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An optimized stability framework for three-dimensional Hartman flow via Chebyshev collocation simulations

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ABSTRACT

Hydrodynamics instability is studied for an electrical conducting fluid against small disturbance between the channels by using normal magnetized force. Chebyshev collocation technique is used to determine the stability of three-dimensional Hartmann flow problem. The distinctive case of the perturbations is calculated. It is also considered that perturbations intensity be determined by just on quantified by different parameters. We have considered one of the flow stabilities conditions to analyze our problem. *QZ* (Qualitat and Zuverlassigkeit) technique is applied to investigate the problem to draw stability curves. α_c, β_c are critical wave numbers in streamwise and span-wise respectively and for a big range of Hartmann value (Ha), we obtained critical Reynolds number (Re_c). It is found that Couette flow is destabilized for a specific value of Magnetize force while with greater or lesser magnitude than the particular one will stabilize the flow. Disturbances with particular oblique angle θ will grow while the others will decay for the three-dimensional disturbance. We observed from over results that for $Ha(>2.0)$ Re_c, increases steadily and when Ha more than 3.886, Re_c decreases fast to the minimum. It is also established that for drop down from $Ha = 3.886$ and Re_c becomes very large. For distinct Hartmann values, there exist two Re_c numbers due to closed contour. The outcomes of current study are utilized in drug-delivery systems and photodynamic therapy.

Introduction

In the fluid mechanics, hydro-dynamic instability is the field of analyzing the stability and instability of fluids flow. The study of hydrodynamic instability objectives to discover whether a given fluid flow is unstable or stable, and therefore, how these instabilities will effect on the expansion of turbulence. Another important field is the magneto-hydrodynamics (MHD) is very effective and have various usages in the research fields. Hannes [1] is a famous Swedish physicist who first invented a new domain of research that is known as MHD fluid. He worked in the research field of plasma physics is unconscionable. Analyzing of hydro-magnetic instabilities of the fluid flow is the most interesting comprehensive investigation. Darzin [2] demonstrated that fluid is unstable due to some disturbance of inertia, viscous stresses, and external forces of fluid. The developing interest of instability has many reasons such as applied magnetic field. The stability of different fluid systems is caused by electromagnetic forces are main fluid systems and interaction of the disturbance are distinct in according to systems of fluid. For plane Couette flow, it was found both in numerical simulation and in experiments (Anselmo et al. [3]; Kraheberger et al. [4]) that very low Reynolds number where the turbulence occurred between 300 and 400. They observed that the previous work of Weibel instability is

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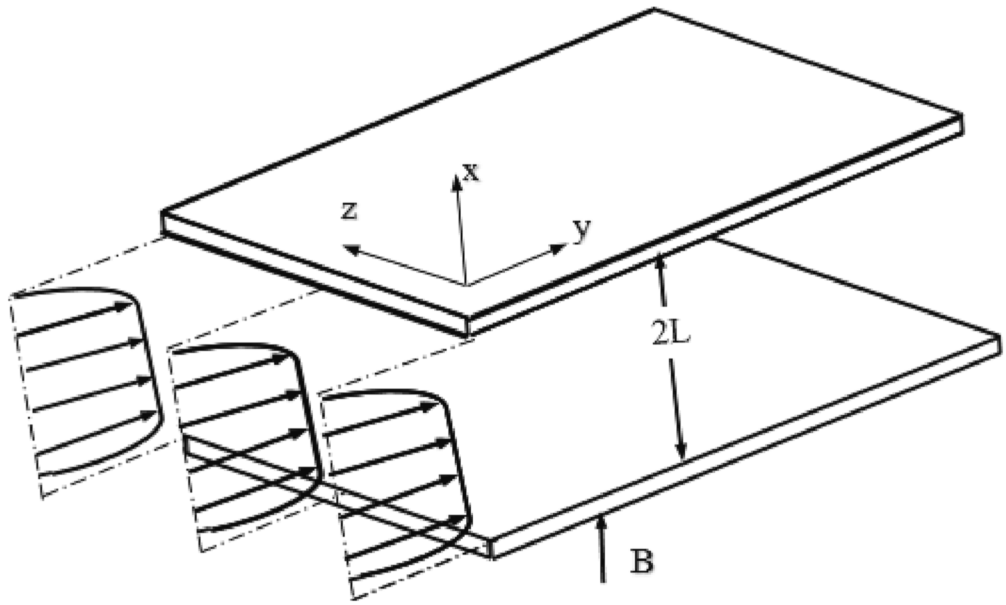

Fig. 1. Schematic diagram.

Table 1
Comparison of results with analysis of studies of Nabil [15] and Takashima [16].

Ha	***Re_c*** Present	Takashima	Nabil	***k_c*** Present	Takashima	Nabil	***N_1*** Present	Takashima	Nabil
0	10E15			0.01			150		
3.887	10,826,409	10,826,766		0.020727	0.020717		150	150	
4	870000.8	869989.8	838,696	0.233069	0.233179	0.2338	120	120	120
5	338531.9	338528.2	334,005	0.64945	0.64950	0.65	110	110	100

because of overestimated number of the observed magnetic fields. Giribabu and Shankar [5] investigated the finite amplitude method by using plane Couette flow. The graphs are presented for least Reynolds numbers ($Re = 5125$) by numerical simulation is presented in steady states. With the help of the following technique was used by Philippou et al. [6] for plane Couette flow, wavy Taylor vortices and a new class of solution was found. The subcritical presence of three-dimensional flows was found for the onset of convection at low critical value Rayleigh number. Harisingh and Rama [7] investigated the shear flow by imposing the perturbation and concluded that results are approximately constant in a streamwise and circular direction. The finite amplitude of 3-D perturbations for Poiseuille flow is unstable in spanwise direction. Couette and Poiseuille flow are investigated by Orszag and Patera [8] and Hussain et al. [9,10]) found that as the wire was set in the flow in plane Couette flow, streamwise vortices could be generated for *Re* about 200. It is found by imposing the perturbation did not verify this approximation. Barkley [11] solved the plane Couette pattern instability by a numerical method when a ribbon is placed in the middle of the plan. He concluded that the results are about comparable when he used experimentally investigation. It has been obtained that by using the magnetize force is acting as a stabilizing factor, and some experimental data verifying this result.

This steady is an electrical conductive in infinite two planes by applying transverse magnetize force has been investigated that the results are showing by Lock [12] and Hussain et al. [13] observed that by applying Couette flow both stabilizing and destabilizing effect are appearing in the results. They determined the stability system is neglecting the velocity disturbance for different number of *Ha.* Zhang et al. [14] worked for the stability for the range of $0 < Ha < 3.91$ and destabilized by the magnetic field for the range of $3.91 < Ha < 5.4$ and stabilized again reversely for the range of $Ha > 5.4$. Nabil et al. [15] presented the accurate results for the same flow and observed that flow is stabled when magnetic field is less than 3.9, destabilized for $3.88 \geqslant Ha \geqslant 5.542$ and then stabilized again for $Ha > 5.542$. Takashima [16] presented Couette flow with magnetic force impact. Some more studies in this topic can be seen in refs. [17–22].

The aim of current investigation is to observe a three-dimensional flow of parallel plate with applications of stability framework. The problem entertained the impact of magnetic force for Hartmann flow. The growth and disturbance of 3D problem is presented. Current model endorsed modifications in work of [14] and [15] for three-dimensional flow. Chebyshev collocation method with a different expansion is presented. The obtained results are more suitable as compared to [15]. Different expansion was used to solve this problem numerically with the help of QZ algorithm.

Mathematical formulation

Electrical conducting Hartmann flow is used to investigate the stability of this problem. Incompressible, viscous fluid properties are used with the help of perpendicular magnetize force. Electrically conductive fluid is moving from the parallel channel by setting 2L distance between the channels. The fluid is flowing inside the channel with basic velocity U_0. We consider the flow in y-direction for the channel. The change in the velocity is appearing in x-axis of channel. Magnetize force is applying the normal direction of the channel that is in *x*-axis as presented in the schematic diagram of Fig. 1.

The flow is assumed to be induction less. The governing equations with non-dimensional parameters are stated as

$$\nabla \cdot v = 0 \tag{1}$$

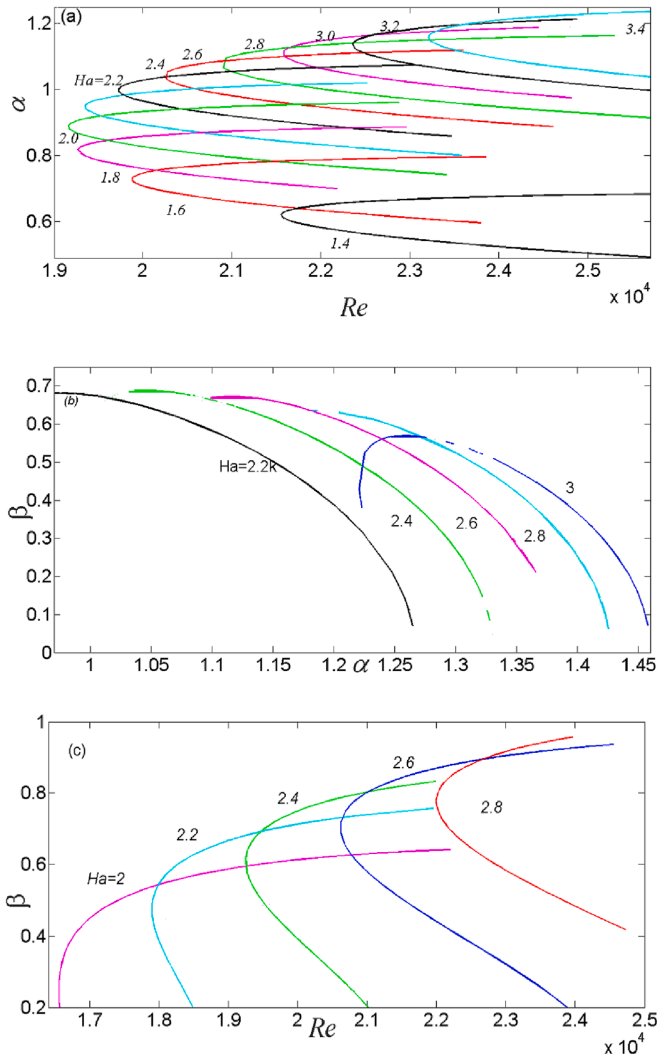

Fig. 2. Neutral curves of stability for (a) and, (b) and (c) and when.

$$\partial_t v + (v.\nabla)v = -\nabla p + \frac{1}{\mathrm{Re}}\Delta v + N(j \times \hat{x}) \tag{2}$$

In above equation, the Ohm law is represented by $\boldsymbol{j}$ which can be described as

$$j = -\nabla\phi + v \times \hat{x}, \tag{3}$$

and charge equation is represented by

$$\nabla \cdot j = 0. \tag{4}$$

The interaction number $N = \sigma LB^2/\rho V$ (proportion of inertial & electromagnetic forces) and $Re = VL/\bar{v}$ (proportion of viscous force and inertial force are in ratio from) are two non-dimensional parameters. Interaction parameter N is the ratio electromagnetic forces and inertia and used in equation (2). Ha is represented by Hartmann number which is also depends on the strength of the magnetic field. The kinematic viscosity represents by the symbol $\bar{v}$. The boundary conditions considered for our system are shown in eq. (5).

$$\partial u/\partial x = \partial v/\partial y = 0, \qquad \partial\phi/\partial x = u = 0, at x = \pm L. \tag{5}$$

where v, ϕ, j, and p are represented as three-dimensional velocity vector, electric potential, current density and pressure V, σVB, VLB and ρV^2, respectively. The fluid's material properties σ and ρ are density and electrical conductivity of the fluid and B is the magnetic field perpendicular to the channel.

Analysis of linear instability

The perturbation term for three dimensional Hartmann flow are represented as $u = u'$, $w = w'$, $v = \bar{v} + v'$, $\phi = \phi'$ and $p = p'$. By applying the new quantities of (u, v, p, ϕ) into equations (1) to (4) as:

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \tag{6}$$

$$\frac{\partial u'}{\partial t} + \bar{v}\frac{\partial u'}{\partial y} = -\frac{\partial p'}{\partial x} + \frac{1}{Re}\nabla^2 u' \tag{7}$$

$$\frac{\partial v'}{\partial t} + u'\frac{\partial \bar{v}}{\partial x} + \bar{v}\frac{\partial v'}{\partial y} = -\frac{\partial p'}{\partial y} + \frac{1}{Re}\nabla^2 v' - \frac{Ha^2}{Re}v' \tag{8}$$

$$\frac{\partial w'}{\partial t} + \bar{v}\frac{\partial w'}{\partial y} = -\frac{\partial p'}{\partial z} + \frac{1}{Re}\nabla^2 w' - \frac{Ha^2}{Re}w' \tag{9}$$

$$\frac{\partial^2 \phi'}{\partial x^2} + \frac{\partial^2 \phi'}{\partial y^2} + \frac{\partial^2 \phi'}{\partial z^2} + \frac{\partial w'}{\partial y} - \frac{\partial v'}{\partial z} = 0 \tag{10}$$

Calculating the basic velocity as:

$$\bar{v} = \frac{\sinh(Ha \cdot y)}{\cosh Ha} \tag{11}$$

Expression used in (6–10) can be further transformed as normal modes

$$(u', v', p', \phi') = [\hat{u}(x), \hat{v}(x), \hat{p}(x), \hat{\phi}(x)]e^{i(\alpha y + \beta z - \omega t)} \tag{12}$$

Here complex phase velocity $\omega = \omega_r + i\omega_i$, here ω_r is represented by oscillation frequency and ω_i is denoted by amplification frequency, α and β are y and z directional which are represented as wave numbers. (ω_i) and (ω_r) imaginary and real from represented as amplification and oscillation frequency. Applying the equation (12) in equation (6–10). The further simplification can be written as

$$D\hat{u} + i\alpha\hat{v} + i\beta\hat{w} = 0 \tag{13a}$$

$$-i\omega\hat{u} + i\alpha\bar{v}\hat{u} = -D\hat{p} + \frac{1}{Re}(D^2 - \alpha^2 - \beta^2)\hat{u} \tag{13b}$$

$$-i\omega\hat{v} + i\alpha\bar{v}\hat{v} + D\bar{v}\hat{u} = -i\alpha\hat{p} + \frac{1}{Re}(D^2 - \alpha^2 - \beta^2)\hat{v} + N(-i\beta\phi - \hat{v}) \tag{13c}$$

$$-i\omega\hat{w} + i\alpha\bar{v}\hat{w} = -i\beta\hat{p} + \frac{1}{Re}(D^2 - \alpha^2 - \beta^2)\hat{w} + N(-i\alpha\hat{\phi} - \hat{w}) \tag{13d}$$

$$-(D^2 - \alpha^2 - \beta^2)\hat{\phi} + i\alpha\hat{w} - i\beta\hat{v} = 0 \tag{13e}$$

where D is represented as d/dx.

To write the systems (13-13e) in simplified form as

$$(\alpha\bar{v} - \omega)(D^2\hat{u} - k^2\hat{u}) - \alpha D^2\bar{v}\hat{u} - ND^2\hat{u} = -\frac{i}{Re}(D^4\hat{u} - 2k^2D^2\hat{u} + k^4\hat{u}) \tag{14}$$

The velocity in above equation in x axis and $k = (\alpha^2 + \beta^2)^{1/2}$ is represented as wave number. $\theta = cos^{-1}(\alpha/k)$ represented as angle of flow direction of the wave spead, which is also known as Oblique angle.

By applying Squire's Theorem, we can rewrite Eq. (14) as

$$(\bar{v} - \omega)(D^2\hat{u} - k^2\hat{u}) - D^2\bar{v}\hat{u} - \frac{kN}{\alpha^2}D^2\hat{u} = -\frac{i}{kRe}(D^4\hat{u} - 2k^2D^2\hat{u} + k^4\hat{u}) \tag{15}$$

The equation (15) is simplified form of (14), Similaly boundary equation in (5) is simplified as

$$\hat{u} = 0 \qquad D\hat{u} = 0 at\ x = \pm L. \tag{16}$$

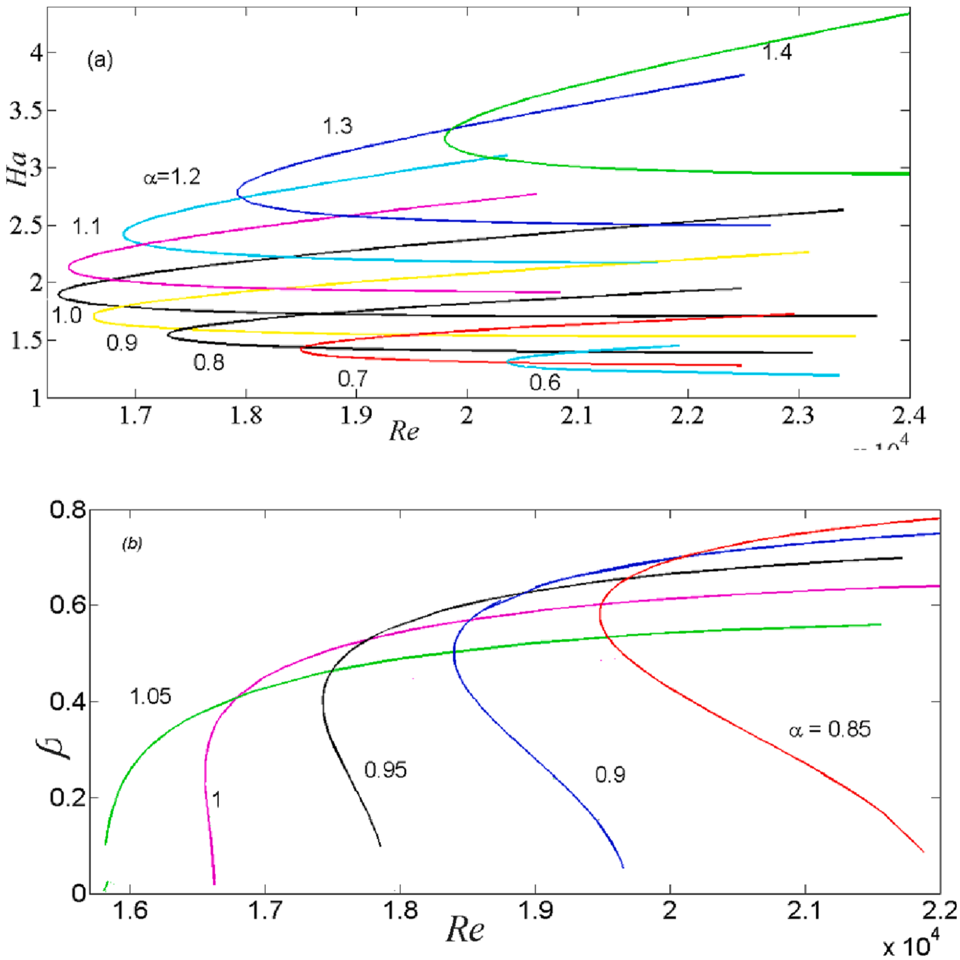

Fig. 3. Neutral curves of stability for various (a). and (b). and.

Numerical scheme

The numerical outcomes are presented via Chebyshev pseudospectral method. This scheme is based on performing the simulations for function presented via Eqs. (15) and (16) in $N_1 + 1$ terms. The velocity $\widehat{u}$ and its derivatives used in (15–16) can be written in Chebyshev series expressed as follows:

$$\begin{aligned} u_m &= \sum_{n=0}^{N_1} a_n T_{nm} \\ u_m' &= \sum_{n=0}^{N_1} a_n T_{nm}' \qquad (m = 1, \cdots, N_1) \\ u_m'' &= \sum_{n=0}^{N_1} a_n T_{nm}'' \\ u_m^{(4)} &= \sum_{n=0}^{N_1} a_n T_{nm}^{(4)} \end{aligned} \tag{17}$$

Here, $T_n(x)$ is denoted as Chebyshev nth expansion of first kind, a_n is also the polynomial denoted as coefficient term, and $T_{nm}', \ldots, T_{nm}^{(4)}$ is expressed with Chebyshev polynomial with same collocation grid points as follow.

$$x_i = \cos\left(m\frac{\pi}{N_1}\right) m = 0, 1, 2, \ldots, N_1 \tag{18}$$

Using $\widehat{u}$ and its derivatives from equation (17), it can be represented as

$$\sum_{n=0}^{N1}\left\{T^{(4)} + \left[-k\frac{\overline{Ha}}{\alpha} - 2k - i\overline{v}k\overline{Re}\right]T'' + \left[k^2 + ik\overline{Re}\overline{v} + ik\overline{Re}D^2\overline{v}\right]T\right\}a_n$$

$$= \overline{\omega}\sum_{n=0}^{N_1}\left\{-ik\overline{Re}\left[T'' - k^2 T\right]\right\}a_n \tag{19}$$

The parameter used in our problem are redesigned as $\overline{Ha} = (\alpha/k)Ha, \overline{Re} = (\alpha/k)Re$, and $\overline{\omega} = \omega/\alpha$. The system satisfied by the expansion coefficients $X = (a_0, a_1, \ldots, a_n)$ is expressed as

$$AX = \omega BX \tag{20}$$

The $X^T = (a_0, a_1, a_3, \ldots, a_n)$ is denoted as transpose of row vector matrix of X and A and B are matrices having the order $(N_1 + 1) \times (N_1 + 1)$..

To get the accurate results, the number of collocation points are used to get our results by setting $N_1 = 100$. System (19) in term of series can be simplified by using QZ technique.

Validation of results

The simulations are verified in Table 1 to ensure the accuracy of solution. The comparison of obtained data is verified with analysis of Nabil [15] and Takashima [16]. Clearly, a good agreement is noted with these studies.

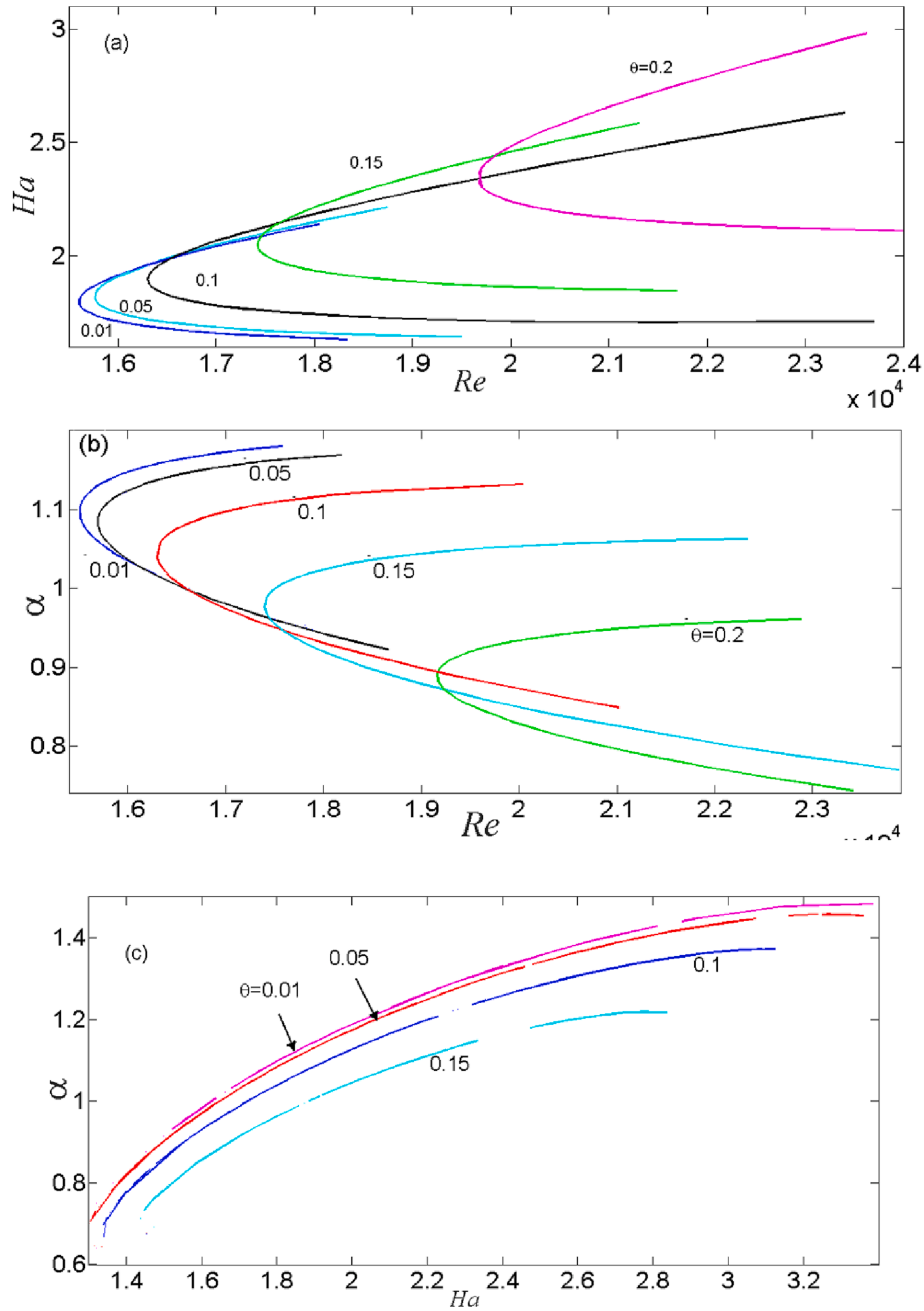

Fig. 4. Neutral curves of stability for various (a) and (b) and (c) and.

Results and discussion

The physical impact of flow parameters is presented in this section. The analysis is performed in view of fixed numerical values parameters like $\alpha = 0.83$, $Ha = 1.4$, $\theta = 0.2\pi$ and $Re = 19170$..

Effects of transverse magnetic field on 3-D instabilities

Fig. 2a presents the effect of Hartmann number *Ha* on 3D flow is investigated when $\theta = 0.2\pi$. It has been claimed that with variation of *Ha* from 1.4 to 2, *Re* is decreasing rapidly and flow system is destabilizing rapidly. However, variation of *Ha* from 2 to 3.4, the flow system destabilize steadily. It is further analyzed that for continues increasing range of *Ha* from 1.4 to 3.4, two kind of stability outcomes are noted. First the flow is destabilizing than gradually starts stabilizing. The lowest values of Re_c appearing at $\alpha = 0.83$, $Ha = 2$ and $Re = 19170$. Upon enhancing *Ha*, the flow is destabilized gradually. For minor increase in starting range in *Ha* , the longitude and altitude area also increasing steadily. It has been known that for fluctuation of *Ha* from 2 to onward, the critical Reynolds number increases gradually. It can be concluded that the normal magnetic field destabilize and stabilize effect for various range of *Ha*. For given flow system, applications of magnetic field destabilizes for $1.4 \leqslant Ha \leqslant 2$. Small Hartmann number with given values of Reynolds number is observed to be 19170. Fig. 2b reports different neutral curves to analyses the stability for space parameter. The observations reveal that an increase in *Ha* with range 2.2 to 3, a

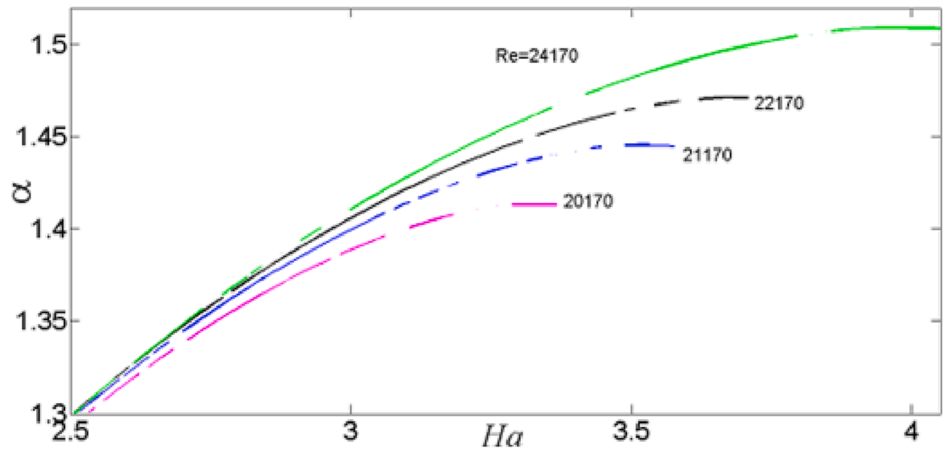

Fig. 5. Neutral curves for various values of.

destabilizing flow pattern is resulted. The upper side of different curve has a special shape and intersecting at one point from up side of the each curve. A gradual destabilizing pattern for enhancing *Ha* is noticed. Contribution of *Ha* for various values of *Re* and β is observed via Fig. 2c. For small change in *Ha*, the area in the unstable region become smaller. The shape of each curve is interesting and intersecting at one point in each curve. For leading values of *Re*, *Ha* also increases.

Effects of wave number on 3-D instabilities

The investigation regarding the instability of Hartman flow for different wave number is studied. Fig. 3 shows different neutral curves for $\theta = 0.2\pi$ with different wave number. Each curve showing the two effects for constant *k*. An unstable area is noted for *k*. Fig. 3b is sketched *Re* and β for distinct values of α. Different neutral curves are drawn to analyses the stability for space parameter. It is observed that an increase in α from 1.05 to 0.85 lead to stabilize the flow. The upper side of different curve has a special shape and intersecting at one point of each curve. Furthermore, for small increase in α, the flow is destabilizing gradually. The area between curves is increasing for enhancing wave number. It is further observed that a minor increasing fluctuation in *Re* is yield out for enriching wave number. The influence of α for Hartmann flow satisfies the stability of system.

Effects of θ on 3-D instabilities

The instability of Hartman flow for θ is important and predicted in various aspect. Fig. 4a implies the different neutral curves for θ along *Re* and *Ha* axes. Each curve showing two effects of *k*. The inside area become unstable whereas the outside regime of each curve become more stable. For small change in θ, a decreasing unstable region is noticed. The curves are diagonally shifting its position to the right direction. Fig. 4b is prepared to analyze the stability for space parameter for various values of θ. An increase in θ, the flow is stabilizing rapidly. The lower side of curves has been intersecting at one point of each curve. Moreover, for small increase in θ stabilize the flow rapidly. The area between the curves is increasing rapidly as the θ is increasing gradually. The influence of α for the Hartmann flow satisfies the stability of given system. The curves are diagonally shifting its position to the right direction. Fig. 4c has been drawn different neutral curves are drawn to analyses the stability for space parameter for various values of θ for *Ha* with. α axis. We observed from this graph that the increase in the value of θ for the range 0.01–0.15 the flow is stabilizing rapidly as increase in θ. Two different observations are claimed for $k = 0.792$. The distance between curves is small for small *Ha* with small increment. Upon small increase in θ, the flow starts stabilizing rapidly. The influence of θ for Hartmann flow satisfies the stability when *Ha* is larger.

Effects of Re on neutral curves

Fig. 5 presents the different neutral curves between *Ha* and α for various values of *Re*. When *Re* is varies from 20,170 to 24,170, an unstable region steadily increases.

Conclusions

This study presents an analysis of the linear stability of 3D Hartmann flow in magnetohydrodynamics under the influence of normal magnetic force. The simulations were performed using the Chebyshev collocation technique. The study compared and discussed the instabilities of the fluid under different embedded physical parameters and growth rates. The following key observations were made:

- The transverse magnetic field has a stabilizing effect on the Hartmann flow.
- The normal magnetic field plays a crucial role in reducing the unstable region.
- The presence of a magnetized field enhances energy dissipation efficiency.
- The normal magnetized field induces a decreasing change in the velocity gradient along the span-wise direction and controls the unstable region.
- The magnetic field contributes to a more nominal energy dissipation, as the Lorentz force acts on the kinetic force of the electrical conducting fluid.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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