\mathcal{N}}, \omega_{\aleph_{\mathcal{N}}})) \rangle$ is defined for $s_{p(\mathcal{N})} \in S$ and $\pounds \in \left[-\frac{1}{2h}, \frac{1}{2h}\right)$, then C$q$-ROFTLS is defined as:

$$T = \{(s_{p(\mathfrak{l})}, \pounds), (\mathfrak{I}'_T(\mathfrak{l}), \aleph'_T(\mathfrak{l})) | l \in L\}$$

where $\mathfrak{I}'_T, \aleph'_T$ are, respectively, termed as 2-tuple linguistic complex-valued membership and non-membership functions and are defined as:

$$\mathfrak{I}'_T(\mathfrak{l}) = \mathfrak{I}_T(\mathfrak{l}) e^{i2\pi\omega_{\mathfrak{I}_T(\mathfrak{l})}}; \quad \aleph'_T(\mathfrak{l}) = \aleph_T(\mathfrak{l}) e^{i2\pi\omega_{\aleph_T(\mathfrak{l})}}$$

where $\mathfrak{I}, \aleph \in [0, 1]$, $\quad \mathfrak{I}^q + \aleph^q \in [0, 1]$ and $\omega_{\mathfrak{I}}^q + \omega_{\aleph}^q \in [0, 1]$.

Definition 7 Let $\mathcal{N} = \langle (s_{p(\mathcal{N})}, \pounds), ((\mathfrak{I}_{\mathcal{N}}, \omega_{\mathfrak{I}_{\mathcal{N}}}), (\aleph_{\mathcal{N}}, \omega_{\aleph_{\mathcal{N}}})) \rangle$ be a Cq-ROFTLN. The score function $\digamma$ of a Cq-ROFTLN $\mathcal{N}$, can be represented as:

$$\digamma(\mathcal{N}) = \frac{\Delta^{-1}(s_{p(\mathcal{N})}, \pounds) \times ((1 + \mathfrak{I}_{\mathcal{N}}^q - \aleph_{\mathcal{N}}^q) + (1 + \omega_{\mathfrak{I}_{\mathcal{N}}}^q - \omega_{\aleph_{\mathcal{N}}}^q))}{4}. \tag{4}$$

Example 1 Let $\mathcal{N}_1 = \langle (s_4, 0), ((0.30, 0.45), (0.34, 0.52)) \rangle$, $\mathcal{N}_2 = \langle (s_2, 0), ((0.30, 0.40), (0.50, 0.20)) \rangle$ and and $\mathcal{N}_3 = \langle (s_3, 0), ((0.50, 0.28), (0.20, 0.70)) \rangle$ are three Cq-ROFTLNs which are derived from LTS $S = \{s_j | j \in \{0, \dots, 6\}\}$ and $q = 4$, then by utilizing Eq. (4), we find the score of given Cq-ROFTLNs as:

$$F(\mathcal{N}_1) = 0.3271,\ F(\mathcal{N}_2) = 0.1641 \text{ and } F(\mathcal{N}_3) = 0.2284.$$

This shows that

$$F(\mathcal{N}_1) > F(\mathcal{N}_3) > F(\mathcal{N}_2) \Rightarrow \mathcal{N}_1 \succ \mathcal{N}_3 \succ \mathcal{N}_2.$$

There are some cases when scores of two or more Cq-ROFTLNs are same. Here, score function fails to rank Cq-ROFTLNs. This situation is handled by accuracy function, given as follows:

Definition 8 The accuracy function of a Cq-ROFTLN $\mathcal{N} = \langle (s_{p(\mathcal{N})}, \pounds), ((\mathfrak{J}_{\mathcal{N}}, \omega_{\mathfrak{J}_{\mathcal{N}}}), (\aleph_{\mathcal{N}}, \omega_{\aleph_{\mathcal{N}}})) \rangle$ is defined as:

$$\beth(\mathcal{N}) = \frac{\Delta^{-1}(s_{p(\mathcal{N})}, \pounds) \times ((\mathfrak{J}_{\mathcal{N}}^q + \aleph_{\mathcal{N}}^q) + (\omega_{\mathfrak{J}_{\mathcal{N}}}^q + \omega_{\aleph_{\mathcal{N}}}^q))}{2}. \tag{5}$$

On the basis of the score and accuracy functions, the order relation between any two Cq-ROFTLNs can be described as follows:

Definition 9 Let $\mathcal{N}_1 = \langle (s_{p_1}, \pounds_1), ((\mathfrak{J}_1, \omega_{\mathfrak{J}_1}), (\aleph_1, \omega_{\aleph_1})) \rangle$ and $\mathcal{N}_2 = \langle (s_{p_2}, \pounds_2), ((\mathfrak{J}_2, \omega_{\mathfrak{J}_2}), (\aleph_2, \omega_{\aleph_2})) \rangle$ be two Cq-ROFTLNs; then these two Cq-ROFTLNs can be compared according to the following rules:

(1) if $F(\mathcal{N}_1) > F(\mathcal{N}_2)$, then $\mathcal{N}_1 \succ \mathcal{N}_2$;
(2) if $F(\mathcal{N}_1) = F(\mathcal{N}_2)$, then:

- if $\beth(\mathcal{N}_1) > \beth(\mathcal{N}_2)$, then $\mathcal{N}_1 \succ \mathcal{N}_2$;
- if $\beth(\mathcal{N}_1) = \beth(\mathcal{N}_2)$, then $\mathcal{N}_1 \sim \mathcal{N}_2$.

Some operational laws to compute the Cq-ROFTLNs are presented below:

Definition 10 Let $\mathcal{N}_1 = \langle (s_{p_1}, \pounds_1), ((\mathfrak{J}_1, \omega_{\mathfrak{J}_1}), (\aleph_1, \omega_{\aleph_1})) \rangle$ and $\mathcal{N}_2 = \langle (s_{p_2}, \pounds_2), ((\mathfrak{J}_2, \omega_{\mathfrak{J}_2}), (\aleph_2, \omega_{\aleph_2})) \rangle$ be two Cq-ROFTLNs, then its operational laws are defined as:

1. $\mathcal{N}_1 \oplus \mathcal{N}_2 = \left\langle \Delta(\Delta^{-1}(s_{p_1}, \pounds_1) + \Delta^{-1}(s_{p_2}, \pounds_2)), \left((\sqrt[q]{\mathfrak{J}_1^q + \mathfrak{J}_2^q - \mathfrak{J}_1^q \mathfrak{J}_2^q}, \sqrt[q]{\omega_{\mathfrak{J}_1}^q + \omega_{\mathfrak{J}_2}^q - \omega_{\mathfrak{J}_1}^q \omega_{\mathfrak{J}_2}^q}), (\aleph_1 \aleph_2, \omega_{\aleph_1} \omega_{\aleph_2}) \right) \right\rangle$;
2. $\mathcal{N}_1 \otimes \mathcal{N}_2 = \left\langle \Delta(\Delta^{-1}(s_{p_1}, \pounds_1) \times \Delta^{-1}(s_{p_2}, \pounds_2)), \left((\mathfrak{J}_1 \mathfrak{J}_2, \omega_{\mathfrak{J}_1} \omega_{\mathfrak{J}_2}), (\sqrt[q]{\aleph_1^q + \aleph_2^q - \aleph_1^q \aleph_2^q}, \sqrt[q]{\omega_{\aleph_1}^q + \omega_{\aleph_2}^q - \omega_{\aleph_1}^q \omega_{\aleph_2}^q}) \right) \right\rangle$;
3. $\lambda \mathcal{N}_1 = \left\langle \Delta(\lambda \times \Delta^{-1}(s_{p_1}, \pounds_1)), \left((\sqrt[q]{1 - (1 - \mathfrak{J}_1^q)^\lambda}, \sqrt[q]{1 - (1 - \omega_{\mathfrak{J}_1}^q)^\lambda}), (\aleph_1^\lambda, \omega_{\aleph_1}^\lambda) \right) \right\rangle$, $\lambda > 0$;
4. $\mathcal{N}_1^\lambda = \left\langle \Delta(\Delta^{-1}(s_{p_1}, \pounds_1))^\lambda, \left((\mathfrak{J}_1^\lambda, \omega_{\mathfrak{J}_1}^\lambda), (\sqrt[q]{1 - (1 - \aleph_1^q)^\lambda}, \sqrt[q]{1 - (1 - \omega_{\aleph_1}^q)^\lambda}) \right) \right\rangle$, $\lambda > 0$.

Theorem 1 *Let* $\mathcal{N}_1 = \langle (s_{p_1}, \pounds_1), ((\mathfrak{J}_1, \omega_{\mathfrak{J}_1}), (\aleph_1, \omega_{\aleph_1})) \rangle$ *and* $\mathcal{N}_2 = \langle (s_{p_2}, \pounds_2), ((\mathfrak{J}_2, \omega_{\mathfrak{J}_2}), (\aleph_2, \omega_{\aleph_2})) \rangle$ *be two Cq-ROFTLNs, and* $\lambda, \lambda_1, \lambda_2 \in [0, 1]$, *then*:

(1) $(\mathcal{N}_1^{\lambda_1})^{\lambda_2} = \mathcal{N}_1^{\lambda_1\lambda_2}$;
(2) $\lambda(\mathcal{N}_1 \oplus \mathcal{N}_2) = \lambda\mathcal{N}_1 \oplus \lambda\mathcal{N}_2$;
(3) $\lambda_1\mathcal{N}_1 \oplus \lambda_2\mathcal{N}_1 = (\lambda_1 + \lambda_2)\mathcal{N}_1$;
(4) $(\mathcal{N}_1 \otimes \mathcal{N}_2)^{\lambda} = \mathcal{N}_1^{\lambda} \otimes \mathcal{N}_2^{\lambda}$;
(5) $\mathcal{N}_1^{\lambda_1} \otimes \mathcal{N}_1^{\lambda_2} = \mathcal{N}_1^{\lambda_1+\lambda_2}$.

Proof These properties can be proved trivially. □

Definition 11 Let $\mathcal{N}_1 = \langle(s_{p_1}, \pounds_1), ((\mathfrak{I}_1, \omega_{\mathfrak{I}_1}), (\aleph_1, \omega_{\aleph_1}))\rangle$ and $\mathcal{N}_2 = \langle(s_{p_2}, \pounds_2), ((\mathfrak{I}_2, \omega_{\mathfrak{I}_2}), (\aleph_2, \omega_{\aleph_2}))\rangle$ be two Cq-ROFTLNs, then there are the following relations:

(1) $\mathcal{N}_1^c = \langle(s_{p_1}, \pounds_1), ((\aleph_1, \omega_{\aleph_1}), (\mathfrak{I}_1, \omega_{\mathfrak{I}_1}))\rangle$;
(2) $\mathcal{N}_1 \subseteq \mathcal{N}_2$ if $(s_{p_1}, \pounds_1) \leq (s_{p_2}, \pounds_2)$, $\mathfrak{I}_1 \leq \mathfrak{I}_2$, $\omega_{\mathfrak{I}_1} \leq \omega_{\mathfrak{I}_2}$ and $\aleph_1 \geq \aleph_2$, $\omega_{\aleph_1} \geq \omega_{\aleph_2}$;
(3) $\mathcal{N}_1 = \mathcal{N}_2$ iff $\mathcal{N}_1 \subseteq \mathcal{N}_2$ and $\mathcal{N}_2 \subseteq \mathcal{N}_1$;
(4) $\mathcal{N}_1 \cap \mathcal{N}_2 = \langle \min((s_{p_1}, \pounds_1), (s_{p_2}, \pounds_2)), ((\min(\mathfrak{I}_1, \mathfrak{I}_2), \min(\omega_{\mathfrak{I}_1}, \omega_{\mathfrak{I}_2})), (\max(\aleph_1, \aleph_2), \max(\omega_{\aleph_1}, \omega_{\aleph_2})))\rangle$;
(5) $\mathcal{N}_1 \cup \mathcal{N}_2 = \langle \max((s_{p_1}, \pounds_1), (s_{p_2}, \pounds_2)), ((\max(\mathfrak{I}_1, \mathfrak{I}_2), \max(\omega_{\mathfrak{I}_1}, \omega_{\mathfrak{I}_2})), (\min(\aleph_1, \aleph_2), \min(\omega_{\aleph_1}, \omega_{\aleph_2})))\rangle$.

4 Some C*q*-ROFTLMM aggregation operators

Cq-ROFTLNs can effectively represent the fuzzy information and the traditional MM operator can only process the crisp numbers. It is crucially important to generalize MM operator to process Cq-ROFTLNs. In this section, some aggregation operators are introduced to fuse Cq-ROFTLNs. Firstly, MM operator is generalized under Cq-ROFTL information and some Cq-ROFTLMM aggregation operators are presented.

4.1 C*q*-ROFTLMM operator

This subsection generalizes the MM operator with q-ROFTLSs in complex scenario to propose the Cq-ROFTLMM operator, and describes its specific properties.

Definition 12 Let $\mathcal{N}_j = \langle(s_j, \pounds_j), ((\mathfrak{I}_j, \omega_{\mathfrak{I}_j}), (\aleph_j, \omega_{\aleph_j}))\rangle (j = 1, 2, \ldots, z)$ be a set of Cq-ROFTLNs and $\psi = (\psi_1, \psi_2, \ldots, \psi_z) \in \mathbb{R}^z$ be a parameters vector, then the Cq-ROFTLMM operator is given as:

$$\mathrm{C}q\text{-ROFTLMM}^{\psi}(\mathcal{N}_1, \mathcal{N}_2, \ldots, \mathcal{N}_z) = \left(\frac{1}{z!} \oplus_{\varrho \in \mathbb{S}_z} \otimes_{j=1}^{z} \mathcal{N}_{\varrho(j)}^{\psi_j}\right)^{\frac{1}{\sum_{j=1}^{z} \psi_j}}$$

where $\varrho_j (j = 1, 2, \ldots, z)$ is any permutation of $(1, 2, \ldots, z)$, and $\mathbb{S}_z$ is the set of all permutations of $(1, 2, \ldots, z)$.

Utilizing operational laws of the Cq-ROFTLNs, we can derive Theorem 2.

Theorem 2 *Let $\mathcal{N}_J = \langle (s_J, \pounds_J), ((\mathfrak{I}_J, \omega_{\mathfrak{I}_J}), (\aleph_J, \omega_{\aleph_J}))\rangle (J = 1, 2, \ldots, z)$ be a set of Cq-ROFTLNs and $\psi = (\psi_1, \psi_2, \ldots, \psi_z) \in \mathbb{R}^z$ be a parameters vector. Then their aggregated result by applying the Cq-ROFTLMM operator is also a Cq-ROFTLN, and*

$$\mathrm{C}q\text{-ROFTLMM}^{\psi}(\mathcal{N}_1, \mathcal{N}_2, \ldots, \mathcal{N}_z) = \left\langle \Delta\left(\left(\frac{1}{z!}\sum_{\varrho\in\mathbb{S}_z}\prod_{J=1}^{z}\left(\Delta^{-1}\left(s_{\varrho(J)}, \pounds_{\varrho(J)}\right)\right)^{\psi_J}\right)^{\frac{1}{\sum_{J=1}^{z}\psi_J}}\right), \right.$$
$$\left.\left(\begin{array}{l}\left(\left(\sqrt[q]{1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{J=1}^{z}\mathfrak{I}_{\varrho(J)}^{q\psi_J}\right)\right)^{\frac{1}{z!}}}\right)^{\frac{1}{\sum_{J=1}^{z}\psi_J}}, \left(\sqrt[q]{1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{J=1}^{z}\omega_{\mathfrak{I}_{\varrho(J)}}^{q\psi_J}\right)\right)^{\frac{1}{z!}}}\right)^{\frac{1}{\sum_{J=1}^{z}\psi_J}}\right), \\ \left(\sqrt[q]{1-\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{J=1}^{z}\left(1-\aleph_{\varrho(J)}^{q}\right)^{\psi_J}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{J=1}^{z}\psi_J}}}, \sqrt[q]{1-\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{J=1}^{z}\left(1-\omega_{\aleph_{\varrho(J)}}^{q}\right)^{\psi_J}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{J=1}^{z}\psi_J}}}\right)\end{array}\right)\right\rangle. \tag{6}$$

$$\mathcal{N}_{\varrho(J)}^{\psi_J} = \left\langle \Delta\left(\Delta^{-1}\left(s_{\varrho(J)}, \pounds_{\varrho(J)}\right)\right)^{\psi_J}, \left(\left(\mathfrak{I}_{\varrho(J)}^{\psi_J}, \omega_{\mathfrak{I}_{\varrho(J)}}^{\psi_J}\right), \left(\sqrt[q]{1-\left(1-\aleph_{\varrho(J)}^{q}\right)^{\psi_J}}, \sqrt[q]{1-\left(1-\omega_{\aleph_{\varrho(J)}}^{q}\right)^{\psi_J}}\right)\right)\right\rangle,$$

$$\otimes_{J=1}^{z}\mathcal{N}_{\varrho(J)}^{\psi_J} = \left\langle \Delta\left(\prod_{J=1}^{z}\left(\Delta^{-1}\left(s_{\varrho(J)}, \pounds_{\varrho(J)}\right)\right)^{\psi_J}\right), \left(\left(\prod_{J=1}^{z}\mathfrak{I}_{\varrho(J)}^{\psi_J}, \prod_{J=1}^{z}\omega_{\mathfrak{I}_{\varrho(J)}}^{\psi_J}\right), \left(\sqrt[q]{1-\prod_{J=1}^{z}\left(1-\aleph_{\varrho(J)}^{q}\right)^{\psi_J}}, \sqrt[q]{1-\prod_{J=1}^{z}\left(1-\omega_{\aleph_{\varrho(J)}}^{q}\right)^{\psi_J}}\right)\right)\right\rangle.$$

Proof

Thereafter,

$$\oplus_{\varrho\in\mathbb{S}_z}\otimes_{J=1}^{z}\mathcal{N}_{\varrho(J)}^{\psi_J} = \left\langle \Delta\left(\sum_{\varrho\in\mathbb{S}_z}\prod_{J=1}^{z}\left(\Delta^{-1}\left(s_{\varrho(J)}, \pounds_{\varrho(J)}\right)\right)^{\psi_J}\right), \right.$$
$$\left.\left(\begin{array}{l}\left(\sqrt[q]{1-\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{J=1}^{z}\mathfrak{I}_{\varrho(J)}^{q\psi_J}\right)}, \sqrt[q]{1-\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{J=1}^{z}\omega_{\mathfrak{I}_{\varrho(J)}}^{q\psi_J}\right)}\right) \\ \left(\prod_{\varrho\in\mathbb{S}_z}\sqrt[q]{1-\prod_{J=1}^{z}\left(1-\aleph_{\varrho(J)}^{q}\right)^{\psi_J}}, \prod_{\varrho\in\mathbb{S}_z}\sqrt[q]{1-\prod_{J=1}^{z}\left(1-\omega_{\aleph_{\varrho(J)}}^{q}\right)^{\psi_J}}\right)\end{array}\right)\right\rangle,$$

$$\frac{1}{z!}\oplus_{\varrho\in\mathbb{S}_z}\otimes_{J=1}^{z}\mathcal{N}_{\varrho(J)}^{\psi_J}\left\langle \Delta\left(\frac{1}{z!}\sum_{\varrho\in\mathbb{S}_z}\prod_{J=1}^{z}\left(\Delta^{-1}\left(s_{\varrho(J)}, \pounds_{\varrho(J)}\right)\right)^{\psi_J}\right), \right.$$
$$\left.\left(\begin{array}{l}\left(\sqrt[q]{1-\prod_{\varrho\in S_z}\left(1-\prod_{J=1}^{z}\mathfrak{I}_{\varrho(J)}^{q\psi_J}\right)^{\frac{1}{z!}}}, \sqrt[q]{1-\prod_{\varrho\in S_z}\left(1-\prod_{J=1}^{z}\omega_{\mathfrak{I}_{\varrho(J)}}^{q\psi_J}\right)^{\frac{1}{z!}}}\right), \\ \left(\left(\prod_{\varrho\in\mathbb{S}_z}\sqrt[q]{1-\prod_{J=1}^{z}\left(1-\aleph_{\varrho(J)}^{q}\right)^{\psi_J}}\right)^{\frac{1}{z!}}, \left(\prod_{\varrho\in\mathbb{S}_z}\sqrt[q]{1-\prod_{J=1}^{z}\left(1-\omega_{\aleph_{\varrho(J)}}^{q}\right)^{\psi_J}}\right)^{\frac{1}{z!}}\right)\end{array}\right)\right\rangle.$$

Therefore,

$$\left(\frac{1}{z!}\oplus_{\varrho\in\mathbb{S}_z}\otimes_{\jmath=1}^{z}\mathcal{N}_{\varrho(\jmath)}^{\psi_\jmath}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}=\Bigg\langle \Delta\left(\frac{1}{z!}\sum_{\varrho\in\mathbb{S}_z}\prod_{\jmath=1}^{z}\left(\Delta^{-1}\left(s_{\varrho(\jmath)},\pounds_{\varrho(\jmath)}\right)\right)^{\psi_\jmath}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}},$$

$$\left(\begin{array}{l}\left(\left(\sqrt[q]{1-\left(\prod_{\varrho\in S_z}\left(1-\prod_{\jmath=1}^{z}\mathfrak{I}_{\varrho(\jmath)}^{q\psi_\jmath}\right)\right)^{\frac{1}{z!}}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}},\left(\sqrt[q]{1-\left(\prod_{\varrho\in S_z}\left(1-\prod_{\jmath=1}^{z}\omega_{\mathfrak{I}_{\varrho(\jmath)}}^{q\psi_\jmath}\right)\right)^{\frac{1}{z!}}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}\right),\\ \left(\begin{array}{l}\sqrt[q]{1-\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\aleph_{\varrho(\jmath)}^{q}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}},\\ \sqrt[q]{1-\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\omega_{\aleph_{\varrho(\jmath)}}^{q}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}}\end{array}\right)\end{array}\right)\Bigg\rangle.$$

Now, we need to prove that the aggregated value by Cq-ROFTLMM operator is also a Cq-ROFTLN.

$$\leq\left(\sqrt[q]{1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\mathfrak{I}_{\varrho(\jmath)}^{q\psi_\jmath}\right)\right)^{\frac{1}{z!}}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}\leq 1,$$

$$0\leq\left(\sqrt[q]{1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\omega_{\mathfrak{I}_{\varrho(\jmath)}}^{q\psi_\jmath}\right)\right)^{\frac{1}{z!}}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}\leq 1.$$

Similarly,

$$0\leq\sqrt[q]{1-\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\aleph_{\varrho(\jmath)}^{q}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}}\leq 1,$$

$$0\leq\sqrt[q]{1-\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\omega_{\aleph_{\varrho(\jmath)}}^{q}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}}\leq 1.$$

Since $\mathfrak{I}_{\varrho(\jmath)}^{q}+\aleph_{\varrho(\jmath)}^{q}\leq 1$ and $\omega_{\mathfrak{I}_{\varrho(\jmath)}}^{q}+\omega_{\aleph_{\varrho(\jmath)}}^{q}\leq 1$, so

$$\begin{aligned}&\left(\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\mathfrak{I}_{\varrho(\jmath)}^{q\psi_\jmath}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}+1-\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\aleph_{\varrho(\jmath)}^{q}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}\right)\\ &\leq\left(\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\aleph_{\varrho(\jmath)}^{q}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}+1-\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\aleph_{\varrho(\jmath)}^{q}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}\right)\\ &=1.\end{aligned}$$

Similarly,

$$\left(\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{j=1}^{z}\omega_{\mathfrak{I}_{\varrho(j)}}^{q\psi_j}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{j=1}^{z}\psi_j}}+1-\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{j=1}^{z}\left(1-\omega_{\aleph_{\varrho(j)}}^{q}\right)^{\psi_j}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{j=1}^{z}\psi_j}}\right)$$
$$\leq\left(\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{j=1}^{z}\left(1-\omega_{\aleph_{\varrho(j)}}^{q}\right)^{\psi_j}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{j=1}^{z}\psi_j}}+1-\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{j=1}^{z}\left(1-\omega_{\aleph_{\varrho(j)}}^{q}\right)^{\psi_j}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{j=1}^{z}\psi_j}}\right)$$
$$=1.$$

Thus, the aggregated value by Cq-ROFTLMM operator is a Cq-ROFTLN. □

Example 2 Let $\mathcal{N}_1=\langle(s_5,0.05),((0.4,0.3),(0.5,0.7))\rangle$, $\mathcal{N}_2=\langle(s_2,0.02),((0.3,0.6),(0.4,0.2))\rangle$, and $\mathcal{N}_3=\langle(s_4,-0.04),((0.7,0.4),(0.6,0.8))\rangle$ be three Cq-ROFTLNs which are derived from LTS $S=\{s_j | j\in\{0,\dots,6\}\}$. $\psi=(1,1,2)$ is the parameters vector and $q=4$, then

$$\Delta\left(\left(\frac{1}{3!}\begin{bmatrix}\left(\Delta^{-1}(s_5,0.05)\right)^1\times\left(\Delta^{-1}(s_2,0.02)\right)^1\times\left(\Delta^{-1}(s_4,-0.04)\right)^2\\+\left(\Delta^{-1}(s_5,0.05)\right)^1\times\left(\Delta^{-1}(s_4,-0.04)\right)^1\times\left(\Delta^{-1}(s_2,0.02)\right)^2\\+\left(\Delta^{-1}(s_2,0.02)\right)^1\times\left(\Delta^{-1}(s_5,0.05)\right)^1\times\left(\Delta^{-1}(s_4,-0.04)\right)^2\\+\left(\Delta^{-1}(s_2,0.02)\right)^1\times\left(\Delta^{-1}(s_4,-0.04)\right)^1\times\left(\Delta^{-1}(s_5,0.05)\right)^2\\+\left(\Delta^{-1}(s_4,-0.04)\right)^1\times\left(\Delta^{-1}(s_2,0.02)\right)^1\times\left(\Delta^{-1}(s_5,0.05)\right)^2\\+\left(\Delta^{-1}(s_4,-0.04)\right)^1\times\left(\Delta^{-1}(s_5,0.05)\right)^1\times\left(\Delta^{-1}(s_2,0.02)\right)^2\end{bmatrix}\right)^{\frac{1}{(1+1+2)}}\right)=(s_4,-0.0763),$$

$$\begin{pmatrix}\left(\sqrt[4]{1-\begin{pmatrix}(1-0.4^4\times0.3^4\times0.7^8)\times(1-0.4^4\times0.7^4\times0.3^8)\\\times(1-0.3^4\times0.7^4\times0.4^8)\times(1-0.3^4\times0.4^4\times0.7^8)\\\times(1-0.7^4\times0.3^4\times0.4^8)\times(1-0.7^4\times0.4^4\times0.3^8)\end{pmatrix}^{\frac{1}{3!}}}\right)^{\frac{1}{1+1+2}},\\\left(\sqrt[4]{1-\begin{pmatrix}(1-0.3^4\times0.6^4\times0.4^8)\times(1-0.3^4\times0.4^4\times0.6^8)\\\times(1-0.6^4\times0.4^4\times0.3^8)\times(1-0.6^4\times0.3^4\times0.4^8)\\\times(1-0.4^4\times0.6^4\times0.3^8)\times(1-0.4^4\times0.3^4\times0.6^8)\end{pmatrix}^{\frac{1}{3!}}}\right)^{\frac{1}{1+1+2}}\end{pmatrix}=(0.4635,0.4318),$$

$$\begin{pmatrix}\sqrt[4]{1-\left(1-\begin{pmatrix}(1-0.9375^1\times0.9744^1\times0.8704^2)\times(1-0.9375^1\times0.8704^1\times0.9744^2)\\\times(1-0.9744^1\times0.8704^1\times0.9375^2)\times(1-0.9744^1\times0.9375^1\times0.8704^2)\\\times(1-0.8704^1\times0.9375^1\times0.9744^2)\times(1-0.8704^1\times0.9744^1\times0.9375^2)\end{pmatrix}^{\frac{1}{3!}}\right)^{\frac{1}{1+1+2}}},\\\sqrt[4]{1-\left(1-\begin{pmatrix}(1-0.7599^1\times0.9984^1\times0.5904^2)\times(1-0.7599^1\times0.5904^1\times0.9984^2)\\\times(1-0.9984^1\times0.5904^1\times0.7599^2)\times(1-0.9984^1\times0.7599^1\times0.5904^2)\\\times(1-0.5904^1\times0.7599^1\times0.9984^2)\times(1-0.5904^1\times0.9984^1\times0.7599^2)\end{pmatrix}^{\frac{1}{3!}}\right)^{\frac{1}{1+1+2}}}\end{pmatrix}=(0.5192,0.6910).$$

Therefore,

$$\mathrm{C}q\text{-ROFTLMM}^{(1,1,2)}(\mathcal{N}_1,\mathcal{N}_2,\mathcal{N}_3)=\langle(s_4,-0.0763),((0.4635,0.4318),(0.5192,0.6910))\rangle.$$

In below, we will prove some useful properties of Cq-ROFTLMM operator.

Property 1 (Idempotency) *Let* $\mathcal{N}_j=\langle(s_j,\pounds_j),((\mathfrak{I}_j,\omega_{\mathfrak{I}_j}),(\aleph_j,\omega_{\aleph_j}))\rangle$ *be a collection of Cq-ROFTLNs and all are equal, i.e.,* $\mathcal{N}_j=\mathcal{N}$ *for all* $j=1,2,\dots,z$, *then*

$$\mathrm{C}q-\mathrm{ROFTLMM}^{\Psi}(\mathcal{N}_1,\mathcal{N}_2,\ldots,\mathcal{N}_z)=\mathcal{N}. \tag{7}$$

Proof

$$\begin{aligned}\mathrm{C}q\text{-}\mathrm{ROFTLMM}^{\Psi}(\mathcal{N}_1,\mathcal{N}_2,\ldots,\mathcal{N}_z)&=\left(\frac{1}{z!}\sum_{\varrho\in\mathbb{S}_z}\prod_{\jmath=1}^{z}\mathcal{N}_\jmath^{\psi_\jmath}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}\\&=\left(\frac{1}{z!}.z!.\mathcal{N}^{\sum_{\jmath=1}^{z}\psi_\jmath}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}\\&=\mathcal{N}.\end{aligned}$$

□

Property 2 (Monotonicity) *Let* $\mathcal{N}_\jmath=\langle(s_\jmath,\pounds_\jmath),((\mathfrak{I}_\jmath,\omega_{\mathfrak{I}_\jmath}),(\aleph_\jmath,\omega_{\aleph_\jmath}))\rangle$ *and* $\mathcal{N}'_\jmath=\langle(s'_\jmath,\pounds'_\jmath),((\mathfrak{I}'_\jmath,\omega_{\mathfrak{I}'_\jmath}),(\aleph'_\jmath,\omega_{\aleph'_\jmath}))\rangle(\jmath=1,2,\ldots,z)$ *be two sets of Cq-ROFTLNs. If* $(s_\jmath,\pounds_\jmath)\le(s'_\jmath,\pounds'_\jmath),\ \mathfrak{I}_\jmath\le\mathfrak{I}'_\jmath,\omega_{\mathfrak{I}_\jmath}\le\omega_{\mathfrak{I}'_\jmath},\aleph_\jmath\ge\aleph'_\jmath$ *and* $\omega_{\aleph_\jmath}\ge\omega_{\aleph'_\jmath}$, *then*

$$\mathrm{C}q\text{-}\mathrm{ROFTLMM}^{\Psi}(\mathcal{N}_1,\mathcal{N}_2,\ldots,\mathcal{N}_z)\le\mathrm{C}q\text{-}\mathrm{ROFTLMM}^{\Psi}(\mathcal{N}'_1,\mathcal{N}'_2,\ldots,\mathcal{N}'_z). \tag{8}$$

Proof Since

$$(s_{\varrho(\jmath)},\pounds_{\varrho(\jmath)})\le(s'_{\varrho(\jmath)},\pounds'_{\varrho(\jmath)}).$$

Then we have

$$\begin{aligned}&\left(\Delta^{-1}\left(s_{\varrho(\jmath)},\pounds_{\varrho(\jmath)}\right)\right)^{\psi_\jmath}\le\left(\Delta^{-1}(s'_{\varrho(\jmath)},\pounds'_{\varrho(\jmath)})\right)^{\psi_\jmath},\\&\prod_{\jmath=1}^{z}\left(\Delta^{-1}\left(s_{\varrho(\jmath)},\pounds_{\varrho(\jmath)}\right)\right)^{\psi_\jmath}\le\prod_{\jmath=1}^{z}\left(\Delta^{-1}(s'_{\varrho(\jmath)},\pounds'_{\varrho(\jmath)})\right)^{\psi_\jmath},\\&\frac{1}{z!}\sum_{\varrho\in\mathbb{S}_z}\prod_{\jmath=1}^{z}\left(\Delta^{-1}\left(s_{\varrho(\jmath)},\pounds_{\varrho(\jmath)}\right)\right)^{\psi_\jmath}\le\frac{1}{z!}\sum_{\varrho\in\mathbb{S}_z}\prod_{\jmath=1}^{z}\left(\Delta^{-1}(s'_{\varrho(\jmath)},\pounds'_{\varrho(\jmath)})\right)^{\psi_\jmath}.\end{aligned}$$

Similarly, we can also prove

$$\left(\prod_{\jmath=1}^{z}\mathfrak{I}_{\varrho_\jmath}^{q\psi_\jmath}\right)\le\left(\prod_{\jmath=1}^{z}\mathfrak{I}'^{q\psi_\jmath}_{\varrho(\jmath)}\right),\quad\left(\prod_{\jmath=1}^{z}\omega_{\mathfrak{I}_{\varrho_\jmath}}^{q\psi_\jmath}\right)\le\left(\prod_{\jmath=1}^{z}\omega_{\mathfrak{I}'_{\varrho(\jmath)}}^{q\psi_\jmath}\right),$$

and

$$\begin{aligned}\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\mathfrak{I}_{\varrho_\jmath}^{q\psi_\jmath}\right)\right)^{\frac{1}{z!}}\ge\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\mathfrak{I}'^{q\psi_\jmath}_{\varrho_\jmath}\right)\right)^{\frac{1}{z!}},\quad&\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\omega_{\mathfrak{I}_{\varrho_\jmath}}^{q\psi_\jmath}\right)\right)^{\frac{1}{z!}}\\&\ge\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\omega_{\mathfrak{I}'_{\varrho_\jmath}}^{q\psi_\jmath}\right)\right)^{\frac{1}{z!}}.\end{aligned}$$

So, we have

$$\left(\left(\sqrt[q]{1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{j=1}^{z}\mathfrak{I}_{\varrho_j}^{q\psi_j}\right)\right)^{\frac{1}{z!}}}\right)^{\frac{1}{\sum_{j=1}^{z}\psi_j}}\right)\leq\left(\left(\sqrt[q]{1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{j=1}^{z}\mathfrak{I}_{\varrho_j}^{\prime q\psi_j}\right)\right)^{\frac{1}{z!}}}\right)^{\frac{1}{\sum_{j=1}^{z}\psi_j}}\right),$$

$$\left(\left(\sqrt[q]{1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{j=1}^{z}\omega_{\mathfrak{I}_{\varrho_j}}^{q\psi_j}\right)\right)^{\frac{1}{z!}}}\right)^{\frac{1}{\sum_{j=1}^{z}\psi_j}}\right)\leq\left(\left(\sqrt[q]{1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{j=1}^{z}\omega_{\mathfrak{I}'_{\varrho_j}}^{q\psi_j}\right)\right)^{\frac{1}{z!}}}\right)^{\frac{1}{\sum_{j=1}^{z}\psi_j}}\right).$$

That is $\mathfrak{I}_j \leq \mathfrak{I}'_j$ and $\omega_{\mathfrak{I}_j} \leq \omega_{\mathfrak{I}'_j}$. Similarly, we can also get $\aleph_j \geq \aleph'_j$ and $\omega_{\aleph_j} \geq \omega_{\aleph'_j}$. □

Property 3 (Boundedness) *Let* $\mathcal{N}_j = \langle (s_j, \pounds_j), ((\mathfrak{I}_j, \omega_{\mathfrak{I}_j}), (\aleph_j, \omega_{\aleph_j}))\rangle (j = 1, 2, \dots, z)$ *be set of Cq-ROFTNs, then*

$$\mathcal{N}^- = \min_j \mathcal{N}_j = \left\langle \min_j(s_j, \pounds_j), \left((\min_j \mathfrak{I}_j, \min_j \omega_{\mathfrak{I}_j}), (\max_j \aleph_j, \max_j \omega_{\aleph_j})\right)\right\rangle$$

and

$$\mathcal{N}^+ = \max_j \mathcal{N}_j = \left\langle \max_j(s_j, \pounds_j), \left((\max_j \mathfrak{I}_j, \max_j \omega_{\mathfrak{I}_j}), (\min_j \aleph_j, \min_j \omega_{\aleph_j})\right)\right\rangle.$$

We can obtain the proof of boundedness property by utilizing the properties of monotonicity and idempotency.

$$\mathcal{N}^- \leq \mathrm{C}q - \mathrm{ROFTLMM}^{\psi}(\mathcal{N}_1, \mathcal{N}_2, \dots, \mathcal{N}_z) \leq \mathcal{N}^+. \tag{9}$$

According to the parameters vector, we will discuss some cases of Cq-ROFTLMM operator.

Case 1. When $\psi = (1, 0, \dots, 0)$, the Cq-ROFTLMM operator is simplified into the Cq-ROFTL arithmetic averaging (Cq-ROFTLAA) operator.

$$\mathrm{C}q - \mathrm{ROFTLAA}(\mathcal{N}_1, \mathcal{N}_2, \dots, \mathcal{N}_z) = \left\langle \Delta\left(\sum_{j=1}^{z}\left(\frac{1}{z}\times\Delta^{-1}(s_j, \pounds_j)\right)\right),\right.$$
$$\left.\left(\left(\sqrt[q]{1-\prod_{j=1}^{z}(1-\mathfrak{I}_j^q)^{\frac{1}{z}}}, \sqrt[q]{1-\prod_{j=1}^{z}(1-\omega_{\mathfrak{I}_j}^q)^{\frac{1}{z}}}\right), \left(\prod_{j=1}^{z}\aleph_j^{\frac{1}{z}}, \quad \prod_{j=1}^{z}\omega_{\aleph_j}^{\frac{1}{z}}\right)\right)\right\rangle.$$

Case 2. When $\psi = (1, 1, 0, 0, \dots, 0)$, the C$q$-ROFTLMM operator is simplified into the Cq-ROFTL Bonferroni mean (Cq-ROFTLBM) operator.

$$\text{C}q\text{-ROFTLBM}(\mathcal{N}_1,\mathcal{N}_2,\ldots,\mathcal{N}_z)=\left\langle \Delta\left(\left(\frac{1}{z(z-1)}\sum_{\imath,\jmath=1,\imath\neq\jmath}^{z}(\Delta^{-1}(s_\jmath,\pounds_\jmath)\times\Delta^{-1}(s_\jmath,\pounds_\jmath))\right)^{\frac{1}{2}}\right),\right.$$
$$\left.\left(\begin{array}{c}\left(\left(\sqrt[q]{1-\left(\prod_{\imath,\jmath=1,\imath\neq\jmath}^{z}(1-\Im_\imath^q\times\Im_\jmath^q)\right)^{\frac{1}{z(z-1)}}}\right)^{\frac{1}{2}},\left(\sqrt[q]{1-\left(\prod_{\imath,\jmath=1,\imath\neq\jmath}^{z}(1-\omega_{\Im_\imath}^q\times\omega_{\Im_\jmath}^q)\right)^{\frac{1}{z(z-1)}}}\right)^{\frac{1}{2}}\right),\\ \left(\sqrt[q]{1-\left(1-\left(\prod_{\imath,\jmath=1,\imath\neq\jmath}^{z}(1-(1-\aleph_\imath^q)(1-\aleph_\jmath^q))\right)^{\frac{1}{z(z-1)}}\right)^{\frac{1}{2}}},\sqrt[q]{1-\left(1-\left(\prod_{\imath,\jmath=1,\imath\neq\jmath}^{z}\left(1-\left(1-\omega_{\aleph_\imath}^q\right)\left(1-\omega_{\aleph_\jmath}^q\right)\right)\right)^{\frac{1}{z(z-1)}}\right)^{\frac{1}{2}}}\right)\end{array}\right)\right\rangle.$$

Case 3. When $\psi=(\overbrace{1,1,\ldots,1}^{k},\overbrace{0,0,\ldots,0}^{z-k})$, the C$q$-ROFTLMM operator is simplified into the Cq-ROFTL Maclaurin symmetric mean (Cq-ROFTLMSM) operator.

$$\text{C}q\text{-ROFTLMSM}(\mathcal{N}_1,\mathcal{N}_2,\ldots,\mathcal{N}_z)=\left\langle \Delta\left(\left(\frac{1}{C_z^k}\sum_{1\leq\imath_1<\ldots<\imath_k\leq z}\prod_{\jmath=1}^{k}\Delta^{-1}(s_{\imath_\jmath},\pounds_{\imath_\jmath})\right)^{\frac{1}{k}}\right),\right.$$
$$\left.\left(\begin{array}{c}\left(\left(\sqrt[q]{1-\left(\prod_{1\leq\imath_1<\ldots<\imath_k\leq z}\left(1-\prod_{\jmath=1}^{k}\Im_{\imath_\jmath}^q\right)\right)^{\frac{1}{C_z^k}}}\right)^{\frac{1}{k}},\left(\sqrt[q]{1-\left(\prod_{1\leq\imath_1<\ldots<\imath_k\leq z}\left(1-\prod_{\jmath=1}^{k}\omega_{\Im_{\imath_\jmath}}^q\right)\right)^{\frac{1}{C_z^k}}}\right)^{\frac{1}{k}}\right),\\ \left(\sqrt[q]{1-\left(1-\left(\prod_{1\leq\imath_1<\ldots<\imath_k\leq z}\left(1-\prod_{\jmath=1}^{k}\left(1-\aleph_{\imath_\jmath}^q\right)\right)\right)^{\frac{1}{C_z^k}}\right)^{\frac{1}{k}}},\sqrt[q]{1-\left(1-\left(\prod_{1\leq\imath_1<\ldots<\imath_k\leq z}\left(1-\prod_{\jmath=1}^{k}\left(1-\omega_{\aleph_{\imath_\jmath}}^q\right)\right)\right)^{\frac{1}{C_z^k}}\right)^{\frac{1}{k}}}\right)\end{array}\right)\right\rangle.$$

Case 4. If $\psi=(1,1,\ldots,1)$ or $(\frac{1}{z},\frac{1}{z},\ldots,\frac{1}{z})$, the C$q$-ROFTLMM operator is simplified to the Cq-ROFTL geometric averaging (Cq-ROFTLGA) operator.

$$\text{C}q\text{-ROFTLGA}(\mathcal{N}_1,\mathcal{N}_2,\ldots,\mathcal{N}_z)=\left\langle \Delta\left(\left(\prod_{\jmath=1}^{z}\Delta^{-1}(s_\jmath,\pounds_\jmath)\right)^{\frac{1}{z}}\right),\right.$$
$$\left.\left(\left(\left(\prod_{\jmath=1}^{z}\Im_\jmath\right)^{\frac{1}{z}},\ \left(\prod_{\jmath=1}^{z}\omega_{\Im_\jmath}\right)^{\frac{1}{z}}\right),\left(\sqrt[q]{1-\prod_{\jmath=1}^{z}(1-\aleph_\jmath^q)^{\frac{1}{z}}},\sqrt[q]{1-\prod_{\jmath=1}^{z}\left(1-\omega_{\aleph_\jmath}^q\right)^{\frac{1}{z}}}\right)\right)\right\rangle.$$

4.2 Cq-ROFTLWMM operator

In subsection 4.1, we can see that Cq-ROFTLMM operator cannot consider weights of attributes. In most practical problems, particularly in MAGDM problems, the weights of attributes play a vital role in the aggregation process. To overcome the limitation of Cq-ROFTLMM, it is necessary to introduce the Cq-ROFTLWMM operator as given.

Definition 13 Let $\mathcal{N}_\jmath=\langle(s_\jmath,\pounds_\jmath),((\Im_\jmath,\omega_{\Im_\jmath}),(\aleph_\jmath,\omega_{\aleph_\jmath}))\rangle(\jmath=1,2,\ldots,z)$ be a set of Cq-ROFTLNs with weight vector $\varpi=(\varpi_1,\varpi_2,\ldots,\varpi_z)^T$, $\varpi_\jmath\in[0,1]$, $\sum_{\jmath=1}^{z}\varpi_\jmath=1$ and $\psi=(\psi_1,\psi_2,\ldots\psi_z)\in\mathbb{R}^z$ be a parameters vector, then the Cq-ROFTLWMM operator is described as follows:

$$\mathrm{C}q\text{-ROFTLWMM}_{\varpi}^{\Psi}(\mathcal{N}_1,\mathcal{N}_2,\ldots,\mathcal{N}_z)=\left(\frac{1}{z!}\oplus_{\varrho\in\mathbb{S}_z}\otimes_{\jmath=1}^{z}\left(z\varpi_{\varrho(\jmath)}\mathcal{N}_{\varrho(\jmath)}\right)^{\psi_\jmath}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}.$$

On the basis of operational laws of Cq-ROFTLNs, we can demonstrate Theorem 3.

Theorem 3 *Let $\mathcal{N}_\jmath=\langle(s_\jmath,\pounds_\jmath),((\mathfrak{I}_\jmath,\omega_{\mathfrak{I}_\jmath}),(\aleph_\jmath,\omega_{\aleph_\jmath}))(\jmath=1,2,\ldots,z)$ be a set of Cq-ROFTLNs, their aggregated result by utilizing the Cq-ROFTLWMM operator is also a Cq-ROFTLN, then*

$$\mathrm{C}q\text{-ROFTLWMM}_{\varpi}^{\Psi}(\mathcal{N}_1,\mathcal{N}_2,\ldots,\mathcal{N}_z)=\left\langle \Delta\left(\left(\frac{1}{z!}\sum_{\varrho\in\mathbb{S}_z}\prod_{\jmath=1}^{z}\left(z\varpi_{\varrho(\jmath)}\Delta^{-1}\left(s_{\varrho(\jmath)},\pounds_{\varrho(\jmath)}\right)\right)^{\psi_\jmath}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}\right),\right.$$
$$\left.\left(\begin{array}{l}\left(\begin{array}{l}\left(\sqrt[q]{1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\left(1-\mathfrak{I}_{\varrho(\jmath)}^{q}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}},\\ \left(\sqrt[q]{1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\left(1-\omega_{\mathfrak{I}_{\varrho(\jmath)}}^{q}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}\end{array}\right),\\ \left(\begin{array}{l}\sqrt[q]{1-\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\left(\aleph_{\varrho(\jmath)}^{q}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}},\\ \sqrt[q]{1-\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\left(\omega_{\aleph_{\varrho(\jmath)}}^{q}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}}\end{array}\right)\end{array}\right)\right\rangle. \tag{10}$$

Proof

$$z\varpi_{\varrho(\jmath)}\mathcal{N}_{\varrho(\jmath)}=\left\langle \Delta\left(z\varpi_{\varrho(\jmath)}\Delta^{-1}\left(s_{\varrho(\jmath)},\pounds_{\varrho(\jmath)}\right)\right),\left(\begin{array}{l}\left(\sqrt[q]{1-\left(1-\mathfrak{I}_{\varrho(\jmath)}^{q}\right)^{z\varpi_{\varrho(\jmath)}}},\sqrt[q]{1-\left(1-\omega_{\mathfrak{I}_{\varrho(\jmath)}}^{q}\right)^{z\varpi_{\varrho(\jmath)}}}\right),\\ \left(\aleph_{\varrho(\jmath)}^{z\varpi_{\varrho(\jmath)}},\quad \omega_{\aleph_{\varrho(\jmath)}}^{z\varpi_{\varrho(\jmath)}}\right)\end{array}\right)\right\rangle,$$

$$\left(z\varpi_{\varrho(\jmath)}\mathcal{N}_{\varrho(\jmath)}\right)^{\psi_\jmath}=\left\langle \Delta\left(z\varpi_{\varrho(\jmath)}\Delta^{-1}\left(s_{\varrho(\jmath)},\pounds_{\varrho(\jmath)}\right)\right)^{\psi_\jmath},\left(\begin{array}{l}\left(\left(\sqrt[q]{1-(1-\mathfrak{I}_{\varrho(\jmath)}^{q})^{z\varpi_{\varrho(\jmath)}}}\right)^{\psi_\jmath},\left(\sqrt[q]{1-(1-\omega_{\mathfrak{I}_{\varrho(\jmath)}}^{q})^{z\varpi_{\varrho(\jmath)}}}\right)^{\psi_\jmath}\right),\\ \left(\sqrt[q]{1-\left(1-\aleph_{\varrho(\jmath)}^{qz\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}},\quad \sqrt[q]{1-\left(1-\omega_{\aleph_{\varrho(\jmath)}}^{qz\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}}\right)\end{array}\right)\right\rangle.$$

Thereafter,

$$\otimes_{\jmath=1}^{z}\left(z\varpi_{\varrho(\jmath)}\mathcal{N}_{\varrho(\jmath)}\right)^{\psi_\jmath}=\Bigg\langle\Delta\Bigg(\prod_{\jmath=1}^{z}\left(z\varpi_{\varrho(\jmath)}\Delta^{-1}\left(s_{\varrho(\jmath)},\pounds_{\varrho(\jmath)}\right)\right)^{\psi_\jmath}\Bigg),$$
$$\left(\begin{array}{l}\left(\prod_{\jmath=1}^{z}\left(\sqrt[q]{1-\left(1-\mathfrak{I}_{\varrho_{(\jmath)}}^{q}\right)^{z\varpi_{\varrho(\jmath)}}}\right)^{\psi_\jmath},\prod_{\jmath=1}^{z}\left(\sqrt[q]{1-\left(1-\omega_{\mathfrak{I}_{\varrho_{(\jmath)}}}^{q}\right)^{z\varpi_{\varrho(\jmath)}}}\right)^{\psi_\jmath}\right),\\ \left(\sqrt[q]{1-\prod_{\jmath=1}^{z}\left(1-\aleph_{\varrho_{(\jmath)}}^{qz\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}},\sqrt[q]{1-\prod_{\jmath=1}^{z}\left(1-\omega_{\aleph_{\varrho_{(\jmath)}}}^{qz\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}}\right)\end{array}\right)\Bigg\rangle$$

$$\oplus_{\varrho\in\mathbb{S}_z}\otimes_{\jmath=1}^{z}\left(z\varpi_{\varrho(\jmath)}\mathcal{N}_{\varrho(\jmath)}\right)^{\psi_\jmath}=\Bigg\langle\Delta\Bigg(\sum_{\varrho\in\mathbb{S}_z}\prod_{\jmath=1}^{z}\left(z\varpi_{\varrho(\jmath)}\Delta^{-1}\left(s_{\varrho(\jmath)},\pounds_{\varrho(\jmath)}\right)\right)^{\psi_\jmath}\Bigg),$$
$$\left(\begin{array}{l}\left(\left(\sqrt[q]{1-\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\left(1-\mathfrak{I}_{\varrho_{(\jmath)}}^{q}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)}\right),\left(\sqrt[q]{1-\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\left(1-\omega_{\mathfrak{I}_{\varrho_{(\jmath)}}}^{q}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)}\right)\right),\\ \left(\prod_{\varrho\in\mathbb{S}_z}\sqrt[q]{1-\prod_{\jmath=1}^{z}\left(1-\aleph_{\varrho_{(\jmath)}}^{qz\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}},\quad\prod_{\varrho\in\mathbb{S}_z}\sqrt[q]{1-\prod_{\jmath=1}^{z}\left(1-\omega_{\aleph_{\varrho_{(\jmath)}}}^{qz\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}}\right)\end{array}\right)\Bigg\rangle.$$

Thus,

$$\frac{1}{z!}\oplus_{\varrho\in\mathbb{S}_z}\otimes_{\jmath=1}^{z}\left(z\varpi_{\varrho(\jmath)}\mathcal{N}_{\varrho(\jmath)}\right)^{\psi_\jmath}=\Bigg\langle\Delta\Bigg(\frac{1}{z!}\sum_{\varrho\in\mathbb{S}_z}\prod_{\jmath=1}^{z}\left(z\varpi_{\varrho(\jmath)}\Delta^{-1}\left(s_{\varrho(\jmath)},\pounds_{\varrho(\jmath)}\right)\right)^{\psi_\jmath}\Bigg),$$
$$\left(\begin{array}{l}\left(\begin{array}{l}\sqrt[q]{1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\left(\prod_{\jmath=1}^{z}\left(1-\left(1-\mathfrak{I}_{\varrho_{(\jmath)}}^{q}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)\right)\right)^{\frac{1}{z!}}},\\ \sqrt[q]{1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\left(\prod_{\jmath=1}^{z}\left(1-\left(1-\omega_{\mathfrak{I}_{\varrho_{(\jmath)}}}^{q}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)\right)\right)^{\frac{1}{z!}}}\end{array}\right),\\ \left(\left(\prod_{\varrho\in\mathbb{S}_z}\sqrt[q]{1-\prod_{\jmath=1}^{z}\left(1-\aleph_{\varrho_{(\jmath)}}^{qz\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}}\right)^{\frac{1}{z!}},\left(\prod_{\varrho\in\mathbb{S}_z}\sqrt[q]{1-\prod_{\jmath=1}^{z}\left(1-\omega_{\aleph_{\varrho_{(\jmath)}}}^{qz\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}}\right)^{\frac{1}{z!}}\right)\end{array}\right)\Bigg\rangle.$$

Therefore,

$$\left(\frac{1}{z!}\oplus_{\varrho\in\mathbb{S}_z}\otimes_{\jmath=1}^{z}\left(z\varpi_{\varrho(\jmath)}\mathcal{N}_{\varrho(\jmath)}\right)^{\psi_\jmath}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}$$

$$= \left\langle \Delta\left(\left(\frac{1}{z!}\sum_{\varrho\in\mathbb{S}_z}\prod_{\jmath=1}^{z}\left(z\varpi_{\varrho(\jmath)}\Delta^{-1}\left(s_{\varrho(\jmath)},\pounds_{\varrho(\jmath)}\right)\right)^{\psi_\jmath}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}\right), \left(\begin{array}{l}\left(\begin{array}{l}\left(\sqrt[q]{1-\prod_{\varrho\in\mathbb{S}_z}\left(1-\left(\prod_{\jmath=1}^{z}\left(1-\left(1-\mathfrak{I}_{\varrho_{(\jmath)}}^{q}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}, \\ \left(\sqrt[q]{1-\prod_{\varrho\in\mathbb{S}_z}\left(1-\left(\prod_{\jmath=1}^{z}\left(1-\left(1-\omega_{\mathfrak{I}_{\varrho_{(\jmath)}}}^{q}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}\end{array}\right), \\ \left(\begin{array}{l}\sqrt[q]{1-\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\left(\aleph_{\varrho_{(\jmath)}}^{q}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}}, \\ \sqrt[q]{1-\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\left(\omega_{\aleph_{\varrho_{(\jmath)}}}^{q}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}}\end{array}\right)\end{array}\right)\right\rangle.$$

and we can get following easily,

$$0\leq\left(\sqrt[q]{1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\left(1-\mathfrak{I}_{\varrho_{(\jmath)}}^{q}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}\leq 1,\quad 0\leq\left(\sqrt[q]{1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\left(1-\omega_{\mathfrak{I}_{\varrho_{(\jmath)}}}^{q}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}\leq 1,$$

$$0\leq\sqrt[q]{1-\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\left(\aleph_{\varrho_{(\jmath)}}^{q}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}}\leq 1,\ 0\leq\sqrt[q]{1-\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\left(\omega_{\aleph_{\varrho_{(\jmath)}}}^{q}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}}\leq 1.$$

Therefore,

$$\left(\left(\left(\sqrt[q]{1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\left(1-\mathfrak{I}^q_{\varrho_{(\jmath)}}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}\right)^q+\left(\sqrt[q]{1-\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\left(\aleph^q_{\varrho_{(\jmath)}}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}}\right)^q\right)$$

$$\leq\left(\left(\left(\sqrt[q]{1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\left(1-\mathfrak{I}^q_{\varrho_{(\jmath)}}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}\right)^q+\left(\sqrt[q]{1-\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\left(1-\mathfrak{I}^q_{\varrho_{(\jmath)}}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}}\right)^q\right)=1.$$

Similarly,

$$\left(\left(\left(\sqrt[q]{1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\left(1-\omega^q_{\mathfrak{I}_{\varrho_{(\jmath)}}}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}\right)^q+\left(\sqrt[q]{1-\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\left(\omega^q_{\aleph_{\varrho_{(\jmath)}}}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}}\right)^q\right)$$

$$\leq\left(\left(\left(\sqrt[q]{1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\left(1-\omega^q_{\mathfrak{I}_{\varrho_{(\jmath)}}}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}\right)^q+\left(\sqrt[q]{1-\left(1-\left(\prod_{\varrho\in\mathbb{S}_z}\left(1-\prod_{\jmath=1}^{z}\left(1-\left(1-\omega^q_{\mathfrak{I}_{\varrho_{(\jmath)}}}\right)^{z\varpi_{\varrho(\jmath)}}\right)^{\psi_\jmath}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}}}\right)^q\right)=1$$

□

Thus, the aggregated value by Cq − ROFTLWMM operator is a Cq-ROFTLN.

Example ***3*** Let $\mathcal{N}_1=\langle(s_5,0.05),((0.4,0.3),(0.5,0.7))\rangle$, $\mathcal{N}_2=\langle(s_2,0.02),((0.3,0.6),(0.4,0.2))\rangle$, and $\mathcal{N}_3=\langle(s_4,-0.04),((0.7,0.4),(0.6,0.8))\rangle$ be three Cq-ROFTLNs which are derived from LTS $S=\{s_j|j\in\{0,\dots,6\}\}$. $\psi=(1,1,2)$ is the parameters vector, $q=4$ and $\varpi=(0.32,0.53,0.15)$ be the weighting vector, then we have

$$
\begin{aligned}
&\mathrm{C}q\text{-ROFTLWMM}^{(1,1,2)}_{(0.32,0.53,0.15)}(\mathcal{N}_1,\mathcal{N}_2,\mathcal{N}_3)\\
&= \left\langle \Delta\left(\left(\frac{1}{3!}\left[\begin{array}{l}
0.96\left(\Delta^{-1}(s_5,0.05)\right)^1\times 1.59\left(\Delta^{-1}(s_2,0.02)\right)^1\times 0.45\left(\Delta^{-1}(s_4,-0.04)\right)^2\\
+0.96\left(\Delta^{-1}(s_5,0.05)\right)^1\times 1.59\left(\Delta^{-1}(s_4,-0.04)\right)^1\times 0.45\left(\Delta^{-1}(s_2,0.02)\right)^2\\
+0.96\left(\Delta^{-1}(s_2,0.02)\right)^1\times 1.59\left(\Delta^{-1}(s_5,0.05)\right)^1\times 0.45\left(\Delta^{-1}(s_4,-0.04)\right)^2\\
+0.96\left(\Delta^{-1}(s_2,0.02)\right)^1\times 1.59\left(\Delta^{-1}(s_4,-0.04)\right)^1\times 0.45\left(\Delta^{-1}(s_5,0.05)\right)^2\\
+0.96\left(\Delta^{-1}(s_4,-0.04)\right)^1\times 1.59\left(\Delta^{-1}(s_2,0.02)\right)^1\times 0.45\left(\Delta^{-1}(s_5,0.05)\right)^2\\
+0.96\left(\Delta^{-1}(s_4,-0.04)\right)^1\times 1.59\left(\Delta^{-1}(s_5,0.05)\right)^1\times 0.45\left(\Delta^{-1}(s_2,0.02)\right)^2
\end{array}\right]\right)^{\frac{1}{(1+1+2)}}\right),\right.\\
&\left(\left(\left(\left(\sqrt[4]{1-\left(\begin{array}{l}
(1-0.0246^1\times 0.0128^1\times 0.1162^2)\times(1-0.0246^1\times 0.1162^1\times 0.0128^2)\\
\times(1-0.0128^1\times 0.1162^1\times 0.0246^2)\times(1-0.0128^1\times 0.0246^1\times 0.1162^2)\\
\times(1-0.1162^1\times 0.0246^1\times 0.0128^2)\times(1-0.1162^1\times 0.0128^1\times 0.0246^2)
\end{array}\right)^{\frac{1}{3!}}}\right)^{\frac{1}{1+1+2}}\right.\right.\right.,\\
&\left.\left(\sqrt[4]{1-\left(\begin{array}{l}
(1-0.0078^1\times 0.1980^1\times 0.0116^2)\times(1-0.0078^1\times 0.0116^1\times 0.1980^2)\\
\times(1-0.1980^1\times 0.0116^1\times 0.0078^2)\times(1-0.1980^1\times 0.0078^1\times 0.0116^2)\\
\times(1-0.0116^1\times 0.0078^1\times 0.1980^2)\times(1-0.0116^1\times 0.1980^1\times 0.0078^2)
\end{array}\right)^{\frac{1}{3!}}}\right)^{\frac{1}{1+1+2}}\right),\\
&\left(\sqrt[4]{1-\left(1-\left(\begin{array}{l}
(1-0.9302^1\times 0.9971^1\times 0.6013^2)\times(1-0.9302^1\times 0.6013^1\times 0.9971^2)\\
\times(1-0.9971^1\times 0.9302^1\times 0.6013^2)\times(1-0.9971^1\times 0.6013^1\times 0.9302^2)\\
\times(1-0.6013^1\times 0.9302^1\times 0.9971^2)\times(1-0.6013^1\times 0.9971^1\times 0.9302^2)
\end{array}\right)^{\frac{1}{3!}}\right)^{\frac{1}{1+1+2}}}\right.,\\
&\left.\left.\left.\sqrt[4]{1-\left(1-\left(\begin{array}{l}
(1-0.7458^1\times 1.0000^1\times 0.3308^2)\times(1-0.7458^1\times 0.3308^1\times 1.0000^2)\\
\times(1-1.0000^1\times 0.3308^1\times 0.7458^2)\times(1-1.0000^1\times 0.7458^1\times 0.3308^2)\\
\times(1-0.3308^1\times 0.7458^1\times 1.0000^2)\times(1-0.3308^1\times 1.0000^1\times 0.7458^2)
\end{array}\right)^{\frac{1}{3!}}\right)^{\frac{1}{1+1+2}}}\right)\right)\right\rangle\\
&= \langle (s_3,0.0246),((0.4387,0.4286),(0.6406,0.7717))\rangle.
\end{aligned}
$$

The Cq-ROFTLWMM operator has the characteristics of boundedness and monotonicity, but the idempotency property does not hold.

Property 4 (Monotonicity) *Let* $\mathcal{N}_J = \langle (s_J,\pounds_J),((\mathfrak{I}_J,\omega_{\mathfrak{I}_J}),(\aleph_J,\omega_{\aleph_J}))\rangle$ *and* $\mathcal{N}'_J = \langle (s'_J,\pounds'_J),((\mathfrak{I}'_J,\omega'_{\mathfrak{I}_J}),(\aleph'_J,\omega'_{\aleph_J}))\rangle (J=1,2,\ldots,z)$ *be two sets of Cq-ROFTLNs. If* $(s_J,\pounds_J)\le(s'_J,\pounds'_J)$, $\mathfrak{I}_J\le\mathfrak{I}'_J$, $\omega_{\mathfrak{I}_J}\le\omega'_{\mathfrak{I}_J}$, $\aleph_J\ge\aleph'_J$ *and* $\omega_{\aleph_J}\ge\omega'_{\aleph_J}$, *then*

$$
\mathrm{C}q-\mathrm{ROFTLWMM}^{\Psi}_{\varpi}(\mathcal{N}_1,\mathcal{N}_2,\ldots,\mathcal{N}_z)\le \mathrm{C}q-\mathrm{ROFTLWMM}^{\Psi}_{\varpi}(\mathcal{N}'_1,\mathcal{N}'_2,\ldots,\mathcal{N}'_z). \tag{11}
$$

Property 5 (Boundedness) *Let* $\mathcal{N}_J = \langle (s_J,\pounds_J),((\mathfrak{I}_J,\omega_{\mathfrak{I}_J}),(\aleph_J,\omega_{\aleph_J}))\rangle (J=1,2,\ldots,z)$ *be a collection of Cq-ROFTLNs. If*

$$
\mathcal{N}^- = \min_J \mathcal{N}_J = \left\langle \min_J(s_J,\pounds_J),\left((\min_J \mathfrak{I}_J,\min_J \omega_{\mathfrak{I}_J}),(\max_J \aleph_J,\max_J \omega_{\aleph_J})\right)\right\rangle
$$

and

$$\mathcal{N}^{+} = \max_{\jmath} \mathcal{N}_{\jmath} = \left\langle \max_{\jmath}(s_{\jmath}, \pounds_{\jmath}), \left((\max_{\jmath} \mathfrak{I}_{\jmath}, \max_{\jmath} \omega_{\mathfrak{I}_{\jmath}}), (\min_{\jmath} \aleph_{\jmath}, \min_{\jmath} \omega_{\aleph_{\jmath}}) \right) \right\rangle.$$

Utilizing Property 4, we have

$$\mathcal{N}^{-} \leq \mathrm{C}q - \mathrm{ROFTLWMM}_{\varpi}^{\psi}(\mathcal{N}_1, \mathcal{N}_2, \dots, \mathcal{N}_z) \leq \mathcal{N}^{+}. \tag{12}$$

4.3 C*q*-ROFTLDMM operator

This subsection generalizes the DMM operator with q-ROFTLSs in complex scenario to develop the Cq-ROFTLDMM operator for aggregating Cq-ROFTLNs, and describes its specific properties.

Definition 14 Let $\mathcal{N}_{\jmath} = \langle (s_{\jmath}, \pounds_{\jmath}), ((\mathfrak{I}_{\jmath}, \omega_{\mathfrak{I}_{\jmath}}), (\aleph_{\jmath}, \omega_{\aleph_{\jmath}})) \rangle (\jmath = 1, 2, \dots, z)$ be a set of Cq-ROFTLNs and parameters vector is $\psi = (\psi_1, \psi_2, \dots, \psi_z) \in \mathbb{R}^z$, then

$$\mathrm{C\ q\text{-}ROFTLDMM}^{\psi}(\mathcal{N}_1, \mathcal{N}_2, \dots, \mathcal{N}_z) = \frac{1}{\sum_{\jmath=1}^{z} \psi_{\jmath}} \left(\otimes_{\varrho \in \mathbb{S}_z} \oplus_{\jmath=1}^{z} \psi_{\jmath} \mathcal{N}_{\varrho(\jmath)} \right)^{\frac{1}{z!}}.$$

Depending on the operational laws of Cq-ROFTLNs, we can derive Theorem 4.

Theorem 4 *Let $\mathcal{N}_{\jmath} = \langle (s_{\jmath}, \pounds_{\jmath}), ((\mathfrak{I}_{\jmath}, \omega_{\mathfrak{I}_{\jmath}}), (\aleph_{\jmath}, \omega_{\aleph_{\jmath}})) \rangle (\jmath = 1, 2, \dots, z)$ be a set of Cq-ROFTLNs, and their aggregated result by applying the Cq-ROFTLDMM operator is also a Cq-ROFTLN, then*

$$\begin{aligned}
&\mathrm{C}q - \mathrm{ROFTLDMM}^{\psi}(\mathcal{N}_1, \mathcal{N}_2, \dots, \mathcal{N}_z) = \left\langle \Delta \left(\frac{1}{\sum_{\jmath=1}^{z} \psi_{\jmath}} \left(\prod_{\varrho \in \mathbb{S}_z} \sum_{\jmath=1}^{z} \psi_{\jmath} \left(\Delta^{-1} \left(s_{\varrho_{\jmath}}, \pounds_{\varrho_{\jmath}} \right) \right) \right)^{\frac{1}{z!}} \right), \right. \\
&\left(\left(\sqrt[q]{1 - \left(1 - \left(\prod_{\varrho \in \mathbb{S}_z} \left(1 - \prod_{\jmath=1}^{z} \left(1 - \mathfrak{I}_{\varrho(\jmath)}^{q} \right)^{\psi_{\jmath}} \right) \right)^{\frac{1}{z!}} \right)^{\frac{1}{\sum_{\jmath=1}^{z} \psi_{\jmath}}}}, \right. \right. \\
&\left. \sqrt[q]{1 - \left(1 - \left(\prod_{\varrho \in \mathbb{S}_z} \left(1 - \prod_{\jmath=1}^{z} \left(1 - \omega_{\mathfrak{I}_{\varrho(\jmath)}}^{q} \right)^{\psi_{\jmath}} \right) \right)^{\frac{1}{z!}} \right)^{\frac{1}{\sum_{\jmath=1}^{z} \psi_{\jmath}}}} \right), \\
&\left(\left(\sqrt[q]{1 - \left(\prod_{\varrho \in \mathbb{S}_z} \left(1 - \left(\prod_{\jmath=1}^{z} \aleph_{\varrho(\jmath)}^{q\psi_{\jmath}} \right) \right) \right)^{\frac{1}{z!}}} \right)^{\frac{1}{\sum_{\jmath=1}^{z} \psi_{\jmath}}}, \right. \\
&\left. \left. \left. \left(\sqrt[q]{1 - \left(\prod_{\varrho \in \mathbb{S}_z} \left(1 - \left(\prod_{\jmath=1}^{z} \omega_{\aleph_{\varrho(\jmath)}}^{q\psi_{\jmath}} \right) \right) \right)^{\frac{1}{z!}}} \right)^{\frac{1}{\sum_{\jmath=1}^{z} \psi_{\jmath}}} \right) \right) \right\rangle.
\end{aligned} \tag{13}$$

Example 4 Let $\mathcal{N}_1 = \langle (s_5, 0.05), ((0.4, 0.3), (0.5, 0.7)) \rangle$, $\mathcal{N}_2 = \langle (s_2, 0.02), ((0.3, 0.6), (0.4, 0.2)) \rangle$, and $\mathcal{N}_3 = \langle (s_4, -0.04), ((0.7, 0.4), (0.6, 0.8)) \rangle$ be three Cq-ROFTLNs which are derived from LTS $S = \{s_j | j \in \{0, \dots, 6\}\}$. $\psi = (1, 1, 2)$ is the parameters vector and $q = 4$, then

$$\Delta\left(\left(\frac{1}{(1+1+2)}\begin{bmatrix} 1\left(\Delta^{-1}(s_5, 0.05)\right) + 1\left(\Delta^{-1}(s_2, 0.02)\right) + 2\left(\Delta^{-1}(s_4, -0.04)\right) \\ \times 1\left(\Delta^{-1}(s_5, 0.05)\right) + 1\left(\Delta^{-1}(s_4, -0.04)\right) + 2\left(\Delta^{-1}(s_2, 0.02)\right) \\ \times 1\left(\Delta^{-1}(s_2, 0.02)\right) + 1\left(\Delta^{-1}(s_5, 0.05)\right) + 2\left(\Delta^{-1}(s_4, -0.04)\right) \\ \times 1\left(\Delta^{-1}(s_2, 0.02)\right) + 1\left(\Delta^{-1}(s_4, -0.04)\right) + 2\left(\Delta^{-1}(s_5, 0.05)\right) \\ \times 1\left(\Delta^{-1}(s_4, -0.04)\right) + 1\left(\Delta^{-1}(s_2, 0.02)\right) + 2\left(\Delta^{-1}(s_5, 0.05)\right) \\ \times 1\left(\Delta^{-1}(s_4, -0.04)\right) + 1\left(\Delta^{-1}(s_5, 0.05)\right) + 2\left(\Delta^{-1}(s_2, 0.02)\right) \end{bmatrix}^{\frac{1}{3!}}\right)\right) = (s_4, -0.0479),$$

$$\left(\begin{array}{c} \sqrt[4]{1 - \left(1 - \left(\begin{array}{c} (1 - 0.9744^1 \times 0.9919^1 \times 0.7599^2) \times (1 - 0.9744^1 \times 0.7599^1 \times 0.9919^2) \times \\ (1 - 0.9919^1 \times 0.7599^1 \times 0.9744^2) \times (1 - 0.9919^1 \times 0.9744^1 \times 0.7599^2) \times \\ (1 - 0.7599^1 \times 0.9744^1 \times 0.9919^2) \times (1 - 0.7599^1 \times 0.9919^1 \times 0.9744^2) \end{array}\right)^{\frac{1}{3!}}\right)^{\frac{1}{1+1+2}}}, \\ \sqrt[4]{1 - \left(1 - \left(\begin{array}{c} (1 - 0.9919^1 \times 0.8704^1 \times 0.9744^2) \times (1 - 0.9919^1 \times 0.9744^1 \times 0.8704^2) \times \\ (1 - 0.8704^1 \times 0.9744^1 \times 0.9919^2) \times (1 - 0.8704^1 \times 0.9919^1 \times 0.9744^2) \times \\ (1 - 0.9744^1 \times 0.9919^1 \times 0.8704^2) \times (1 - 0.9744^1 \times 0.8704^1 \times 0.9919^2) \end{array}\right)^{\frac{1}{3!}}\right)^{\frac{1}{1+1+2}}} \end{array}\right) = (0.5528, 0.4826),$$

$$\left(\begin{array}{c} \left(\sqrt[4]{1 - \left(\begin{array}{c} (1 - 0.5^4 \times 0.4^4 \times 0.6^8) \times (1 - 0.5^4 \times 0.6^4 \times 0.4^8) \times \\ (1 - 0.4^4 \times 0.6^4 \times 0.5^8) \times (1 - 0.4^4 \times 0.5^4 \times 0.6^8) \times \\ (1 - 0.6^4 \times 0.5^4 \times 0.4^8) \times (1 - 0.6^4 \times 0.4^4 \times 0.5^8) \end{array}\right)^{\frac{1}{3!}}}\right)^{\frac{1}{1+1+2}}, \\ \left(\sqrt[4]{1 - \left(\begin{array}{c} (1 - 0.7^4 \times 0.2^4 \times 0.8^8) \times (1 - 0.7^4 \times 0.8^4 \times 0.2^8) \times \\ (1 - 0.2^4 \times 0.8^4 \times 0.7^8) \times (1 - 0.2^4 \times 0.7^4 \times 0.8^8) \times \\ (1 - 0.8^4 \times 0.7^4 \times 0.2^8) \times (1 - 0.8^4 \times 0.2^4 \times 0.7^8) \end{array}\right)^{\frac{1}{3!}}}\right)^{\frac{1}{1+1+2}} \end{array}\right) = (0.4996, 0.5258).$$

Therefore,

$$\text{C}q\text{-ROFTLDMM}^{(1,1,2)}(\mathcal{N}_1, \mathcal{N}_2, \mathcal{N}_3) = \langle (s_4, -0.0479), ((0.5528, 0.4826), (0.4996, 0.5258)) \rangle.$$

Property 6 (Idempotency) *Let* $\mathcal{N}_j = \langle (s_j, \pounds_j), ((\mathfrak{I}_j, \omega_{\mathfrak{I}_j}), (\aleph_j, \omega_{\aleph_j})) \rangle$ *be a collection of Cq-ROFTLNs and all are equal, i.e.,* $\mathcal{N}_j = \mathcal{N}$ *for all* $j = 1, 2, \dots, z$, *then*

$$\text{C}q\text{-ROFTLDMM}^{\Psi}(\mathcal{N}_1, \mathcal{N}_2, \dots, \mathcal{N}_z) = \mathcal{N}. \tag{14}$$

Property 7 (Monotonicity) *Let* $\mathcal{N}_j = \langle (s_j, \pounds_j), ((\mathfrak{I}_j, \omega_{\mathfrak{I}_j}), (\aleph_j, \omega_{\aleph_j})) \rangle$ *and* $\mathcal{N}'_j = \langle (s'_j, \pounds'_j), ((\mathfrak{I}'_j, \omega'_{\mathfrak{I}_j}), (\aleph'_j, \omega'_{\aleph_j})) \rangle (j = 1, 2, \dots, z)$ *be two sets of Cq-ROFTLNs. If* $(s_j, \pounds_j) \le (s'_j, \pounds'_j)$, $\mathfrak{I}_j \le \mathfrak{I}'_j$, $\omega_{\mathfrak{I}_j} \le \omega'_{\mathfrak{I}_j}$, $\aleph_j \ge \aleph'_j$ *and* $\omega_{\aleph_j} \ge \omega'_{\aleph_j}$, *then*

$$\text{C}q\text{-ROFTLDMM}^{\Psi}(\mathcal{N}_1, \mathcal{N}_2, \dots, \mathcal{N}_z) \le \text{C}q\text{-ROFTLDMM}^{\Psi}(\mathcal{N}'_1, \mathcal{N}'_2, \dots, \mathcal{N}'_z). \tag{15}$$

Property 8 (Boundedness) *Let* $\mathcal{N}_j = \langle (s_j, \pounds_j), ((\mathfrak{I}_j, \omega_{\mathfrak{I}_j}), (\aleph_j, \omega_{\aleph_j})) \rangle (j = 1, 2, \dots, z)$ *be a collection of Cq-ROFTLNs. If*

$$\mathcal{N}^- = \min_j \mathcal{N}_j = \left\langle \min_j (s_j, \pounds_j), \left((\min_j \mathfrak{I}_j, \min_j \omega_{\mathfrak{I}_j}), (\max_j \aleph_j, \max_j \omega_{\aleph_j}) \right) \right\rangle$$

and

$$\mathcal{N}^{+}=\max_{\jmath}\mathcal{N}_{\jmath}=\left\langle \max_{\jmath}(s_{\jmath},\pounds_{\jmath}),\left((\max_{\jmath}\mathfrak{I}_{\jmath},\max_{\jmath}\omega_{\mathfrak{I}_{\jmath}}),(\min_{\jmath}\aleph_{\jmath},\min_{\jmath}\omega_{\aleph_{\jmath}})\right)\right\rangle .$$

Utilizing Properties 6 and 7, we have

$$\mathcal{N}^{-}\leq \text{C}q\text{-ROFTLDMM}^{\psi}(\mathcal{N}_1,\mathcal{N}_2,\dots,\mathcal{N}_z)\leq \mathcal{N}^{+}. \tag{16}$$

Case 1. When $\psi=(1,0,\dots,0)$, the Cq-ROFTLDMM operator is converted into the Cq-ROFTLGA operator.

$$\text{C}q\text{-ROFTLGA}(\mathcal{N}_1,\mathcal{N}_2,\dots,\mathcal{N}_z)=\left\langle \Delta\left(\prod_{\jmath=1}^{z}\left(\Delta^{-1}(s_{\jmath},\pounds_{\jmath})\right)^{\frac{1}{z}}\right),\left(\begin{array}{c}\left(\prod_{\jmath=1}^{z}\mathfrak{I}_{\jmath}^{\frac{1}{z}},\prod_{\jmath=1}^{z}\omega_{\mathfrak{I}_{\jmath}}^{\frac{1}{z}}\right),\\ \left(\sqrt[q]{1-\prod_{\jmath=1}^{z}\left(1-\aleph_{\jmath}^{q}\right)^{\frac{1}{z}}},\sqrt[q]{1-\prod_{\jmath=1}^{z}\left(1-\omega_{\aleph_{\jmath}}^{q}\right)^{\frac{1}{z}}}\right)\end{array}\right)\right\rangle .$$

Case 2. When $\psi=(1,1,0,0,\dots,0)$, the Cq-ROFTLDMM operator is converted into the Cq-ROFTL geometric Bonferroni mean (Cq-ROFTLGBM) operator.

$$\text{C}q\text{-ROFTLGBM}(\mathcal{N}_1,\mathcal{N}_2,\dots,\mathcal{N}_z)=\left\langle \Delta\left(\frac{1}{2}\left(\prod_{\imath,\jmath=1,\imath\neq\jmath}^{z}\left(\Delta^{-1}(s_{\imath},\pounds_{\imath})+\Delta^{-1}(s_{\jmath},\pounds_{\jmath})\right)\right)^{\frac{1}{z(z-1)}}\right),\right.$$
$$\left.\left(\begin{array}{c}\left(\begin{array}{c}\sqrt[q]{1-\left(1-\left(\prod_{\imath,\jmath=1,\imath\neq\jmath}^{z}\left(1-\left(1-\mathfrak{I}_{\imath}^{q}\right)\left(1-\mathfrak{I}_{\jmath}^{q}\right)\right)\right)^{\frac{1}{z(z-1)}}\right)^{\frac{1}{2}}},\\ \sqrt[q]{1-\left(1-\left(\prod_{\imath,\jmath=1,\imath\neq\jmath}^{z}\left(1-\left(1-\omega_{\mathfrak{I}_{\imath}}^{q}\right)\left(1-\omega_{\mathfrak{I}_{\jmath}}^{q}\right)\right)\right)^{\frac{1}{z(z-1)}}\right)^{\frac{1}{2}}}\end{array}\right),\\ \left(\begin{array}{c}\left(\sqrt[q]{1-\left(\prod_{\imath,\jmath=1,\imath\neq\jmath}^{z}\left(1-\aleph_{\imath}^{q}\times\aleph_{\jmath}^{q}\right)\right)^{\frac{1}{z(z-1)}}}\right)^{\frac{1}{2}},\\ \left(\sqrt[q]{1-\left(\prod_{\imath,\jmath=1,\imath\neq\jmath}^{z}\left(1-\omega_{\aleph_{\imath}}^{q}\times\omega_{\aleph_{\jmath}}^{q}\right)\right)^{\frac{1}{z(z-1)}}}\right)^{\frac{1}{2}}\end{array}\right)\end{array}\right)\right\rangle .$$

Case 3. When $\psi=(\overbrace{1,1,\dots,1}^{k},\overbrace{0,0,\dots,0}^{z-k})$, the C$q$-ROFTLDMM operator is converted into the Cq-ROFTL dual MSM (Cq-ROFTLDMSM) operator.

$$\mathrm{C}q\text{-ROFTLGMSM}(\mathcal{N}_1,\mathcal{N}_2,\ldots,\mathcal{N}_z)=\left\langle \Delta\left(\frac{1}{k}\left(\prod_{1\le\iota_1<\ldots<\iota_k\le z}\sum_{\jmath=1}^{k}\Delta^{-1}(s_{\iota_\jmath},\pounds_{\iota_\jmath})\right)^{\frac{1}{C_z^k}}\right),\right.$$

$$\left.\left(\begin{array}{l}\left(\sqrt[q]{1-\left(1-\left(\prod_{1\le\iota_1<\ldots<\iota_k\le z}\left(1-\prod_{\jmath=1}^{k}\left(1-\mathfrak{I}_{\iota_\jmath}^{q}\right)\right)\right)^{\frac{1}{C_z^k}}\right)^{\frac{1}{k}}},\sqrt[q]{1-\left(1-\left(\prod_{1\le\iota_1<\ldots<\iota_k\le z}\left(1-\prod_{\jmath=1}^{k}\left(1-\omega_{\mathfrak{I}_{\iota_\jmath}}^{q}\right)\right)\right)^{\frac{1}{C_z^k}}\right)^{\frac{1}{k}}}\right),\\ \left(\left(\sqrt[q]{1-\left(\prod_{1\le\iota_1<\ldots<\iota_k\le z}\left(1-\prod_{\jmath=1}^{k}\aleph_{\iota_\jmath}^{q}\right)\right)^{\frac{1}{C_z^k}}}\right)^{\frac{1}{k}},\left(\sqrt[q]{1-\left(\prod_{1\le\iota_1<\ldots<\iota_k\le z}\left(1-\prod_{\jmath=1}^{k}\omega_{\aleph_{\iota_\jmath}}^{q}\right)\right)^{\frac{1}{C_z^k}}}\right)^{\frac{1}{k}}\right)\end{array}\right)\right\rangle.$$

Case 4. When $\psi=(1,1,\ldots,1)$ or $(\frac{1}{z},\frac{1}{z},\ldots,\frac{1}{z})$, the C$q$-ROFTLDMM operator is converted into the Cq-ROFTL arithmetic averaging (Cq-ROFTLAA) operator.

$$\mathrm{C}q\text{-ROFTLAA}(\mathcal{N}_1,\mathcal{N}_2,\ldots,\mathcal{N}_z)=\left\langle \Delta\left(\sum_{\jmath=1}^{z}\left(\frac{1}{z}\times\Delta^{-1}(s_\jmath,\pounds_\jmath)\right)\right),\left(\begin{array}{l}\left(\sqrt[q]{1-\prod_{\jmath=1}^{z}\left(1-\mathfrak{I}_\jmath^{q}\right)^{\frac{1}{z}}},\sqrt[q]{1-\prod_{\jmath=1}^{z}\left(1-\omega_{\mathfrak{I}_\jmath}^{q}\right)^{\frac{1}{z}}}\right),\\ \left(\left(\prod_{\jmath=1}^{z}\aleph_\jmath\right)^{\frac{1}{z}},\left(\prod_{\jmath=1}^{z}\omega_{\aleph_\jmath}\right)^{\frac{1}{z}}\right)\end{array}\right)\right\rangle.$$

4.4 Cq-ROFTLWDMM operator

In subsection 4.3, we can see that Cq-ROFTLDMM operator cannot consider weights of attributes. In most practical problems, particularly in MAGDM, the weights of attributes play a significant role in the aggregation process. To overcome the limitation of Cq-ROFTLDMM, it is necessary to introduce the Cq-ROFTLWDMM operator as given.

Definition 15 Let $\mathcal{N}_\jmath=\langle(s_\jmath,\pounds_\jmath),((\mathfrak{I}_\jmath,\omega_{\mathfrak{I}_\jmath}),(\aleph_\jmath,\omega_{\aleph_\jmath}))\rangle(\jmath=1,2,\ldots,z)$ be a set of Cq-ROFTLNs, $\varpi=(\varpi_1,\varpi_2,\ldots,\varpi_z)^T$, $\varpi_\jmath\in[0,1]$, $\sum_{\jmath=1}^{z}\varpi_\jmath=1$ is the weight vector and $\psi=(\psi_1,\psi_2,\ldots,\psi_z)\in\mathbb{R}^z$ is a parameters vector, then

$$\mathrm{C}q\text{-ROFTLWDMM}_{\varpi}^{\psi}(\mathcal{N}_1,\mathcal{N}_2,\ldots,\mathcal{N}_z)=\frac{1}{\sum_{\jmath=1}^{z}\psi_\jmath}\left(\otimes_{\varrho\in\mathbb{S}_z}\oplus_{\jmath=1}^{z}\psi_\jmath(\mathcal{N}_{\varrho(\jmath)})^{z\varpi_{\varrho(\jmath)}}\right)^{\frac{1}{z!}}.$$

Depending on the operational laws of the Cq-ROFTLNs, Theorem 5 can be obtained.

Theorem 5 *Let $\mathcal{N}_J = \langle (s_J, \pounds_J), ((\Im_J, \omega_{\Im_J}), (\aleph_J, \omega_{\aleph_J}))\rangle (J = 1, 2, \ldots, z)$ be a set of Cq-ROFTLNs, their aggregated result by applying the Cq-ROFTLWDMM operator is also a Cq-ROFTLN, then*

$$\begin{aligned}
&Cq\text{-ROFTLWDMM}_{\varpi}^{\psi}(\mathcal{N}_1, \mathcal{N}_2, \ldots, \mathcal{N}_z) = \left\langle \Delta\left(\frac{1}{\sum_{J=1}^{z} \psi_J} \left(\prod_{\rho \in \mathbb{S}_z} \sum_{J=1}^{z} \psi_J \left(\Delta^{-1}\left(s_{\rho(J)}, \pounds_{\rho(J)}\right)\right)^{z\varpi_{\rho(J)}} \right)^{\frac{1}{z!}} \right), \right. \\
&\left(\left(\sqrt[q]{1 - \left(1 - \left(\prod_{\rho \in \mathbb{S}_z} \left(1 - \prod_{J=1}^{z} \left(1 - \left(\Im_{\rho(J)}^{q}\right)^{z\varpi_{\rho(J)}}\right)^{\psi_J}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{J=1}^{z}\psi_J}}}, \right.\right. \\
&\left.\left. \sqrt[q]{1 - \left(1 - \left(\prod_{\rho \in \mathbb{S}_z} \left(1 - \prod_{J=1}^{z} \left(1 - \left(\omega_{\Im_{\rho(J)}}^{q}\right)^{z\varpi_{\rho(J)}}\right)^{\psi_J}\right)\right)^{\frac{1}{z!}}\right)^{\frac{1}{\sum_{J=1}^{z}\psi_J}}} \right), \right. \\
&\left(\left(\sqrt[q]{1 - \left(\prod_{\rho \in \mathbb{S}_z} \left(1 - \prod_{J=1}^{z} \left(1 - \left(1 - \aleph_{\rho(J)}^{q}\right)^{z\varpi_{\rho(J)}}\right)^{\psi_J}\right)\right)^{\frac{1}{z!}}} \right)^{\frac{1}{\sum_{J=1}^{z}\psi_J}}, \right. \\
&\left.\left.\left. \left(\sqrt[q]{1 - \left(\prod_{\rho \in \mathbb{S}_z} \left(1 - \prod_{J=1}^{z} \left(1 - \left(1 - \omega_{\aleph_{\rho(J)}}^{q}\right)^{z\varpi_{\rho(J)}}\right)^{\psi_J}\right)\right)^{\frac{1}{z!}}} \right)^{\frac{1}{\sum_{J=1}^{z}\psi_J}} \right)\right) \right\rangle.
\end{aligned} \tag{17}$$

Example 5 Let $\mathcal{N}_1 = \langle (s_5, 0.05), ((0.4, 0.3), (0.5, 0.7))\rangle$, $\mathcal{N}_2 = \langle (s_2, 0.02), ((0.3, 0.6), (0.4, 0.2))\rangle$, and $\mathcal{N}_3 = \langle (s_4, -0.04), ((0.7, 0.4), (0.6, 0.8))\rangle$ be three Cq-ROFTLNs which are derived from LTS $S = \{s_j | j \in \{0, \ldots, 6\}\}$. $\psi = (1, 1, 2)$ is the parameters vector, $q = 4$ and $\varpi = (0.32, 0.53, 0.15)$ be the weighting vector, then we have

$$
\begin{aligned}
&\text{C}q\text{-ROFTLWDMM}^{(1,1,2)}_{(0.32,0.53,0.15)}(\mathcal{N}_1,\mathcal{N}_2,\mathcal{N}_3)\\
&=\left\langle \Delta\left(\left(\frac{1}{(1+1+2)}\begin{bmatrix}
1\left(\Delta^{-1}(s_5,0.05)\right)^{0.96}+1\left(\Delta^{-1}(s_2,0.02)\right)^{1.59}+2\left(\Delta^{-1}(s_4,-0.04)\right)^{0.45}\\
\times 1\left(\Delta^{-1}(s_5,0.05)\right)^{0.96}+1\left(\Delta^{-1}(s_4,-0.04)\right)^{1.59}+2\left(\Delta^{-1}(s_2,0.02)\right)^{0.45}\\
\times 1\left(\Delta^{-1}(s_2,0.02)\right)^{0.96}+1\left(\Delta^{-1}(s_5,0.05)\right)^{1.59}+2\left(\Delta^{-1}(s_4,-0.04)\right)^{0.45}\\
\times 1\left(\Delta^{-1}(s_2,0.02)\right)^{0.96}+1\left(\Delta^{-1}(s_4,-0.04)\right)^{1.59}+2\left(\Delta^{-1}(s_5,0.05)\right)^{0.45}\\
\times 1\left(\Delta^{-1}(s_4,-0.04)\right)^{0.96}+1\left(\Delta^{-1}(s_2,0.02)\right)^{1.59}+2\left(\Delta^{-1}(s_5,0.05)\right)^{0.45}\\
\times 1\left(\Delta^{-1}(s_4,-0.04)\right)^{0.96}+1\left(\Delta^{-1}(s_5,0.05)\right)^{1.59}+2\left(\Delta^{-1}(s_2,0.02)\right)^{0.45}
\end{bmatrix}\right)^{\frac{1}{3!}}\right),\right.\\
&\left(\left(\left(\sqrt[4]{1-\begin{pmatrix}
(1-0.9704^1\times 0.9995^1\times 0.4738^2)\times(1-0.9704^1\times 0.4738^1\times 0.9995^2)\\
\times(1-0.9995^1\times 0.4738^1\times 0.9704^2)\times(1-0.9995^1\times 0.9704^1\times 0.4738^2)\\
\times(1-0.4738^1\times 0.9704^1\times 0.9995^2)\times(1-0.4738^1\times 0.9995^1\times 0.9704^2)
\end{pmatrix}^{\frac{1}{3!}}}\right)^{\frac{1}{1+1+2}},\right.\right.\\
&\left(\sqrt[q]{1-\begin{pmatrix}
(1-0.9902^1\times 0.9612^1\times 0.8078^2)\times(1-0.9902^1\times 0.8078^1\times 0.9612^2)\\
\times(1-0.9612^1\times 0.8078^1\times 0.9902^2)\times(1-0.9612^1\times 0.9902^1\times 0.8078^2)\\
\times(1-0.8078^1\times 0.9902^1\times 0.9612^2)\times(1-0.8078^1\times 0.9612^1\times 0.9902^2)
\end{pmatrix}^{\frac{1}{3!}}}\right)^{\frac{1}{1+1+2}}\Bigg),\\
&\left(\sqrt[4]{1-\left(1-\begin{pmatrix}
(1-0.0601^1\times 0.0404^1\times 0.0606^2)\times(1-0.0601^1\times 0.0606^1\times 0.0404^2)\\
\times(1-0.0404^1\times 0.0606^1\times 0.0601^2)\times(1-0.0404^1\times 0.0601^1\times 0.0606^2)\\
\times(1-0.0606^1\times 0.0601^1\times 0.0404^2)\times(1-0.0606^1\times 0.0404^1\times 0.0601^2)
\end{pmatrix}^{\frac{1}{3!}}\right)^{\frac{1}{1+1+2}}},\right.\\
&\left.\left.\left.\sqrt[4]{1-\left(1-\begin{pmatrix}
(1-0.2371^1\times 0.0025^1\times 0.2111^2)\times(1-0.2371^1\times 0.2111^1\times 0.0025^2)\\
\times(1-0.0025^1\times 0.2111^1\times 0.2371^2)\times(1-0.0025^1\times 0.2371^1\times 0.2111^2)\\
\times(1-0.2111^1\times 0.2371^1\times 0.0025^2)\times(1-0.2111^1\times 0.0025^1\times 0.2371^2)
\end{pmatrix}^{\frac{1}{3!}}\right)^{\frac{1}{1+1+2}}}\right)\right)\right\rangle\\
&=\langle(s_4,-0.0420),((0.6794,0.5336),(0.4798,0.5060))\rangle.
\end{aligned}
$$

The property of monotonicity and boundedness is fulfilled by Cq-ROFTLWDMM, but the property of idempotency is not satisfied.

Property 9 (Monotonicity) *Let* $\mathcal{N}_j=\langle(s_j,\pounds_j),((\mathfrak{I}_j,\omega_{\mathfrak{I}_j}),(\aleph_j,\omega_{\aleph_j}))\rangle$, $\mathcal{N}'_j=\langle(s'_j,\pounds'_j),((\mathfrak{I}'_j,\omega'_{\mathfrak{I}_j}),(\aleph'_j,\omega'_{\aleph_j}))\rangle(j=1,2,\ldots,z)$ *be two sets of Cq-ROFTLNs and* $(s_j,\pounds_j)\le(s'_j,\pounds'_j)$, $\mathfrak{I}_j\le\mathfrak{I}'_j$, $\omega_{\mathfrak{I}_j}\le\omega'_{\mathfrak{I}_j}$, $\aleph_j\ge\aleph'_j$ *and* $\omega_{\aleph_j}\ge\omega'_{\aleph_j}$, *then*

$$
\text{C}q\text{-ROFTLWDMM}^{\Psi}_{\varpi}(\mathcal{N}_1,\mathcal{N}_2,\ldots,\mathcal{N}_z)\le \text{C}q-\text{ROFTLWDMM}^{\Psi}_{\varpi}(\mathcal{N}'_1,\mathcal{N}'_2,\ldots,\mathcal{N}'_z). \tag{18}
$$

Property 10 (Boundedness) *Let* $\mathcal{N}_j=\langle(s_j,\pounds_j),((\mathfrak{I}_j,\omega_{\mathfrak{I}_j}),(\aleph_j,\omega_{\aleph_j}))\rangle(j=1,2,\ldots,z)$ *be a collection of Cq-ROFTLNs. If*

$$
\mathcal{N}^-=\min_j\mathcal{N}_j=\left\langle \min_j(s_j,\pounds_j),\left((\min_j\mathfrak{I}_j,\min_j\omega_{\mathfrak{I}_j}),(\max_j\aleph_j,\max_j\omega_{\aleph_j})\right)\right\rangle
$$

and

$$
\mathcal{N}^+=\max_j\mathcal{N}_j=\left\langle \max_j(s_j,\pounds_j),\left((\max_j\mathfrak{I}_j,\max_j\omega_{\mathfrak{I}_j}),(\min_j\aleph_j,\min_j\omega_{\aleph_j})\right)\right\rangle.
$$

Utilizing Property 9, we have

$$\mathcal{N}^{-} \leq \mathrm{C}q - \mathrm{ROFTLWDMM}^{\psi}_{\varpi}(\mathcal{N}_1, \mathcal{N}_2, \ldots, \mathcal{N}_z) \leq \mathcal{N}^{+}. \tag{19}$$

5 An algorithm to MAGDM problems with C*q*-ROFTL information

To solve the MAGDM problem under Cq-ROFTL environment, we choose a set of m alternatives $\mathcal{M}_\imath(\imath = 1, 2, \ldots, m)$ and set of z attributes $\hbar_\jmath(\jmath = 1, 2, \ldots, z)$. Let $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_z)$ be the weighting vector of attributes satisfying $\varpi_\jmath \in [0, 1]$ and $\sum_{\jmath=1}^{z} \varpi_\jmath = 1$. Let $E = \{\mathfrak{c}_1, \mathfrak{c}_2, \ldots, \mathfrak{c}_g\}$ be the collection of experts.

Suppose $R^l = (\mathcal{N}^l_{\imath\jmath})_{m\times z}$ $(l = 1, 2, \ldots, g)$ is the individual Cq-ROFTL decision matrix where $\mathcal{N}^l_{\imath\jmath} = \langle (s^l_{\imath\jmath}, \pounds^l_{\imath\jmath}), ((\Im^l_{\imath\jmath}, \omega^l_{\Im_{\imath\jmath}}), (\aleph^l_{\imath\jmath}, \omega^l_{\aleph_{\imath\jmath}}))\rangle$, $s^l_{\imath\jmath} \in S = \{s_0, s_1, \ldots, s_h\}$, $\Im^l_{\imath\jmath}, \aleph^l_{\imath\jmath} \in [0, 1]$, $\omega^l_{\Im_{\imath\jmath}}, \omega^l_{\aleph_{\imath\jmath}} \in [0, 1]$, $(\Im^l_{\imath\jmath})^q + (\aleph^l_{\imath\jmath})^q \leq 1, (\omega^l_{\Im_{\imath\jmath}})^q + (\omega^l_{\aleph_{\imath\jmath}})^q \leq 1, q \geq 1$ is a Cq-ROFTLN provided by the decision expert $\mathfrak{c}_g$ for the alternative $\mathcal{M}_\imath$ according to the attribute $\hbar_\jmath$. To choose the most suitable alternative(s), the proposed operators are used to construct a method for MAGDM in a Cq-ROFTL environment. The main steps are listed in Algorithm 1.

Algorithm 1 The algorithm for the selection of the most important medication.

INPUT: A discrete set of medications (alternatives) $\mathcal{M} = \{\mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_m\}$, a set of attributes $\hbar = \{\hbar_1, \hbar_2, \ldots, \hbar_z\}$, a set of experts $E = \{\mathfrak{c}_1, \mathfrak{c}_2, \ldots, \mathfrak{c}_g\}$ and construction of decision matrix for each expert.
OUTPUT: The selection of optimal medicine.

1. **begin**
2. To solve the group decision-making problem, we choose a set of alternatives $\mathcal{M} = \{\mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_m\}$ and a set of attributes $\hbar = \{\hbar_1, \hbar_2, \ldots, \hbar_z\}$. Let $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_z)^T$ is the set of weights satisfying $0 \leq \varpi_\jmath \leq 1$ and $\sum_{\jmath=1}^{z} \varpi_\jmath = 1$. Suppose $E = \{\mathfrak{c}_1, \mathfrak{c}_2, \ldots, \mathfrak{c}_g\}$ is the collection of experts. Decision matrices given by decision experts $E_l(l = 1, 2, \ldots, g)$ is described as $\mathcal{N}^l_{\imath\jmath} = \langle (s^l_{\imath\jmath}, \pounds^l_{\imath\jmath}), ((\Im^l_{\imath\jmath}, \omega^l_{\Im_{\imath\jmath}}), (\aleph^l_{\imath\jmath}, \omega^l_{\aleph_{\imath\jmath}}))\rangle$.
3. Integrate the individual decision matrices with parameters vector $\psi = (\psi_1, \psi_2, \ldots, \psi_g)$ and $q \geq 1$ into a group decision matrix by utilizing Eq. (6) and Eq. (13).
4. Aggregate all assessment values of the alternative $\mathcal{M}_\imath(\imath = 1, 2, \ldots, m)$ on all attributes $\hbar_\jmath(\jmath = 1, 2, \ldots, z)$ into the overall assessment based on the Eqs. (10) and (17), respectively, by taking a parameters vector $\psi = (\psi_1, \psi_2, \ldots, \psi_z)$ and $q \geq 1$.
5. Determine the scores $\digamma(\mathcal{N}_\imath)(\imath = 1, 2, \ldots, m)$ from Eq. (4) of overall Cq-ROFTLNs $\mathcal{N}_\imath(\imath = 1, 2, \ldots, m)$ to rank all the alternatives $\mathcal{M}_\imath$.
6. According to $\digamma(\mathcal{N}_\imath)(\imath = 1, 2, \ldots, m)$, rank all the feasible alternatives $\mathcal{M}_\imath$.
7. Output the optimal alternative.
8. **end**

Figure 2 illustrates the various steps of the suggested method to MAGDM.

6 Illustrative example

We implement the proposed technique into an illustrative example to demonstrate the effectiveness and validity of the proposed approach with Cq-ROFTL information.

Example 6 Pneumonia is a common disease having four basic symptoms such as fever, chest pain, fatigue, and cough. This disease can be treated with five medicines. These medications have different therapeutic effects on these four symptoms (attributes). To collect the more precise information, the panel of three doctors, $E = \{\mathfrak{c}_1, \mathfrak{c}_2, \mathfrak{c}_3\}$, will give their

Table 1 Cq-ROFTL decision matrix given by $\mathfrak{c}_1$

Alternatives	$\hbar_1$	$\hbar_2$	$\hbar_3$	$\hbar_4$
$\mathcal{M}_1$	$\langle(s_4,0),((0.4,0.3),(0.5,0.6))\rangle$	$\langle(s_2,0),((0.3,0.5),(0.5,0.4))\rangle$	$\langle(s_7,0),((0.6,0.4),(0.2,0.4))\rangle$	$\langle(s_6,0),((0.2,0.8),(0.9,0.4))\rangle$
$\mathcal{M}_2$	$\langle(s_5,0),((0.7,0.5),(0.2,0.4))\rangle$	$\langle(s_3,0),((0.4,0.7),(0.3,0.1))\rangle$	$\langle(s_4,0),((0.8,0.2),(0.1,0.6))\rangle$	$\langle(s_2,0),((0.8,0.2),(0.1,0.5))\rangle$
$\mathcal{M}_3$	$\langle(s_3,0),((0.6,0.2),(0.1,0.5))\rangle$	$\langle(s_4,0),((0.3,0.8),(0.6,0.1))\rangle$	$\langle(s_2,0),((0.3,0.6),(0.8,0.5))\rangle$	$\langle(s_7,0),((0.3,0.2),(0.5,0.4))\rangle$
$\mathcal{M}_4$	$\langle(s_4,0),((0.1,0.6),(0.7,0.2))\rangle$	$\langle(s_2,0),((0.5,0.3),(0.2,0.5))\rangle$	$\langle(s_6,0),((0.5,0.2),(0.3,0.6))\rangle$	$\langle(s_5,0),((0.4,0.7),(0.3,0.1))\rangle$
$\mathcal{M}_5$	$\langle(s_5,0),((0.7,0.4),(0.5,0.7))\rangle$	$\langle(s_4,0),((0.3,0.5),(0.6,0.2))\rangle$	$\langle(s_2,0),((0.5,0.4),(0.3,0.5))\rangle$	$\langle(s_6,0),((0.3,0.3),(0.6,0.5))\rangle$

Table 2 Cq-ROFTL decision matrix given by $\mathfrak{c}_2$

Alternatives	$\hbar_1$	$\hbar_2$	$\hbar_3$	$\hbar_4$
$\mathcal{M}_1$	$\langle(s_6, 0), ((0.3, 0.2), (0.5, 0.7))\rangle$	$\langle(s_4, 0), ((0.4, 0.6), (0.8, 0.5))\rangle$	$\langle(s_3, 0), ((0.4, 0.3), (0.3, 0.5))\rangle$	$\langle(s_5, 0), ((0.3, 0.5), (0.4, 0.3))\rangle$
$\mathcal{M}_2$	$\langle(s_4, 0), ((0.6, 0.4), (0.3, 0.5))\rangle$	$\langle(s_3, 0), ((0.5, 0.8), (0.2, 0.1))\rangle$	$\langle(s_5, 0), ((0.7, 0.1), (0.2, 0.7))\rangle$	$\langle(s_7, 0), ((0.8, 0.3), (0.1, 0.6))\rangle$
$\mathcal{M}_3$	$\langle(s_5, 0), ((0.5, 0.1), (0.2, 0.6))\rangle$	$\langle(s_1, 0), ((0.4, 0.3), (0.5, 0.7))\rangle$	$\langle(s_4, 0), ((0.9, 0.5), (0.4, 0.6))\rangle$	$\langle(s_6, 0), ((0.4, 0.3), (0.4, 0.3))\rangle$
$\mathcal{M}_4$	$\langle(s_2, 0), ((0.2, 0.5), (0.7, 0.3))\rangle$	$\langle(s_6, 0), ((0.6, 0.4), (0.1, 0.5))\rangle$	$\langle(s_5, 0), ((0.4, 0.1), (0.4, 0.7))\rangle$	$\langle(s_3, 0), ((0.5, 0.8), (0.2, 0.1))\rangle$
$\mathcal{M}_5$	$\langle(s_7, 0), ((0.6, 0.3), (0.2, 0.5))\rangle$	$\langle(s_5, 0), ((0.3, 0.7), (0.5, 0.1))\rangle$	$\langle(s_3, 0), ((0.4, 0.5), (0.2, 0.2))\rangle$	$\langle(s_7, 0), ((0.3, 0.5), (0.5, 0.4))\rangle$

Table 3 Cq-ROFTL decision matrix given by $\mathfrak{c}_3$

Alternatives	$\hbar_1$	$\hbar_2$	$\hbar_3$	$\hbar_4$
$\mathcal{M}_1$	$\langle(s_3,0),((0.5,0.4),(0.4,0.5))\rangle$	$\langle(s_5,0),((0.5,0.5),(0.2,0.3))\rangle$	$\langle(s_7,0),((0.5,0.3),(0.2,0.5))\rangle$	$\langle(s_4,0),((0.4,0.6),(0.4,0.3))\rangle$
$\mathcal{M}_2$	$\langle(s_2,0),((0.8,0.6),(0.3,0.5))\rangle$	$\langle(s_6,0),((0.4,0.7),(0.3,0.2))\rangle$	$\langle(s_3,0),((0.7,0.1),(0.1,0.7))\rangle$	$\langle(s_5,0),((0.8,0.2),(0.1,0.5))\rangle$
$\mathcal{M}_3$	$\langle(s_6,0),((0.7,0.3),(0.2,0.4))\rangle$	$\langle(s_3,0),((0.3,0.7),(0.8,0.5))\rangle$	$\langle(s_5,0),((0.2,0.6),(0.5,0.2))\rangle$	$\langle(s_2,0),((0.4,0.2),(0.6,0.8))\rangle$
$\mathcal{M}_4$	$\langle(s_5,0),((0.2,0.7),(0.6,0.1))\rangle$	$\langle(s_2,0),((0.5,0.3),(0.2,0.4))\rangle$	$\langle(s_7,0),((0.4,0.1),(0.4,0.7))\rangle$	$\langle(s_1,0),((0.5,0.7),(0.6,0.5))\rangle$
$\mathcal{M}_5$	$\langle(s_4,0),((0.5,0.4),(0.3,0.5))\rangle$	$\langle(s_7,0),((0.5,0.3),(0.4,0.3))\rangle$	$\langle(s_6,0),((0.9,0.5),(0.4,0.7))\rangle$	$\langle(s_3,0),((0.7,0.3),(0.1,0.5))\rangle$

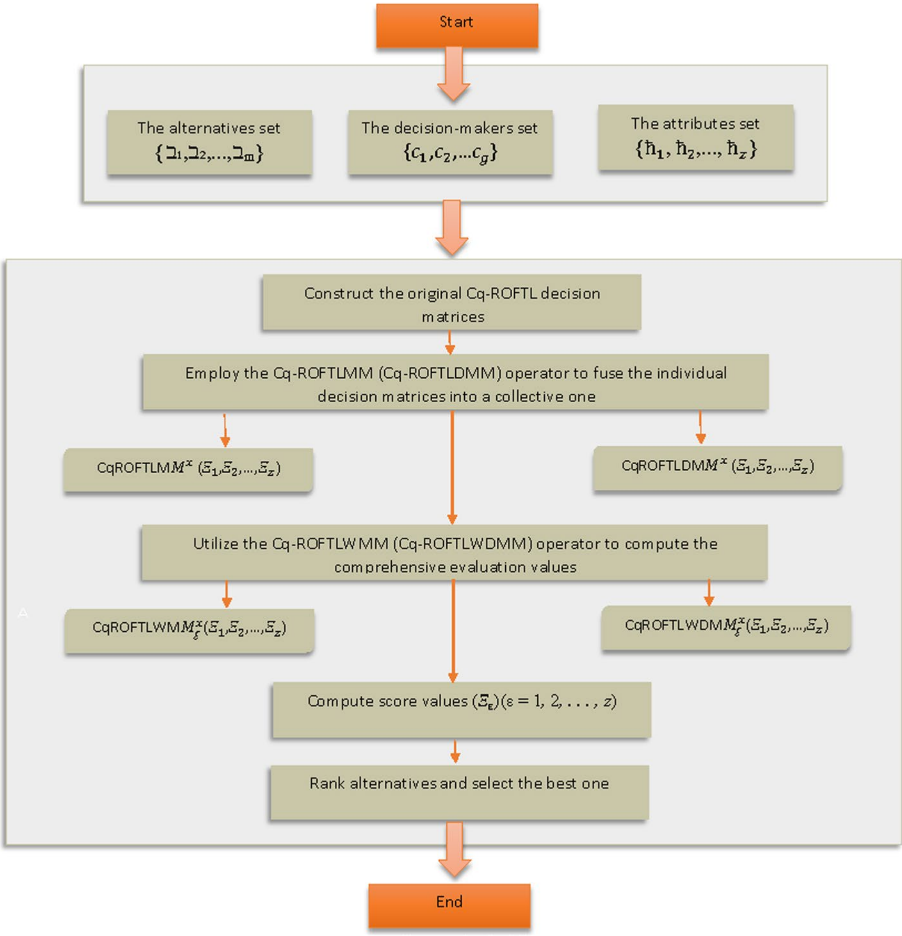

Fig. 2 Decision framework proposed in the context of Cq-ROFWMM and Cq-ROFWDMM operators

opinion about five medicines in the form of LTS $S = \{s_0 =$ extremely poor, $s_1 =$ very poor, $s_2 =$ poor, $s_3 =$ slightly poor, $s_4 =$ fair, $s_5 =$ slightly good, $s_6 =$ good, $s_7 =$ very good, $s_8 =$ extremely good$\}$. Let $\hbar = \{\hbar_1, \hbar_2, \hbar_3, \hbar_4\}$ represent the four symptoms (attributes) and $\mathcal{M} = \{\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3, \mathcal{M}_4, \mathcal{M}_5\}$ denotes the five medicines (alternatives). Furthermore, considering the significant degree of the four symptoms, the doctor provides the weight vector $\varpi = (0.3, 0.1, 0.2, 0.4)$ of the attributes. The panel of doctors presents their evaluation for each medicine (alternative) to their corresponding symptom (attribute) in the form of individual decision matrices $R^l = (\mathcal{N}^l_{ij})_{5\times4} (l = 1, 2, 3)$ which are given in Tables 1, 2, and 3. We apply the recommended technique to get the best medicine for Pneumonia.

Table 4 Fused decision matrix by the Cq-ROFTLMM operator

Alternatives	$\hbar_1$	$\hbar_2$
$\mathcal{M}_1$	$\left\langle \begin{array}{l}(s_4, 0.0253),\\((0.3992, 0.2982), (0.4732, 0.6177))\end{array}\right\rangle$	$\left\langle \begin{array}{l}(s_3, 0.0600),\\((0.3992, 0.5334), (0.6424, 0.4223))\end{array}\right\rangle$
$\mathcal{M}_2$	$\left\langle \begin{array}{l}(s_3, 0.0600),\\((0.6999, 0.4996), (0.2770, 0.4732))\end{array}\right\rangle$	$\left\langle \begin{array}{l}(s_4, -0.0208),\\((0.4334, 0.7334), (0.2770, 0.1550))\end{array}\right\rangle$
$\mathcal{M}_3$	$\left\langle \begin{array}{l}(s_5, -0.0591),\\((0.5997, 0.1946), (0.1815, 0.5192))\end{array}\right\rangle$	$\left\langle \begin{array}{l}(s_2, 0.0473),\\((0.3335, 0.5823), (0.6771, 0.5665))\end{array}\right\rangle$
$\mathcal{M}_4$	$\left\langle \begin{array}{l}(s_3, 0.0600),\\((0.1643, 0.5997), (0.6722, 0.2372))\end{array}\right\rangle$	$\left\langle \begin{array}{l}(s_3, -0.0012),\\((0.5334, 0.3335), (0.1815, 0.4732))\end{array}\right\rangle$
$\mathcal{M}_5$	$\left\langle \begin{array}{l}(s_5, 0.0284),\\((0.5997, 0.3663), (0.3916, 0.5928))\end{array}\right\rangle$	$\left\langle \begin{array}{l}(s_5, 0.0284),\\((0.3668, 0.4940), (0.5192, 0.2372))\end{array}\right\rangle$
	$\hbar_3$	$\hbar_4$
$\mathcal{M}_1$	$\left\langle \begin{array}{l}(s_5, 0.0465),\\((0.4996, 0.3335), (0.2466, 0.4732))\end{array}\right\rangle$	$\left\langle \begin{array}{l}(s_5, -0.0063),\\((0.2982, 0.6331), (0.7332, 0.3429))\end{array}\right\rangle$
$\mathcal{M}_2$	$\left\langle \begin{array}{l}(s_4, -0.0080),\\((0.7334, 0.1330), (0.1550, 0.6722))\end{array}\right\rangle$	$\left\langle \begin{array}{l}(s_4, 0.0314),\\((0.8000, 0.2335), (0.1000, 0.5399))\end{array}\right\rangle$
$\mathcal{M}_3$	$\left\langle \begin{array}{l}(s_3, 0.0600),\\((0.4388, 0.5665), (0.6492, 0.5039))\end{array}\right\rangle$	$\left\langle \begin{array}{l}(s_5, -0.0591),\\((0.3663, 0.2335), (0.5192, 0.6331))\end{array}\right\rangle$
$\mathcal{M}_4$	$\left\langle \begin{array}{l}(s_6, -0.0053),\\((0.4334, 0.1330), (0.3746, 0.6722))\end{array}\right\rangle$	$\left\langle \begin{array}{l}(s_3, -0.0512),\\((0.4664, 0.7334), (0.4630, 0.3768))\end{array}\right\rangle$
$\mathcal{M}_5$	$\left\langle \begin{array}{l}(s_3, 0.0487),\\((0.5970, 0.4664), (0.3278, 0.5672))\end{array}\right\rangle$	$\left\langle \begin{array}{l}(s_5, 0.0114),\\((0.4296, 0.3668), (0.5029, 0.4732))\end{array}\right\rangle$

6.1 Implementation of the proposed methods

To choose the most appropriate medicine for pneumonia, the Cq-ROFTLWMM and Cq-ROFTLWDMM operators are utilized to solve the MAGDM problem with Cq-ROFTLNs according to the following manner:

6.1.1 Decision-making process based on the Cq-ROFTLWMM and Cq-ROFTLWDMM operators

Depending on the individual assessment matrix $R^l = (\mathcal{N}^l_{\iota\jmath})_{5\times 4} (l = 1, 2, 3)$, and $q = 4$, we utilize the Cq-ROFTLMM (Cq-ROFTLDMM) operator, (Suppose $\psi = (1, 2, 1)$), to obtain the group decision matrix R, and the outcomes are represented in Tables 4 and 5.

We now utilize Cq-ROFTLWMM and Cq-ROFTLWDMM operators, (assume $\psi = (1, 1, 1, 1)$ and $q = 4$) to obtain the aggregated assessment values of five alternatives. The outcomes are represented in Table 6.

Score function $\digamma(\mathcal{M}_\iota)$ of aggregated assessment value $\mathcal{M}_\iota (\iota = 1, 2, \ldots, 5)$ can be calculated utilizing Eq. (4) (see Table 7).

Rank the alternatives $\mathcal{M}_\iota$ $(\iota = 1, 2, \ldots, 5)$ according to score index (Table 8).

Table 5 Fused decision matrix by the Cq-ROFTLDMM operator

Alternatives	$\hbar_1$	$\hbar_2$
$\mathcal{M}_1$	$\left\langle \begin{array}{l} (s_4, 0.0403), \\ ((0.4223, 0.3278), (0.4664, 0.5997)) \end{array} \right\rangle$	$\left\langle \begin{array}{l} (s_4, -0.0434), \\ ((0.4223, 0.5399), (0.4739, 0.3992)) \end{array} \right\rangle$
$\mathcal{M}_2$	$\left\langle \begin{array}{l} (s_4, -0.0434), \\ ((0.7181, 0.5192), (0.2659, 0.4664)) \end{array} \right\rangle$	$\left\langle \begin{array}{l} (s_4, -0.0019), \\ ((0.4409, 0.7400), (0.2659, 0.1330)) \end{array} \right\rangle$
$\mathcal{M}_3$	$\left\langle \begin{array}{l} (s_5, -0.0430), \\ ((0.6177, 0.2372), (0.1643, 0.4996)) \end{array} \right\rangle$	$\left\langle \begin{array}{l} (s_3, -0.0440), \\ ((0.3429, 0.6925), (0.6331, 0.3748)) \end{array} \right\rangle$
$\mathcal{M}_4$	$\left\langle \begin{array}{l} (s_4, -0.0434), \\ ((0.1815, 0.6177), (0.6666, 0.1946)) \end{array} \right\rangle$	$\left\langle \begin{array}{l} (s_3, 0.0377), \\ ((0.5399, 0.3429), (0.1643, 0.4664)) \end{array} \right\rangle$
$\mathcal{M}_5$	$\left\langle \begin{array}{l} (s_5, 0.0405), \\ ((0.6177, 0.3746), (0.3297, 0.5670)) \end{array} \right\rangle$	$\left\langle \begin{array}{l} (s_5, 0.0405), \\ ((0.4008, 0.5701), (0.4996, 0.1946)) \end{array} \right\rangle$
	$\hbar_3$	$\hbar_4$
$\mathcal{M}_1$	$\left\langle \begin{array}{l} (s_6, -0.0442), \\ ((0.5192, 0.3429), (0.2335, 0.4664)) \end{array} \right\rangle$	$\left\langle \begin{array}{l} (s_5, -0.0005), \\ ((0.3278, 0.6771), (0.5628, 0.3335)) \end{array} \right\rangle$
$\mathcal{M}_2$	$\left\langle \begin{array}{l} (s_4, -0.0007), \\ ((0.7400, 0.1550), (0.1330, 0.6666)) \end{array} \right\rangle$	$\left\langle \begin{array}{l} (s_5, -0.0453), \\ ((0.8000, 0.2466), (0.1007, 0.5334)) \end{array} \right\rangle$
$\mathcal{M}_3$	$\left\langle \begin{array}{l} (s_4, -0.0434), \\ ((0.7260, 0.5725), (0.5653, 0.4170)) \end{array} \right\rangle$	$\left\langle \begin{array}{l} (s_5, -0.0038), \\ ((0.3746, 0.2466), (0.4996, 0.4939)) \end{array} \right\rangle$
$\mathcal{M}_4$	$\left\langle \begin{array}{l} (s_6, -0.0004), \\ ((0.4409, 0.1550), (0.3663, 0.6666)) \end{array} \right\rangle$	$\left\langle \begin{array}{l} (s_3, -0.0035), \\ ((0.4732, 0.7400), (0.3595, 0.2088)) \end{array} \right\rangle$
$\mathcal{M}_5$	$\left\langle \begin{array}{l} (s_4, -0.0446), \\ ((0.7395, 0.4732), (0.2982, 0.4458)) \end{array} \right\rangle$	$\left\langle \begin{array}{l} (s_5, 0.0395), \\ ((0.5439, 0.4008), (0.3505, 0.4664)) \end{array} \right\rangle$

Table 6 Aggregated assessment values by utilizing Cq-ROFTLWMM and Cq-ROFTLWDMM operators

Alternatives	Cq-ROFTLWMM operator	Cq-ROFTLWDMM operator
$\mathcal{M}_1$	$\left\langle \begin{array}{l} (s_1, -0.0021), \\ ((0.3810, 0.4146), (0.6641, 0.5830)) \end{array} \right\rangle$	$\left\langle \begin{array}{l} (s_7, 0.0040), \\ ((0.5675, 0.6087), (0.3999, 0.4260)) \end{array} \right\rangle$
$\mathcal{M}_2$	$\left\langle \begin{array}{l} (s_1, -0.0181), \\ ((0.6252, 0.3186), (0.4321, 0.5646)) \end{array} \right\rangle$	$\left\langle \begin{array}{l} (s_7, -0.0260), \\ ((0.7254, 0.6870), (0.1702, 0.3736)) \end{array} \right\rangle$
$\mathcal{M}_3$	$\left\langle \begin{array}{l} (s_1, -0.0252), \\ ((0.4104, 0.3404), (0.6963, 0.6397)) \end{array} \right\rangle$	$\left\langle \begin{array}{l} (s_7, -0.0183), \\ ((0.6430, 0.6851), (0.4027, 0.4294)) \end{array} \right\rangle$
$\mathcal{M}_4$	$\left\langle \begin{array}{l} (s_1, -0.0265), \\ ((0.3542, 0.3602), (0.5098, 0.6313)) \end{array} \right\rangle$	$\left\langle \begin{array}{l} (s_7, -0.0455), \\ ((0.5989, 0.5750), (0.3356, 0.3259)) \end{array} \right\rangle$
$\mathcal{M}_5$	$\left\langle \begin{array}{l} (s_1, 0.0039), \\ ((0.4726, 0.4073), (0.5809, 0.5474)) \end{array} \right\rangle$	$\left\langle \begin{array}{l} (s_7, 0.0120), \\ ((0.6645, 0.6179), (0.3517, 0.3770)) \end{array} \right\rangle$

Table 7 Score values of the alternatives

Operators	Score values				
Cq-ROFTLWMM operator	$\digamma(\mathcal{M}_1) = 0.0535$	$\digamma(\mathcal{M}_2) = 0.0542$	$\digamma(\mathcal{M}_3) = 0.0409$	$\digamma(\mathcal{M}_4) = 0.0445$	$\digamma(\mathcal{M}_5) = 0.0604$
Cq-ROFTLWDMM operator	$\digamma(\mathcal{M}_1) = 0.4796$	$\digamma(\mathcal{M}_2) = 0.5263$	$\digamma(\mathcal{M}_3) = 0.4992$	$\digamma(\mathcal{M}_4) = 0.4591$	$\digamma(\mathcal{M}_5) = 0.5112$

Table 8 Ranking results of alternatives

Operators	Ranking	Optimal choice
Cq-ROFTLWMM operator	$\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_1 \succ \mathcal{M}_4 \succ \mathcal{M}_3$	$\mathcal{M}_5$
Cq-ROFTLWDMM operator	$\mathcal{M}_2 \succ \mathcal{M}_5 \succ \mathcal{M}_3 \succ \mathcal{M}_1 \succ \mathcal{M}_4$	$\mathcal{M}_2$

Table 9 Variation of parameter q by Cq-ROFTLWMM operator

Parameter	$\digamma(\mathcal{M}_1)$	$\digamma(\mathcal{M}_2)$	$\digamma(\mathcal{M}_3)$	$\digamma(\mathcal{M}_4)$	$\digamma(\mathcal{M}_5)$	Ranking
$q = 1$	0.0490	0.0545	0.0371	0.0400	0.0575	$\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_1 \succ \mathcal{M}_4 \succ \mathcal{M}_3$
$q = 2$	0.0492	0.0547	0.0368	0.0409	0.0582	$\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_1 \succ \mathcal{M}_4 \succ \mathcal{M}_3$
$q = 3$	0.0513	0.0545	0.0387	0.0428	0.0593	$\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_1 \succ \mathcal{M}_4 \succ \mathcal{M}_3$
$q = 5$	0.0553	0.0539	0.0428	0.0458	0.0614	$\mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_2 \succ \mathcal{M}_4 \succ \mathcal{M}_3$
$q = 7$	0.0578	0.0536	0.0455	0.0475	0.0627	$\mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_2 \succ \mathcal{M}_4 \succ \mathcal{M}_3$
$q = 11$	0.0600	0.0534	0.0480	0.0488	0.0639	$\mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_2 \succ \mathcal{M}_4 \succ \mathcal{M}_3$
$q = 13$	0.0605	0.0534	0.0487	0.0490	0.0642	$\mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_2 \succ \mathcal{M}_4 \succ \mathcal{M}_3$
$q = 17$	0.0610	0.0535	0.0493	0.0492	0.0644	$\mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_2 \succ \mathcal{M}_3 \succ \mathcal{M}_4$
$q = 19$	0.0611	0.0535	0.0495	0.0492	0.0644	$\mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_2 \succ \mathcal{M}_3 \succ \mathcal{M}_4$
$q = 23$	0.0613	0.0535	0.0497	0.0492	0.0644	$\mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_2 \succ \mathcal{M}_3 \succ \mathcal{M}_4$

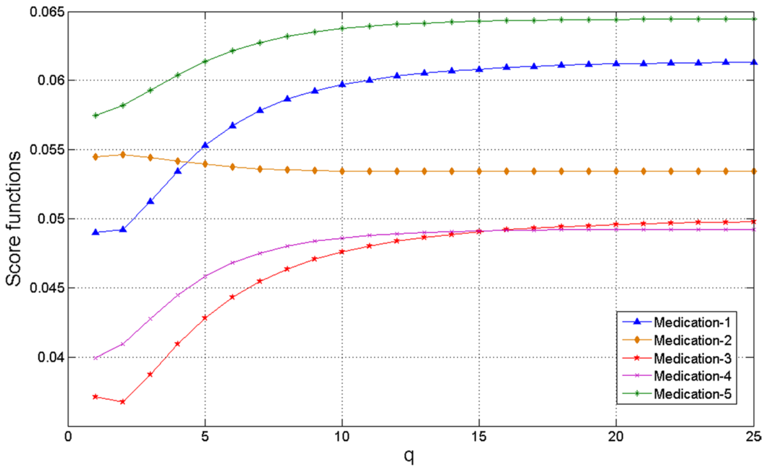

Fig. 3 Scores of alternatives $\mathcal{M}_\iota (\iota = 1, 2, 3, 4, 5)$ when $\psi = (1, 1, 1, 1)$ and $q \in \{1, \ldots, 25\}$ based on the Cq-ROFTLWMM operator

Table 10 Variation of parameter q by Cq-ROFTLWDMM operator

Parameter	$\digamma(\mathcal{M}_1)$	$\digamma(\mathcal{M}_2)$	$\digamma(\mathcal{M}_3)$	$\digamma(\mathcal{M}_4)$	$\digamma(\mathcal{M}_5)$	Ranking
$q = 1$	0.5035	0.5900	0.5119	0.5034	0.5553	$\mathcal{M}_2 \succ \mathcal{M}_5 \succ \mathcal{M}_3 \succ \mathcal{M}_1 \succ \mathcal{M}_4$
$q = 2$	0.5005	0.5806	0.5211	0.4956	0.5509	$\mathcal{M}_2 \succ \mathcal{M}_5 \succ \mathcal{M}_3 \succ \mathcal{M}_1 \succ \mathcal{M}_4$
$q = 3$	0.4906	0.5527	0.5128	0.4765	0.5312	$\mathcal{M}_2 \succ \mathcal{M}_5 \succ \mathcal{M}_3 \succ \mathcal{M}_1 \succ \mathcal{M}_4$
$q = 5$	0.4699	0.5049	0.4857	0.4460	0.4947	$\mathcal{M}_2 \succ \mathcal{M}_5 \succ \mathcal{M}_3 \succ \mathcal{M}_1 \succ \mathcal{M}_4$
$q = 7$	0.4563	0.4754	0.4649	0.4303	0.4726	$\mathcal{M}_2 \succ \mathcal{M}_5 \succ \mathcal{M}_3 \succ \mathcal{M}_1 \succ \mathcal{M}_4$
$q = 11$	0.4447	0.4473	0.4441	0.4191	0.4535	$\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_1 \succ \mathcal{M}_3 \succ \mathcal{M}_4$
$q = 13$	0.4425	0.4405	0.4391	0.4172	0.4495	$\mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_2 \succ \mathcal{M}_3 \succ \mathcal{M}_4$
$q = 17$	0.4405	0.4330	0.4336	0.4156	0.4458	$\mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_3 \succ \mathcal{M}_2 \succ \mathcal{M}_4$
$q = 19$	0.4401	0.4309	0.4321	0.4153	0.4449	$\mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_3 \succ \mathcal{M}_2 \succ \mathcal{M}_4$
$q = 23$	0.4397	0.4282	0.4303	0.4149	0.4440	$\mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_3 \succ \mathcal{M}_2 \succ \mathcal{M}_4$

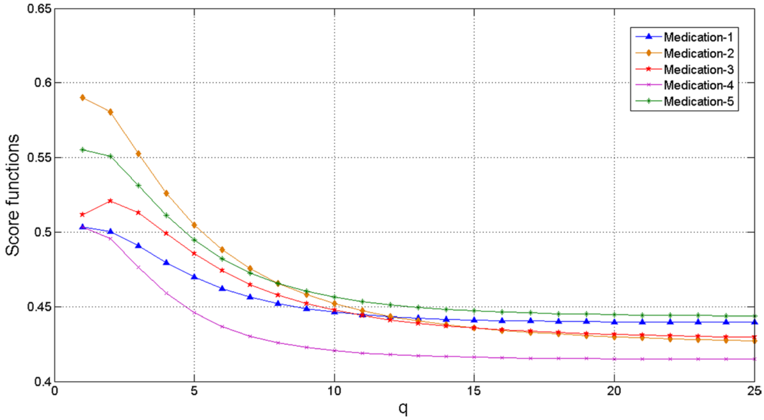

Fig. 4 Scores of alternative $\mathcal{M}_\imath (\imath = 1, 2, 3, 4, 5)$ when $\psi = (1, 1, 1, 1)$ and $q \in \{1, \ldots, 25\}$ based on the Cq-ROFTLWDMM operator

6.2 Variation of parameters

In the ranking results, the parameters q and ψ play an important role. So, in the following subsections, on the basis of Cq-ROFTLWMM and Cq-ROFTLWDMM operators, we study the effect of parameters q and ψ on the score functions as well as ranking results.

6.2.1 Influence of parameter *q*

A parameter analysis is designed to investigate the effect of varying values of parameter q on the aggregation and ranking results. The proposed approach is a general technique that enables DMs to expand their decision evaluation space based on parameter q. In the proposed approach, the parameter q has a major impact on the decision results. Let $\psi = (1, 1, 1, 1)$ to ensure generality. Changing the value of the parameter q provides various ranking results, as shown in Tables 9 and 10. When the value of parameter q varies, the score of each alternative varies, and the ranking results of the alternative are roughly

Table 11 Score functions by Cq-ROFTLWMM operator ($q = 4$)

Parameter	$F(\mathcal{M}_1)$	$F(\mathcal{M}_2)$	$F(\mathcal{M}_3)$	$F(\mathcal{M}_4)$	$F(\mathcal{M}_5)$	Ranking
(1,0,0,0)	0.0728	0.0705	0.0629	0.0596	0.0757	$\mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_2 \succ \mathcal{M}_3 \succ \mathcal{M}_4$
(1,1,0,0)	0.0647	0.0650	0.0547	0.0544	0.0702	$\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_1 \succ \mathcal{M}_3 \succ \mathcal{M}_4$
(1,1,1,0)	0.0592	0.0601	0.0473	0.0486	0.0652	$\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_1 \succ \mathcal{M}_4 \succ \mathcal{M}_3$
(1,1,1,1)	0.0535	0.0542	0.0409	0.0445	0.0604	$\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_1 \succ \mathcal{M}_4 \succ \mathcal{M}_3$
(1,0,2,0)	0.0706	0.0708	0.0615	0.0587	0.0762	$\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_1 \succ \mathcal{M}_3 \succ \mathcal{M}_4$
(2,0,3,0)	0.0755	0.0760	0.0681	0.0629	0.0823	$\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_1 \succ \mathcal{M}_3 \succ \mathcal{M}_4$
(1,2,1,2)	0.0563	0.0570	0.0442	0.0466	0.0630	$\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_1 \succ \mathcal{M}_4 \succ \mathcal{M}_3$
(2,3,2,3)	0.0544	0.0554	0.0424	0.0452	0.0619	$\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_1 \succ \mathcal{M}_4 \succ \mathcal{M}_3$

the same. When the value of parameter q varies, the best alternatives are $\mathcal{M}_5$ and $\mathcal{M}_2$, as shown in Fig. 3 and Fig. 4, respectively. Fundamentally, q should be chosen as the smallest number whose total of the q-th powers of MD and q-th powers of NMD is equal to or less than 1. As we can see from the Tables 9 and 10, different scores and ranking results are derived by different values of q utilizing Cq-ROFWMM and Cq-ROFWDMM operators,

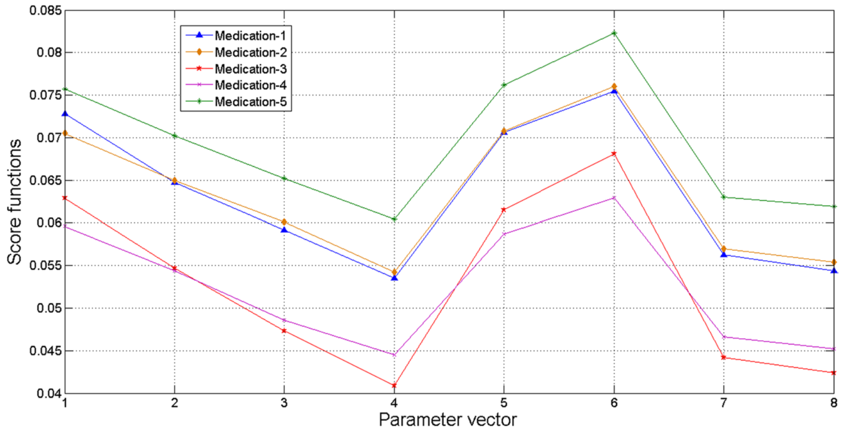

Fig. 5 Scores of alternatives $\mathcal{M}_\imath(\imath = 1, 2, \ldots, 5)$ according to ψ and $q = 4$ based on the Cq-ROFTLWMM operator

Table 12 Score functions by Cq-ROFTLWDMM operator ($q = 4$)

Parameter	$F(\mathcal{M}_1)$	$F(\mathcal{M}_2)$	$F(\mathcal{M}_3)$	$F(\mathcal{M}_4)$	$F(\mathcal{M}_5)$	Ranking
(1,0,0,0)	0.4288	0.4631	0.4187	0.4002	0.4587	$\mathcal{M}_2 \succ \mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_3 \succ \mathcal{M}_4$
(1,1,0,0)	0.4540	0.4765	0.4516	0.4341	0.4818	$\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_1 \succ \mathcal{M}_3 \succ \mathcal{M}_4$
(1,1,1,0)	0.4690	0.4949	0.4806	0.4487	0.4974	$\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_3 \succ \mathcal{M}_1 \succ \mathcal{M}_4$
(1,1,1,1)	0.4796	0.5263	0.4992	0.4591	0.5112	$\mathcal{M}_2 \succ \mathcal{M}_5 \succ \mathcal{M}_3 \succ \mathcal{M}_1 \succ \mathcal{M}_4$
(1,0,2,0)	0.4469	0.4719	0.4452	0.4256	0.4765	$\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_1 \succ \mathcal{M}_3 \succ \mathcal{M}_4$
(2,0,3,0)	0.4436	0.4679	0.4408	0.4237	0.4732	$\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_1 \succ \mathcal{M}_3 \succ \mathcal{M}_4$
(1,2,1,2)	0.4757	0.5199	0.4933	0.4555	0.5070	$\mathcal{M}_2 \succ \mathcal{M}_5 \succ \mathcal{M}_3 \succ \mathcal{M}_1 \succ \mathcal{M}_4$
(2,3,2,3)	0.4775	0.5232	0.4961	0.4634	0.5123	$\mathcal{M}_2 \succ \mathcal{M}_5 \succ \mathcal{M}_3 \succ \mathcal{M}_1 \succ \mathcal{M}_4$

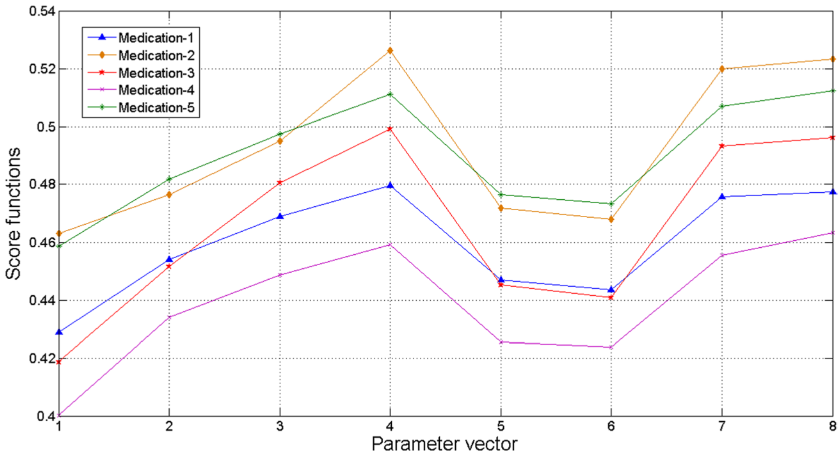

Fig. 6 Scores of alternatives $\mathcal{M}_\imath(\imath = 1, 2, \ldots, 5)$ according to ψ and $q = 4$ based on the Cq-ROFTLWDMM operator

which also demonstrates the elasticity of our proposed method. In Table 9 and Fig. 3, when $q = \{1, 2, 3\}$ the ranking result is $\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_1 \succ \mathcal{M}_4 \succ \mathcal{M}_3$. When $q = \{5, 7, 11, 13\}$ the ranking result is $\mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_2 \succ \mathcal{M}_4 \succ \mathcal{M}_3$ while for $q = \{17, 19, 23\}$ the ranking result is $\mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_2 \succ \mathcal{M}_3 \succ \mathcal{M}_4$. In Table 10 and Fig. 4, when $q = \{1, 2, 3, 5, 7\}$ the ranking result is $\mathcal{M}_2 \succ \mathcal{M}_5 \succ \mathcal{M}_3 \succ \mathcal{M}_1 \succ \mathcal{M}_4$. When $q = 11$ and $q = 13$ the ranking results are $\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_1 \succ \mathcal{M}_3 \succ \mathcal{M}_4$ and $\mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_2 \succ \mathcal{M}_3 \succ \mathcal{M}_4$, respectively. When $q = \{17, 19, 23\}$ the ranking result is $\mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_3 \succ \mathcal{M}_2 \succ \mathcal{M}_4$.

6.2.2 Influence of parameter ψ

Parameter vector can be considered as DMs risk preference and makes Cq-ROFTLMM operator more flexible. To additionally show the viability and applicability of the introduced approach, we utilize the developed technique to solve MAGDM problems. Different aggregation results can be obtained by changing the parameter vector ψ. Let $q = 4$ to ensure generality then the influence of the several values of parameter ψ can be seen in Tables 11 and 12. By fixing the parameter $\psi = (1, 0, 0, 0)$ the ranking results of the proposed Cq-ROFTLWMM and Cq-ROFTLWDMM approaches are $\mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_2 \succ \mathcal{M}_3 \succ \mathcal{M}_4$ and $\mathcal{M}_2 \succ \mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_3 \succ \mathcal{M}_4$ as shown in Tables 11 and 12, respectively. In addition, we find out that when $\psi = (1, 1, 0, 0)$, $\psi = (1, 0, 2, 0)$ or $\psi = (2, 0, 3, 0)$ the ranking result is $\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_1 \succ \mathcal{M}_3 \succ \mathcal{M}_4$ which is different from the ranking results when $\psi = (1, 1, 1, 0)$, $\psi = (1, 1, 1, 1)$, $\psi = (1, 2, 1, 2)$ or $\psi = (2, 3, 2, 3)$ by utilizing Cq-ROFTLWMM operator. In view of the scores in Tables 11 and 12, and from Fig. 5 and Fig. 6, respectively, we observe that the ranking results by the Cq-ROFTLWMM and Cq-ROFTLWDMM operators are nearly the same, however, the optimal alternatives are $\mathcal{M}_2$ and $\mathcal{M}_5$.

6.3 Validity analysis of the developed approach

In this subsection, two actual decision-making situations are addressed using the existing MAGDM approaches, and the results are discussed to demonstrate the validity of the new

Table 13 CIFTL decision matrix given by $\mathfrak{c}_1$

Alternatives	$\hbar_1$	$\hbar_2$	$\hbar_3$	$\hbar_4$
$\mathcal{M}_1$	$\langle (s_5, 0), ((0.5, 0.3), (0.4, 0.3)) \rangle$	$\langle (s_2, 0), ((0.4, 0.5), (0.5, 0.4)) \rangle$	$\langle (s_1, 0), ((0.3, 0.5), (0.6, 0.3)) \rangle$	$\langle (s_2, 0), ((0.1, 0.4), (0.6, 0.1)) \rangle$
$\mathcal{M}_2$	$\langle (s_8, 0), ((0.7, 0.1), (0.2, 0.6)) \rangle$	$\langle (s_9, 0), ((0.3, 0.6), (0.4, 0.1)) \rangle$	$\langle (s_5, 0), ((0.7, 0.2), (0.2, 0.4)) \rangle$	$\langle (s_4, 0), ((0.1, 0.2), (0.2, 0.5)) \rangle$
$\mathcal{M}_3$	$\langle (s_5, 0), ((0.3, 0.7), (0.5, 0.1)) \rangle$	$\langle (s_4, 0), ((0.2, 0.7), (0.6, 0.1)) \rangle$	$\langle (s_3, 0), ((0.1, 0.8), (0.6, 0.1)) \rangle$	$\langle (s_9, 0), ((0.8, 0.1), (0.1, 0.6)) \rangle$
$\mathcal{M}_4$	$\langle (s_6, 0), ((0.2, 0.8), (0.7, 0.1)) \rangle$	$\langle (s_7, 0), ((0.6, 0.1), (0.2, 0.7)) \rangle$	$\langle (s_8, 0), ((0.5, 0.3), (0.1, 0.6)) \rangle$	$\langle (s_5, 0), ((0.6, 0.2), (0.2, 0.5)) \rangle$
$\mathcal{M}_5$	$\langle (s_1, 0), ((0.3, 0.5), (0.4, 0.2)) \rangle$	$\langle (s_6, 0), ((0.1, 0.5), (0.5, 0.3)) \rangle$	$\langle (s_6, 0), ((0.8, 0.4), (0.1, 0.3)) \rangle$	$\langle (s_3, 0), ((0.7, 0.2), (0.1, 0.6)) \rangle$

Table 14 CIFTL decision matrix given by $\mathfrak{c}_2$

Alternatives	$\hbar_1$	$\hbar_2$	$\hbar_3$	$\hbar_4$
$\mathcal{M}_1$	$\langle(s_6,0),((0.6,0.5),(0.2,0.3))\rangle$	$\langle(s_6,0),((0.3,0.5),(0.4,0.1))\rangle$	$\langle(s_4,0),((0.8,0.2),(0.1,0.5))\rangle$	$\langle(s_5,0),((0.5,0.2),(0.1,0.5))\rangle$
$\mathcal{M}_2$	$\langle(s_3,0),((0.4,0.2),(0.3,0.6))\rangle$	$\langle(s_5,0),((0.5,0.2),(0.3,0.4))\rangle$	$\langle(s_6,0),((0.5,0.4),(0.2,0.5))\rangle$	$\langle(s_2,0),((0.2,0.8),(0.3,0.1))\rangle$
$\mathcal{M}_3$	$\langle(s_4,0),((0.3,0.6),(0.5,0.2))\rangle$	$\langle(s_7,0),((0.4,0.3),(0.2,0.6))\rangle$	$\langle(s_9,0),((0.7,0.2),(0.1,0.6))\rangle$	$\langle(s_9,0),((0.4,0.6),(0.3,0.2))\rangle$
$\mathcal{M}_4$	$\langle(s_3,0),((0.6,0.2),(0.1,0.5))\rangle$	$\langle(s_4,0),((0.3,0.7),(0.6,0.1))\rangle$	$\langle(s_5,0),((0.1,0.2),(0.8,0.5))\rangle$	$\langle(s_6,0),((0.3,0.2),(0.5,0.4))\rangle$
$\mathcal{M}_5$	$\langle(s_8,0),((0.7,0.4),(0.1,0.5))\rangle$	$\langle(s_3,0),((0.3,0.5),(0.6,0.3))\rangle$	$\langle(s_1,0),((0.2,0.4),(0.3,0.2))\rangle$	$\langle(s_3,0),((0.3,0.3),(0.4,0.5))\rangle$

Table 15 CIFTL decision matrix given by $\mathfrak{c}_3$

Alternatives	$\hbar_1$	$\hbar_2$	$\hbar_3$	$\hbar_4$
$\mathcal{M}_1$	$\langle(s_4,0),((0.3,0.7),(0.6,0.2))\rangle$	$\langle(s_3,0),((0.7,0.2),(0.1,0.6))\rangle$	$\langle(s_2,0),((0.6,0.2),(0.2,0.7))\rangle$	$\langle(s_5,0),((0.5,0.3),(0.3,0.4))\rangle$
$\mathcal{M}_2$	$\langle(s_9,0),((0.7,0.2),(0.1,0.4))\rangle$	$\langle(s_8,0),((0.3,0.5),(0.6,0.4))\rangle$	$\langle(s_9,0),((0.4,0.5),(0.5,0.3))\rangle$	$\langle(s_2,0),((0.1,0.7),(0.5,0.1))\rangle$
$\mathcal{M}_3$	$\langle(s_3,0),((0.3,0.6),(0.5,0.1))\rangle$	$\langle(s_6,0),((0.6,0.1),(0.3,0.7))\rangle$	$\langle(s_3,0),((0.1,0.6),(0.7,0.1))\rangle$	$\langle(s_1,0),((0.6,0.1),(0.2,0.4))\rangle$
$\mathcal{M}_4$	$\langle(s_4,0),((0.6,0.1),(0.2,0.6))\rangle$	$\langle(s_2,0),((0.3,0.4),(0.5,0.3))\rangle$	$\langle(s_5,0),((0.5,0.2),(0.3,0.4))\rangle$	$\langle(s_6,0),((0.4,0.2),(0.2,0.6))\rangle$
$\mathcal{M}_5$	$\langle(s_9,0),((0.3,0.6),(0.4,0.2))\rangle$	$\langle(s_3,0),((0.1,0.4),(0.6,0.3))\rangle$	$\langle(s_4,0),((0.7,0.4),(0.1,0.3))\rangle$	$\langle(s_7,0),((0.7,0.2),(0.1,0.6))\rangle$

Table 16 Decision results by different methods

Approaches	Parameters	$F(\mathcal{M}_1)$	$F(\mathcal{M}_2)$	$F(\mathcal{M}_3)$	$F(\mathcal{M}_4)$	$F(\mathcal{M}_5)$	Ranking
CIFWA (Garg and Rani 2019)	No parameter	0.0439	0.0751	0.0584	0.0512	0.0522	$\mathcal{M}_2 \succ \mathcal{M}_3 \succ \mathcal{M}_5 \succ \mathcal{M}_4 \succ \mathcal{M}_1$
CIFWBM (Garg and Rani 2020)	$r = 1, s = 1$	0.0423	0.0689	0.0543	0.0484	0.0496	$\mathcal{M}_2 \succ \mathcal{M}_3 \succ \mathcal{M}_5 \succ \mathcal{M}_4 \succ \mathcal{M}_1$
CPFWA (Ma et al. 2021)	No parameter	0.0441	0.0761	0.0589	0.0521	0.0518	$\mathcal{M}_2 \succ \mathcal{M}_3 \succ \mathcal{M}_4 \succ \mathcal{M}_5 \succ \mathcal{M}_1$
CPFWBM	$r = 1, s = 1$	0.0420	0.0710	0.0554	0.0512	0.0495	$\mathcal{M}_2 \succ \mathcal{M}_3 \succ \mathcal{M}_4 \succ \mathcal{M}_5 \succ \mathcal{M}_1$
Cq-ROFTLWMM	$\psi = (1, 0, 0, 0), q = 1$	0.0440	0.0750	0.0588	0.0500	0.0523	$\mathcal{M}_2 \succ \mathcal{M}_3 \succ \mathcal{M}_5 \succ \mathcal{M}_4 \succ \mathcal{M}_1$
Cq-ROFTLWMM	$\psi = (1, 1, 0, 0), q = 1$	0.0412	0.0696	0.0546	0.0488	0.0492	$\mathcal{M}_2 \succ \mathcal{M}_3 \succ \mathcal{M}_5 \succ \mathcal{M}_4 \succ \mathcal{M}_1$
Cq-ROFTLWMM	$\psi = (1, 0, 0, 0), q = 2$	0.0443	0.0759	0.0594	0.0525	0.0520	$\mathcal{M}_2 \succ \mathcal{M}_3 \succ \mathcal{M}_4 \succ \mathcal{M}_5 \succ \mathcal{M}_1$
Cq-ROFTLWMM	$\psi = (1, 1, 0, 0), q = 2$	0.0415	0.0709	0.0550	0.0507	0.0491	$\mathcal{M}_2 \succ \mathcal{M}_3 \succ \mathcal{M}_4 \succ \mathcal{M}_5 \succ \mathcal{M}_1$

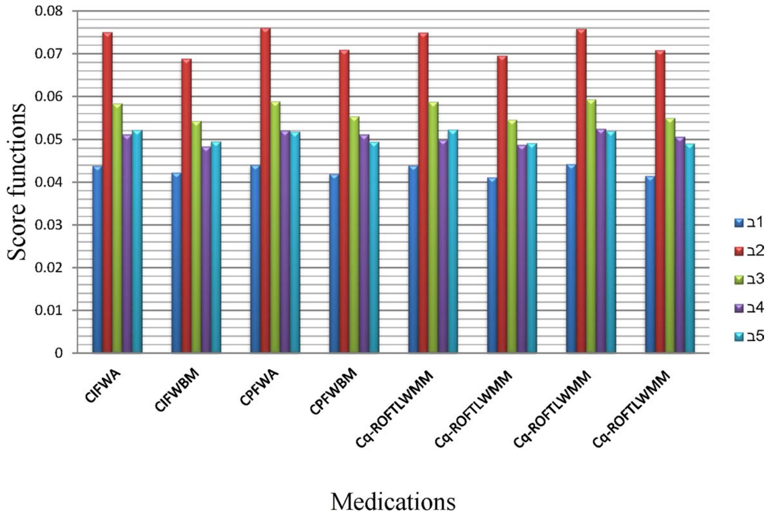

Fig. 7 Comparison with other existing approaches

approach. We utilize the CIFTLWA, CIFTLWBM, CPFTLWA, CPFTLWBM approaches, and the Cq-ROFTLWMM approach presented in this paper to solve several examples and to demonstrate the practicability of our proposed approach.

Example 7 A food company wants to track various inventory goods. This firm makes five different types of food, i.e., pies ($\mathcal{M}_1$), muffins ($\mathcal{M}_2$), pastries ($\mathcal{M}_3$), buns ($\mathcal{M}_4$), and doughnuts ($\mathcal{M}_5$). Three DMs $\mathfrak{c}_1$, $\mathfrak{c}_2$, and $\mathfrak{c}_3$ are invited to evaluate the performance of the five alternatives based on four attributes, i.e., cost price ($\hbar_1$), storage facilities ($\hbar_2$), sale price level ($\hbar_3$), and staleness level ($\hbar_4$). The weight vector of attributes is $\varpi = (0.30, 0.24, 0.29, 0.17)$ (suppose $\psi = (1, 1, 2)$). Each DM uses CIFTLNs to present their preference information for alternatives $\mathcal{M}_\imath (\imath = 1, 2, \ldots, 5)$, and three CIFTL assessment matrices are provided in Tables 13, 14, and 15.

We utilize the CIFTLWA, CIFTLWBM, CPFTLWA, CPFTLWBM approaches, and the Cq-ROFTLWMM approach proposed in this paper to solve Example 2 and show the decision results in Table 16 and Fig. 7. Table 16 illustrates that the ranking results derived by the existing approaches are the same as that derived by our proposed approach, which proves the validity of our developed approach.

Example 8 Consider the medicine of Pneumonia selection problem shown in Example 4. We will compare our proposed operators to existing tools, such as the Cq-ROFTLWA and Cq-ROFTLWG operators to solve this example and demonstrate the effectiveness of our proposed method. Also, we will compare with Cq-ROFTLWBM operator, Cq-ROFTLWGBM operator, Cq-ROFTLWMSM operator, and Cq-ROFTLWDMSM operator. Table 17 and Fig. 8 show the comparison results.

We can obtain different ranking results from the above, it is evident that the optimal alternative is $\mathcal{M}_2$ or $\mathcal{M}_5$, indicating the reliability and superiority of the suggested operators for MAGDM problems with Cq-ROFTLNs. In the MAGDM process, Cq-ROFTLS is becoming more comprehensive and gives more information.

Table 17 Decision results by different methods

Approaches	Parameters	$F(\mathcal{M}_1)$	$F(\mathcal{M}_2)$	$F(\mathcal{M}_3)$	$F(\mathcal{M}_4)$	$F(\mathcal{M}_5)$	Ranking
Cq-ROFTLWA	No parameter	0.6502	0.6793	0.6287	0.6129	0.6683	$\mathcal{M}_2 \succ \mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_3 \succ \mathcal{M}_4$
Cq-ROFTLWG	No parameter	0.2194	0.2652	0.2045	0.2151	0.2462	$\mathcal{M}_2 \succ \mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_4 \succ \mathcal{M}_3$
Cq-ROFTLWBM	$r = 1, s = 1$	0.6240	0.6422	0.5972	0.5866	0.6472	$\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_1 \succ \mathcal{M}_3 \succ \mathcal{M}_4$
Cq-ROFTLWGBM	$r = 1, s = 1$	0.2373	0.2931	0.2314	0.2461	0.2668	$\mathcal{M}_2 \succ \mathcal{M}_5 \succ \mathcal{M}_4 \succ \mathcal{M}_1 \succ \mathcal{M}_3$
Cq-ROFTLWMSM	$t = 3, q = 4$	0.6108	0.6306	0.5762	0.5710	0.6367	$\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_1 \succ \mathcal{M}_3 \succ \mathcal{M}_4$
Cq-ROFTLWDMSM	$t = 3, q = 4$	0.2465	0.3037	0.2458	0.2714	0.2733	$\mathcal{M}_2 \succ \mathcal{M}_5 \succ \mathcal{M}_4 \succ \mathcal{M}_1 \succ \mathcal{M}_3$
Cq-ROFTLWMM	$\psi = (1, 0, 0, 0), q = 4$	0.0728	0.0705	0.0629	0.0596	0.0757	$\mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_2 \succ \mathcal{M}_3 \succ \mathcal{M}_4$
Cq-ROFTLWMM	$\psi = (1, 1, 1, 1), q = 4$	0.0535	0.0542	0.0409	0.0445	0.0604	$\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_1 \succ \mathcal{M}_4 \succ \mathcal{M}_3$
Cq-ROFTLWDMM	$\psi = (1, 0, 0, 0), q = 4$	0.4288	0.4631	0.4187	0.4002	0.4587	$\mathcal{M}_2 \succ \mathcal{M}_5 \succ \mathcal{M}_1 \succ \mathcal{M}_3 \succ \mathcal{M}_4$
Cq-ROFTLWDMM	$\psi = (1, 1, 1, 1), q = 4$	0.4796	0.5263	0.4992	0.4591	0.5112	$\mathcal{M}_2 \succ \mathcal{M}_5 \succ \mathcal{M}_3 \succ \mathcal{M}_1 \succ \mathcal{M}_4$

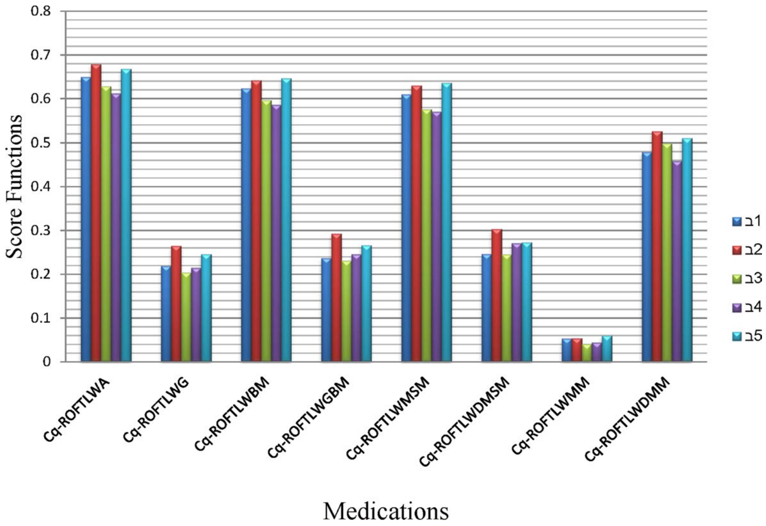

Fig. 8 Comparison with other existing approaches

- The interrelationship between arguments is not taken into consideration during the fusion process, which is a limitation of Cq-ROFTLWA and Cq-ROFTLWG operators. In other words, they presume that all attributes are distinct, which might not be valid. To choose the optimal medication for pneumonia, we consider the attribute values of each alternative as well as the interrelationship between attributes. As a result, the Cq-ROFTLWA and Cq-ROFTLWG techniques are insufficient to address this issue. Since our proposed operator can capture interrelationships among multi-input arguments, it is becoming more feasible to solve this problem than the Cq-ROFTLWA and Cq-ROFTLWG approaches.
- The Cq-ROFTLWBM and Cq-ROFTLWGBM operators represent the correlation between attributes, the ranking results of our proposed approach and both operators are the same, which indicates the validity and reliability of the proposed approach. Cq-ROFTLWBM and Cq-ROFTLWGBM operators, on the other hand, can only describe the correlation between two attribute values and cannot represent the correlation between multiple input arguments. In addition, the Cq-ROFWBM and Cq-ROFTLWGBM operators need to input two parameters, and our method has a vector of parameters. Therefore, the developed approach is general and more flexible than the Cq-ROFTLBM operator.
- As both Cq-ROFTLMSM and the proposed operators can capture the interrelationship among multi-input arguments, the ranking results of both approaches are the same, which proves that the proposed method is more effective and valid. Furthermore, Cq-ROFTLWMSM and Cq-ROFTLWDMSM operators are unable to reduce the influence of excessively high or low arguments on ranking results, whereas our proposed technique can. Cq-ROFTLMSM is a special case of our proposed operator, demonstrating that the suggested operator is more effective and superior to the Cq-ROFTLMSM operator.

6.4 Advantages of the proposed work

Various aggregation operators have various functions, and the DM can select appropriate aggregation operators based on the actual decision-making environment. As a result, when compared to other operators, our proposed operators have some advantages and superiorities.

The following are some of the advantages of our developed approach:

- Since the proposed approach successfully handle the dependency of the multi-input arguments, so this approach is preferable to others. In addition, based on the parameter vector ψ, several existing operators can be considered special cases of Cq-ROFTLMM operator. As a result, we can determine that the suggested operators are significantly powerful and have potential applications.
- The Cq-ROFTLMM operator provides more flexible and powerful information fusion, making it easier to solve complex MAGDM problems. It uses the MM operator to interpret attribute interrelationships and can be effectively utilized in a variety of cases by allocating preference values with different values of parameter ψ. As a result, the Cq-ROFTLMM operator can be used effectively in the information fusion process because of its excellent flexibility to represent the correlation between multi-input arguments.
- The constructed framework uses Cq-ROFTLS as the information representation and Cq-ROFTLS can provide more comprehensive assessment details because it combines the best features of the Cq-ROFS and 2TLS. Moreover, Cq-ROFTLS can tackle practical problems both quantitatively and qualitatively. Therefore, the proposed approach is clear and has less loss of data.

Table 18 compares the characteristics of various operators.

7 Conclusions

MAGDM in the linguistic environment is an essential part of modern decision-making research. While addressing MAGDM problems, the information aggregation operators play a crucial role. Cq-ROFS is a useful tool for dealing with the evaluations given by DMs in MAGDM problems without any loss of data. 2TL terms can describe the information given by DMs in natural language. In this paper, a new Cq-ROFTLS has been presented by combining the concepts of Cq-ROFS and 2TLS. The Cq-ROFTLMM aggregation operator is different from others, not only because it accepts Cq-ROFTLNs, but also considered the interdependent phenomena among the multi-input arguments. Some novel Cq-ROFTLMM aggregation operators, such as Cq-ROFTLMM, Cq-ROFTLWMM, Cq-ROFTLDMM, and Cq-ROFTLWDMM operators, have been put forward for computing Cq-ROFTLNs. Further, some important properties and special cases of the developed aggregation operators have also been discussed with the vector of the parameter. Subsequently, a novel approach to MAGDM based on Cq-ROFTLMM aggregation operators has been presented in an efficient way under Cq-ROFTL circumstances. Further, a numerical instance related to the choice of the best medication for pneumonia has been presented to illustrate the effectiveness of the proposed concepts in decision-making. Moreover, from obtaining results we

Table 18 The characteristic comparison among different operators

Aggregation operators	Whether the operator can capture the inter-relationship between input values	Whether consider the interactions between the MD and NMD	Makes method flexible by parameter value	Represents two dimensional information
Cq-ROFTLWA	×	×	×	✓
Cq-ROFTLWG	×	×	×	✓
Cq-ROFTLWBM	✓	✓	✓	✓
Cq-ROFTLWGBM	✓	✓	✓	✓
Cq-ROFTLWMSM	✓	✓	✓	✓
Cq-ROFTLWDMSM	✓	✓	✓	✓
Cq-ROFTLWMM	✓	✓	✓	✓
Cq-ROFTLWDMM	✓	✓	✓	✓

concluded that this strategy is feasible, powerful, adaptable, and fulfilling expressiveness, it also takes into account all the interrelationships among various attributes and reduces the negative impact of biased attribute values.

Though the proposed approach develops a novel notion to explain ambiguous and imprecise information and expands its theory and application in real-life, the following two limitations need to be investigated further. The first limitation is to employ proper procedures for determining attribute and expert weights. The proposed approach supposes that DMs provide the weights of attributes and DMs in advance. The weights provided by experts examine only the subjectively and disregard the weight information given by the decision matrix. As a result, the associative weight method for determining attribute weight should be considered for dealing with actual decision issues. It is obvious that the changing parameters can result in different rankings, consequently, determining the best parameter value for the provided algorithm is the second limitation. In the future, MM and DMM operators will be extended to: (1) complex interval-valued q-rung orthopair fuzzy 2-tuple linguistic sets; (2) complex simplified interval-valued q-rung orthopair fuzzy 2-tuple linguistic sets; and (3) complex hesitant q-rung orthopair fuzzy 2-tuple linguistic sets.

Declarations

Conflict of interest The authors declare no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by the author.

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